Mismatched Complementary-on-Receive Filtering of Diverse FM Waveform Subsets

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Motivation

- Complementary coding was first proposed by Golay nearly 60 years ago … yet the practical limitations due to Doppler (well-known) and transmitter distortion (less well appreciated) have largely consigned it to the realm of pure theory.
- However, it was recently **experimentally demonstrated** **[1]** that complementary subsets of random FM waveforms provide greater robustness to these limitations when the subsets are "pre-summed" after standard matched filtering.
- Here we take an alternative approach whereby **arbitrary** random FM waveforms are pulse compressed using subsets of optimal mismatched filters (MMFs) that are jointly designed to provide a complementary property when their responses are pre-summed.
- This approach is denoted as Mismatched Complementary-on-Receive Filtering (MiCRFt)
  - Pronounced the same as Sherlock Holmes' older brother (Mycroft)

[1] C.A. Mohr, P.M. McCormick, S.D. Blunt, "Optimized complementary waveform subsets within an FM noise radar CPI," *IEEE Radar Conf.*, Oklahoma, City, OK, Apr. 2018.

Discretizing an FM waveform

- Let $s(t)$ define an arbitrary FM waveform with pulsewidth T and 3-dB bandwidth B. To achieve a high-fidelity discretized representation with minimal aliasing*, set the sampling period to

$$T_S = \frac{T}{K(BT)} = \frac{T}{N}$$

where K is the "oversampling" (relative to 3-dB bandwidth) and N is the resulting number of discretized samples.

- The result is the vector $\mathbf{s} = [s_1\ s_2\ \cdots\ s_N]^T$, which can be used to form the $((M+1)N-1) \times MN$ Toeplitz matrix

$$\mathbf{A} = \begin{bmatrix} s_1 & 0 & \cdots & 0 \\ \vdots & s_1 & & \vdots \\ s_N & \vdots & \ddots & 0 \\ 0 & s_N & & s_1 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 & s_N \end{bmatrix}$$

where MN is the length of the filter constructed from this formulation.

*** A time-limited pulse has theoretically infinite bandwidth, thus some aliasing is unavoidable**

The Least Squares (LS) Mismatched Filter (MMF)

- Using $\mathbf{A}$ to perform convolution between discretized waveform $\mathbf{s}$ and discrete filter $\mathbf{h}$, set the desired response to be

$$\mathbf{A}\mathbf{h} = \mathbf{e}_m$$

where $\mathbf{e}_m$ is a length $(M+1)N-1$ elementary vector with a 1 in the mth element and zero elsewhere

- This formulation has the well-known LS MMF solution [2]:

$$\mathbf{h} = (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H\mathbf{e}_m$$

for $(\bullet)^H$ the Hermitian operation

[2] M.H. Ackroyd, F. Ghani, "Optimal mismatched filters for sidelobe suppression," *IEEE Trans. Aerospace & Electronic Systems*, vol. AES-9, no. 2, pp. 214-218, Mar. 1973.

Accounting for FM

- Here the "oversampling" in **s** becomes problematic and can produce a super-resolution effect causing severe mismatch loss
- To compensate, the **A** matrix is beam-spoiled to become $\tilde{\mathbf{A}}$
- This ensures the mismatch filter has the same nominal resolution as the matched filter
  - Beam-spoiling is performed by zeroing out the K–1 rows above & below the mth row, for oversampling factor K **[3]**

$$\left.\mathbf{A} = \begin{bmatrix} s_1 & 0 & \cdots & 0 \\ \vdots & s_1 & \ddots & \vdots \\ s_N & \vdots & \ddots & 0 \\ 0 & s_N & & s_1 \\ \vdots & & \ddots & \vdots \\ 0 & \cdots & 0 & s_N \end{bmatrix}\right\} \tilde{\mathbf{A}}$$

*m*th row

replace K–1 rows with 0's

[3] D. Henke, P. McCormick, S.D. Blunt, T. Higgins, "Practical aspects of optimal mismatch filtering and adaptive pulse compression," *IEEE Intl. Radar Conf.*, Arlington, VA, May 2015.

Accounting for FM

- Using this beam-spoiled matrix as well as a diagonal loading term to improve stability and control mismatch loss, the solution becomes [3]

$$\mathbf{h} = \left(\tilde{\mathbf{A}}^H\tilde{\mathbf{A}} + \delta\mathbf{I}\right)^{-1}\tilde{\mathbf{A}}^H\mathbf{e}_m$$

where

- δ is a diagonal loading factor and $\mathbf{I}$ is an $MN \times MN$ identity matrix,
- $\tilde{\mathbf{A}}$ is the beam-spoiled $\mathbf{A}$ matrix with K-1 rows above and below the mth row replaced with zeros

[3] D. Henke, P. McCormick, S.D. Blunt, T. Higgins, "Practical aspects of optimal mismatch filtering and adaptive pulse compression," *IEEE Intl. Radar Conf.*, Arlington, VA, May 2015.

Mismatch Complementary-on-Receive Filtering

- Given a contiguous subset of Q diverse FM waveforms, MiCRFt is formulated by expanding the LS problem as

$$\sum_{q=1}^{Q} \widetilde{\mathbf{A}}_q \mathbf{h}_q = Q\,\mathbf{e}_m$$

where the scaling by Q accounts for the gain when pre-summing the Q MMF responses.

- Then rearrange into the single matrix-vector equation

$$\widetilde{\mathbf{B}}\bar{\mathbf{h}} = Q\,\mathbf{e}_m$$

in which $\bar{\mathbf{h}} = \left[\mathbf{h}_1^T\ \mathbf{h}_2^T\ \cdots\ \mathbf{h}_Q^T\right]^T$ is an $MNQ \times 1$ composite of MMFs and the combined matrix $\widetilde{\mathbf{B}} = [\widetilde{\mathbf{A}}_1\ \widetilde{\mathbf{A}}_2\ \cdots\ \widetilde{\mathbf{A}}_Q]$ has dimensionality $((M+1)N-1) \times MNQ$

Accounting for Range Straddling

- Unavoidable aliasing – reduced but not eliminated by "oversampling" – leads to **range straddling** **[3-5]** => <u>loss in SNR and hinders sidelobe suppression</u>
- MiCRFt can further compensate for range straddling by introducing L equally-spaced delay offsets $\ell T_S/L$, for $\ell = 0, 1, \cdots, L-1$, when discretizing $s(t)$.
- The subsequent beam-spoiled, Toeplitz matrices constructed from these delay-offset versions can then be used to formulate the MiCRFt LS problem as

$$\sum_{q=1}^{Q} \tilde{\mathbf{A}}_{q,\ell}\,\mathbf{h}_q = Q\,\mathbf{e}_m$$

for the ℓth delay offset. Note that $\mathbf{h}_q$ does <u>not</u> vary with ℓ (i.e. desire an invariant filter response).

[4] A.M. Klein, M.T. Fujita, "Detection performance of hard-limited phase-coded signals," *IEEE Aerospace & Electronic Systems*, vol. AES-15, no. 6, pp. 795-802, Nov. 1979.

[5] M.A. Richards, J.A. Scheer, W.A. Holm, *Principles of Modern Radar: Basic Principles*, SciTech Publishing, pp. 786-787, 2010.

Range-Straddle Compensated MiCRFt

- Then collect the L delay-offset matrices for each of the Q waveforms into the $((M+1)N-1)L \times MNQ$ matrix

$$\tilde{\mathbf{C}} = \begin{bmatrix} \tilde{\mathbf{A}}_{1,0} & \tilde{\mathbf{A}}_{2,0} & \cdots & \tilde{\mathbf{A}}_{Q,0} \\ \tilde{\mathbf{A}}_{1,1} & \tilde{\mathbf{A}}_{2,1} & \cdots & \tilde{\mathbf{A}}_{Q,1} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\mathbf{A}}_{1,L-1} & \tilde{\mathbf{A}}_{2,L-1} & \cdots & \tilde{\mathbf{A}}_{Q,L-1} \end{bmatrix}$$

so that the complete LS problem becomes

$$\tilde{\mathbf{C}}\bar{\mathbf{h}} = Q\,\bar{\mathbf{e}}_m$$

where $\bar{\mathbf{e}}_m = [\mathbf{e}_m^T\ \mathbf{e}_m^T\ \cdots\ \mathbf{e}_m^T]^T$ is a length $((M+1)N-1)L$ vector that is a concatenation of L replicas of the elementary vector.

- The solution to which is

$$\bar{\mathbf{h}} = Q\left(\tilde{\mathbf{C}}^H\tilde{\mathbf{C}} + \delta\bar{\mathbf{I}}\right)^{-1}\tilde{\mathbf{C}}^H\bar{\mathbf{e}}_m$$

for $\bar{\mathbf{I}}$ an identity $MNQ \times MNQ$ matrix.

MiCRFt Solution

- The set of Q MMF filters of length-MN realized by

$$\bar{\mathbf{h}} = Q\left(\tilde{\mathbf{C}}^H\tilde{\mathbf{C}} + \delta\bar{\mathbf{I}}\right)^{-1}\tilde{\mathbf{C}}^H\bar{\mathbf{e}}_m$$

provide complementary sidelobe cancellation when applied to the echoes induced by their associated waveform and then pre-summed in slow-time.

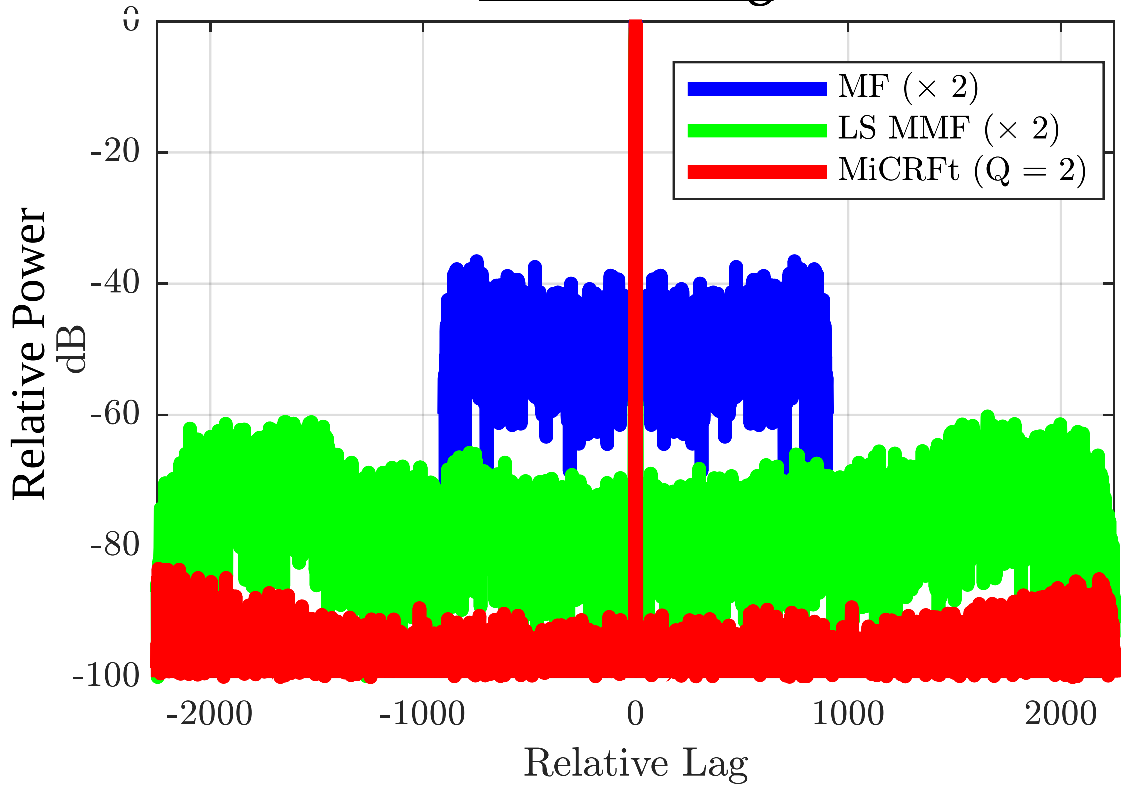
Comments:

- The high dimensionality above presently precludes real-time operation
- Pre-summing incurs a trade-off that reduces the Doppler space

Simulation - No Straddling

- Two random FM waveforms based on [6] with $BT = 300$ and $K = 3$ were generated
- To ensure a fair and consistent comparison with previous LS MMF, all MMFs use the same number of beam-spoiled rows (K above and below), as well as $M = 4$ and $\delta = 1$
- For MF, LS MMF, and MiCRFt ($L = 1$), the two filters responses were pre-summed to provide a direct comparison

Response for noiseless point scatterer, **no straddling**

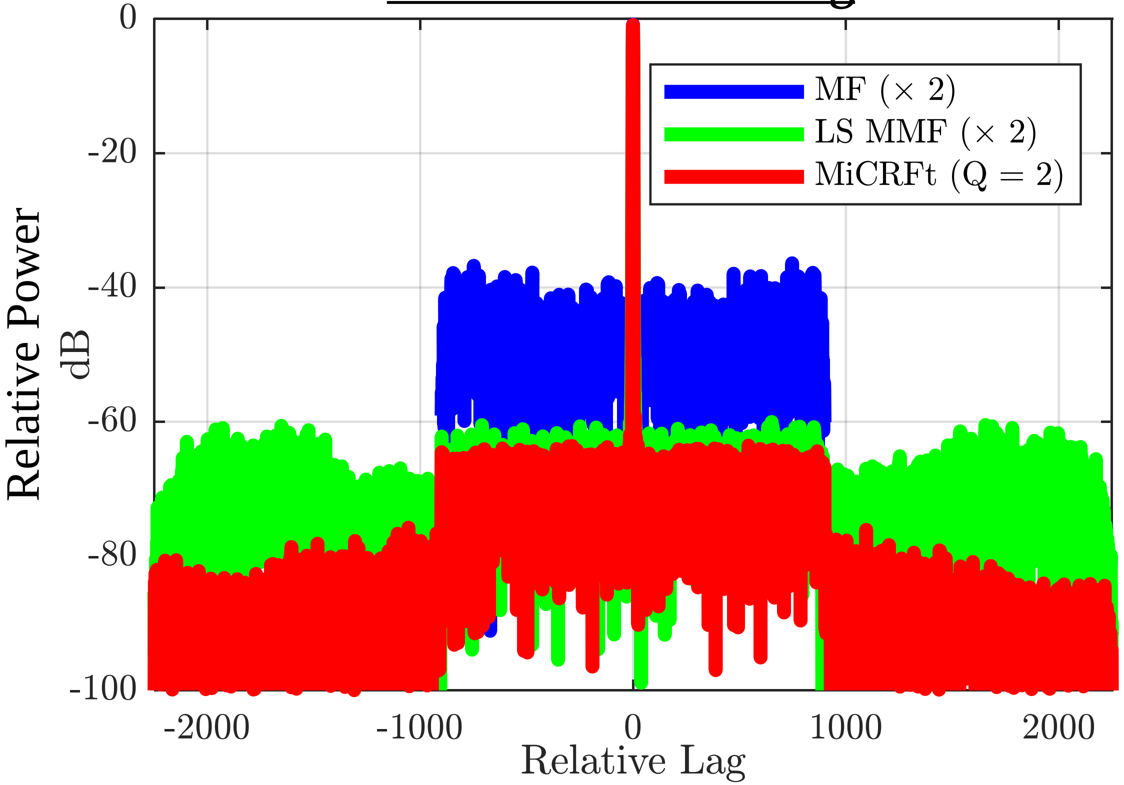


[6] C.A. Mohr, P.M. McCormick, S.D. Blunt, C. Mott, "Spectrally-efficient FM noise radar waveforms optimized in the logarithmic domain," *IEEE Radar Conf.*, Oklahoma City, OK, Apr. 2018.

Simulation – Worst-Case Range Straddling

- The same filters were used on the maximally straddled version of the waveforms ($0.5T_S$) to show worst-case degradation
- Clearly, the MiCRFt peak sidelobe level is much higher (about 16 dB) … though it is still below –60 dB

Response for noiseless point scatterer, worst-case straddling



Simulation – Worst-Case w/ Compensation

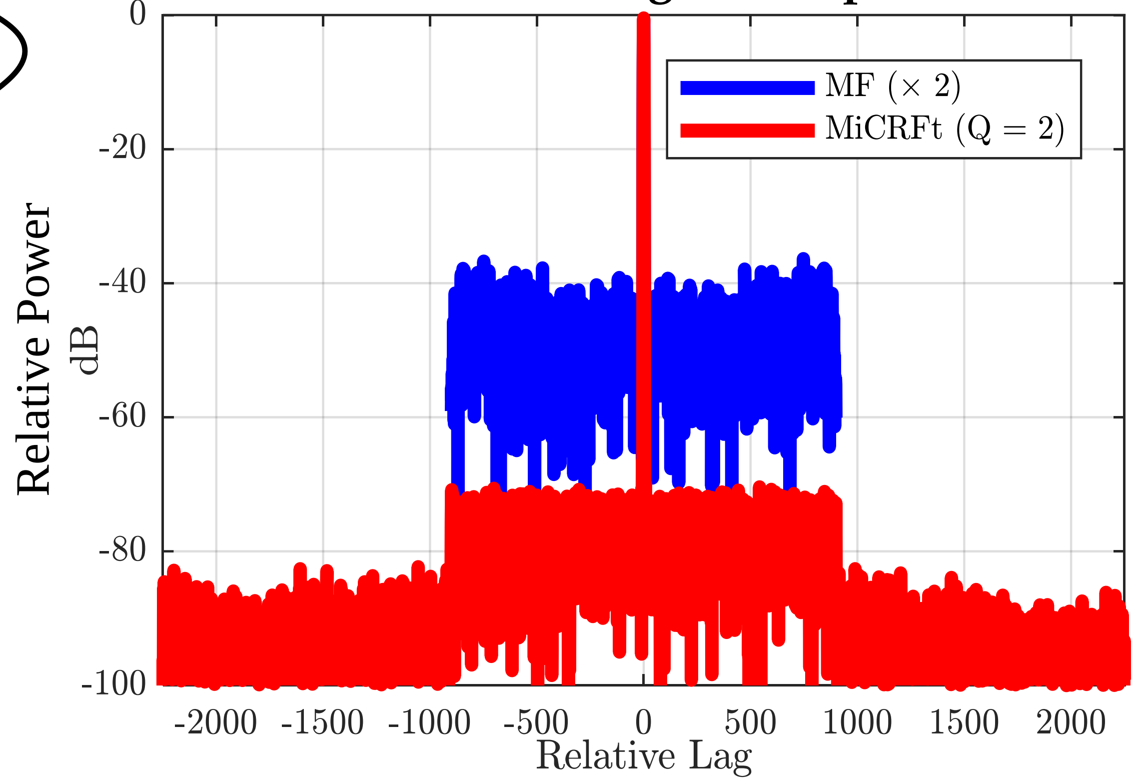
- The worst-case straddled ($0.5T_S$) version of each waveform was added to MiCRFt ($L = 2$) and the new worst-case straddled response (now $0.25T_S$) examined

$$\bar{\mathbf{h}} = Q\left(\tilde{\mathbf{C}}^H\tilde{\mathbf{C}} + \delta\bar{\mathbf{I}}\right)^{-1}\tilde{\mathbf{C}}^H\bar{\mathbf{e}}_m$$

$$\tilde{\mathbf{C}} = \begin{bmatrix} \tilde{\mathbf{A}}_{1,0} & \tilde{\mathbf{A}}_{2,0} & \cdots & \tilde{\mathbf{A}}_{Q,0} \\ \tilde{\mathbf{A}}_{1,1} & \tilde{\mathbf{A}}_{2,1} & \cdots & \tilde{\mathbf{A}}_{Q,1} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\mathbf{A}}_{1,L-1} & \tilde{\mathbf{A}}_{2,L-1} & \cdots & \tilde{\mathbf{A}}_{Q,L-1} \end{bmatrix}$$

- Over 7 dB in peak sidelobe suppression is regained (down to almost –70 dB)

Response for noiseless point scatterer, **worst-case straddling w/ compensation**



Simulation – Range Straddling Comparison

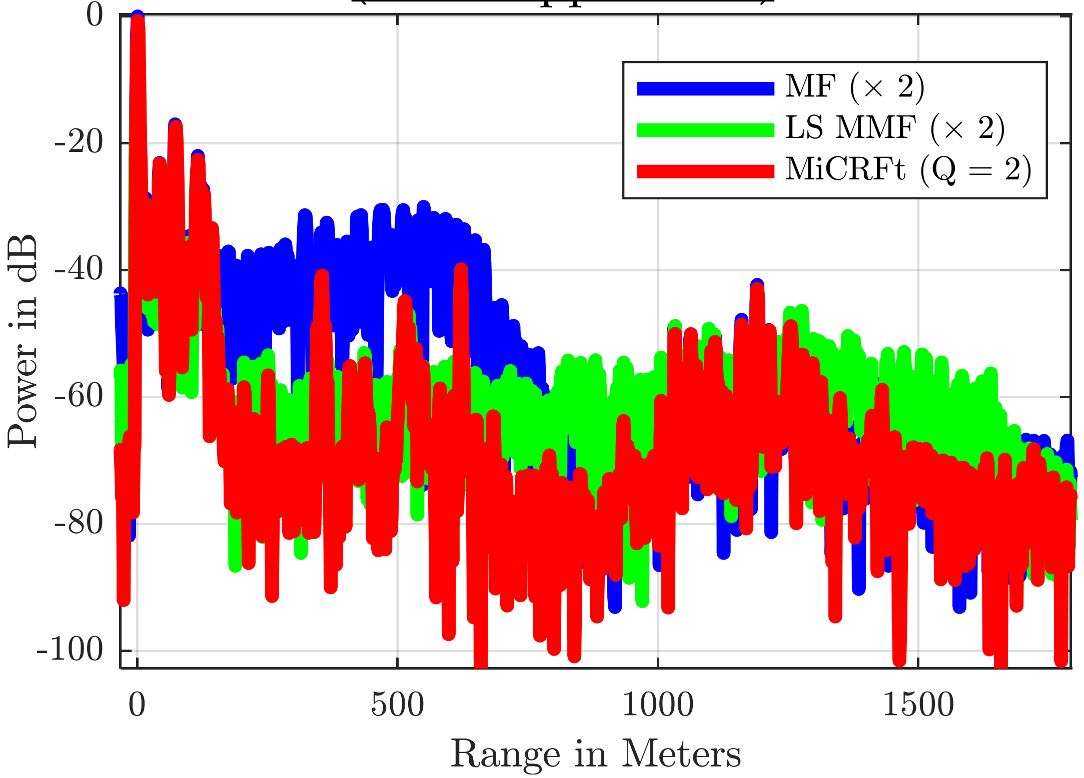
- Typically the performance of a filter degrades as the range straddling increases (increasing mismatch)
- However, for MiCRFt $L = 2$ the mismatch loss is rather flat as a function of the amount of range straddling
- In other words, the MiCRFt filters can be made relatively invariant to straddling

Filter Type	**Total Mismatch Loss**		
	offset = $0T_s$	**$0.25T_s$**	**$0.5T_s$**
MF	0 dB	0.2 dB	0.8 dB
LS MMF	1.0 dB	1.1 dB	1.4 dB
MiCRFt (L = 1)	0.2 dB	0.4 dB	0.9 dB
MiCRFt (L = 2)	0.5 dB	0.3 dB	0.5 dB

Open-Air Experimental results

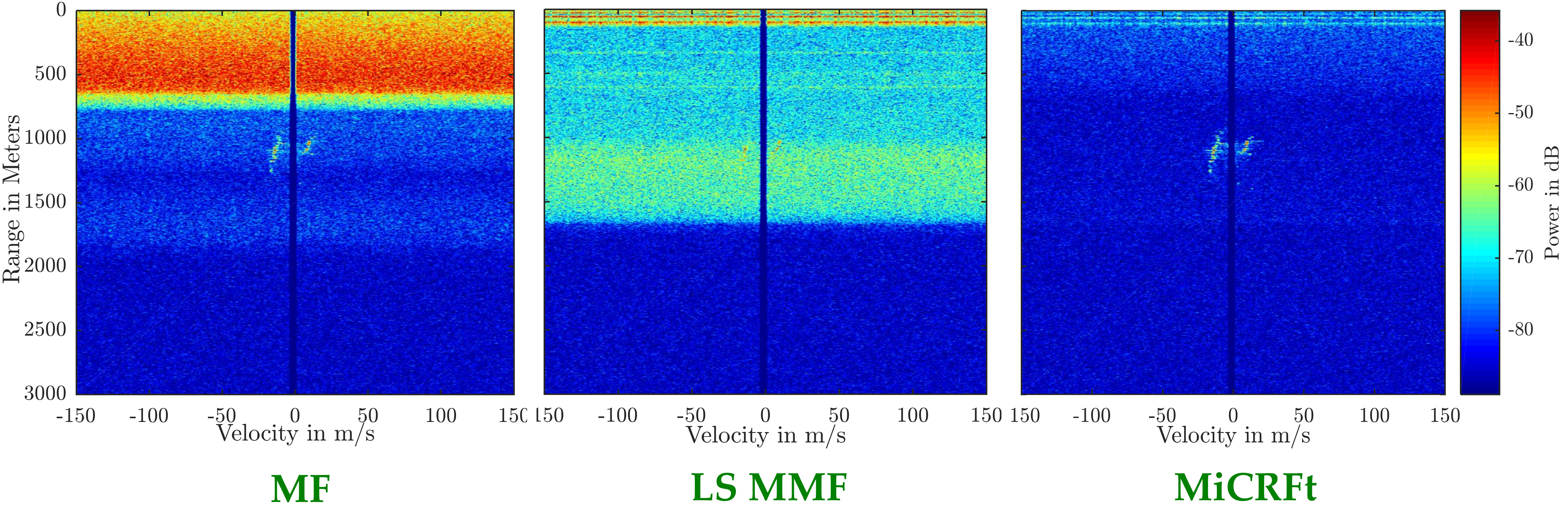
- Two random FM waveforms based on [6] with $BT = 150$ and $K = 3$ were generated
- MMFs generated using the same parameters used previously, and $L = 2$
- After pre-summing, LS MMF response suppresses sidelobes relative to MF … but also extends them
- MiCRFt also has extended sidelobes, but they are much lower due to complementary cancellation

Open-air measurement after pre-summing (zero-Doppler slice)



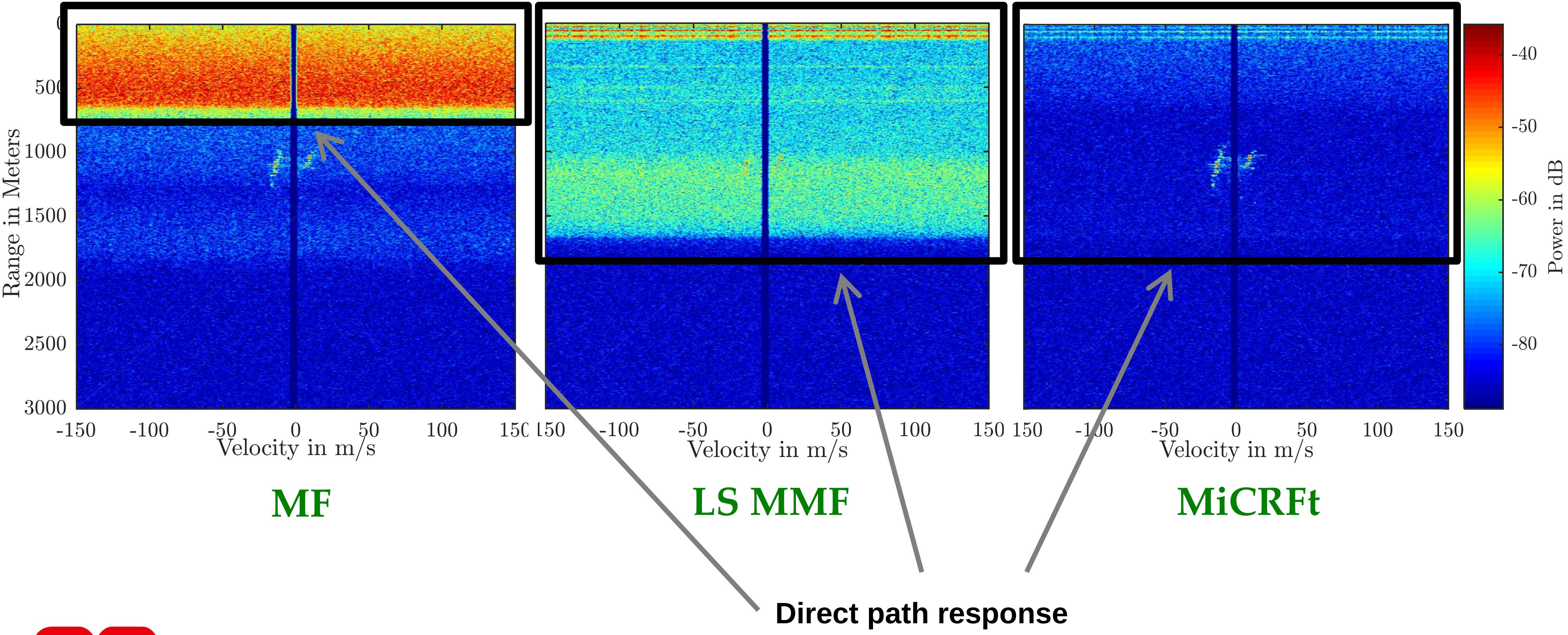
Range sidelobe modulation (RSM) occurs when sidelobes change over the CPI, causing a smearing in Doppler

After projection-based clutter cancelation (stationary platform)



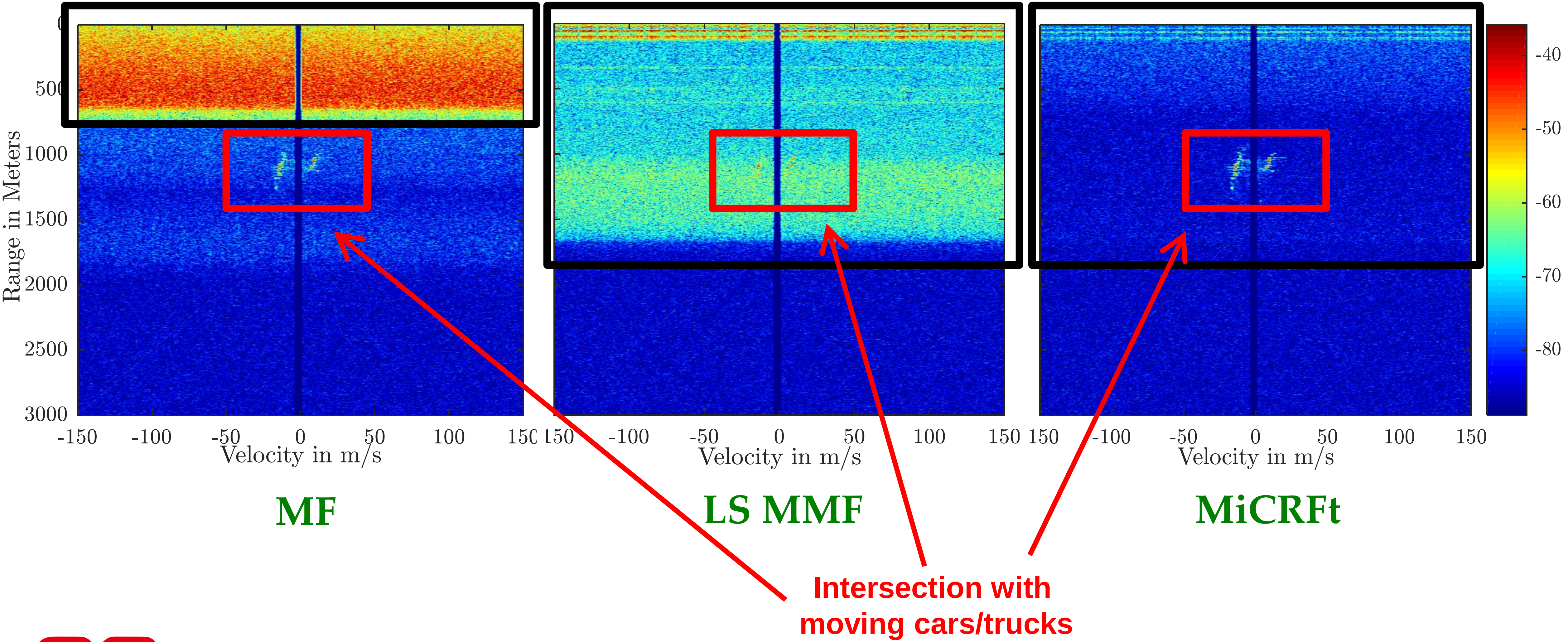
Separate transmit and receive antennas were used, so there is a direct path leakage that dominates

After projection-based clutter cancelation (stationary platform)

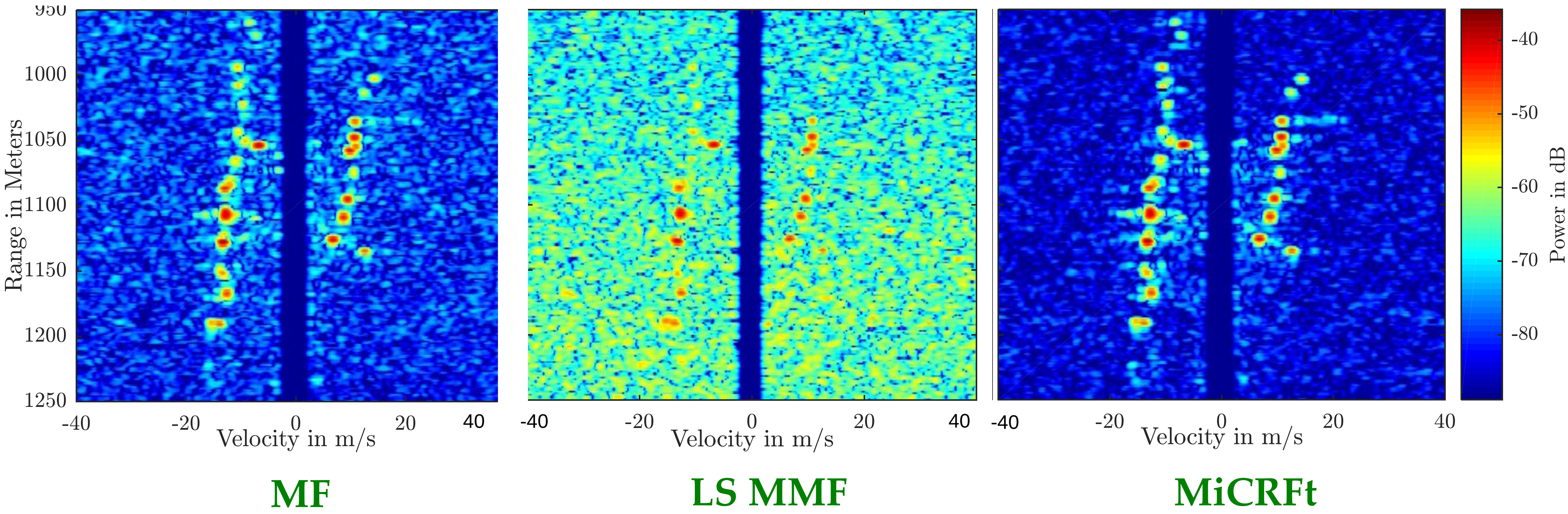


Open-Air Results – Range Doppler Response (1000 waveforms)

After projection-based clutter cancelation (stationary platform)



After projection-based clutter cancelation (stationary platform)



Close-up of the traffic intersection: MiCRFt has significantly enhanced visibility of all moving targets

Conclusions

- Mismatch Complementary-on-Receive filtering (MiCRFt) exploits the increased degrees of freedom provided by arbitrary random FM waveforms

- These added degrees of freedom offers a significant reduction in range sidelobes and range sidelobe modulation (RSM) of clutter relative to the LS MMF and the matched filter

- The trade-offs for this enhanced sensitivity are high computational complexity (reduction currently being investigated) and reduction in the maximum unambiguous Doppler