Mismatched Complementary-on-Receive Filtering of Diverse FM Waveform Subsets

Christian C. Jones and Shannon D. Blunt

Radar Systems & Remote Sensing Lab (RSL), University of Kansas

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Motivation

• Complementary coding was first proposed by Golay nearly 60 years ago … yet the practical limitations due to Doppler (well-known) and transmitter distortion (less well appreciated) have largely consigned it to the realm of pure theory.

• However, it was recently experimentally demonstrated [1] that complementary subsets of random FM waveforms provide greater robustness to these limitations when the subsets are “pre-summed” after standard matched filtering.

• Here we take an alternative approach whereby arbitrary random FM waveforms are pulse compressed using subsets of optimal mismatched filters (MMFs) that are jointly designed to provide a complementary property when their responses are pre-summed.

• This approach is denoted as Mismatched Complementary-on-Receive Filtering (MiCRFt)
  – Pronounced the same as Sherlock Holmes’ older brother (Mycroft)

Discretizing an FM waveform

• Let \( s(t) \) define an arbitrary FM waveform with pulsewidth \( T \) and 3-dB bandwidth \( B \). To achieve a high-fidelity discretized representation with minimal aliasing\(^*\), set the sampling period to

\[
T_s = \frac{T}{K(BT)} = \frac{T}{N}
\]

where \( K \) is the “oversampling” (relative to 3-dB bandwidth) and \( N \) is the resulting number of discretized samples.

• The result is the vector \( \mathbf{s} = [s_1, s_2, \ldots, s_N]^T \), which can be used to form the

\[
( (M+1)N-1 ) \times MN \text{ Toeplitz matrix}
\]

\[
\mathbf{A} = \begin{bmatrix}
  s_1 & 0 & \cdots & 0 \\
  \vdots & s_1 & \ddots & \vdots \\
  s_N & \vdots & \ddots & 0 \\
  0 & s_N & \cdots & s_1 \\
  \vdots & \vdots & \ddots & \vdots \\
  0 & \cdots & 0 & s_N
\end{bmatrix}
\]

where \( MN \) is the length of the filter constructed from this formulation.

\(^*\) A time-limited pulse has theoretically infinite bandwidth, thus some aliasing is unavoidable.
• Using $A$ to perform convolution between discretized waveform $s$ and
discrete filter $h$, set the desired response to be

$$Ah = e_m$$

where $e_m$ is a length $(M +1)N −1$ elementary vector with a 1 in
the $m$th element and zero elsewhere

• This formulation has the well-known LS MMF solution [2]:

$$h = (A^H A)^{-1} A^H e_m$$

for $(\cdot)^H$ the Hermitian operation

Accounting for FM

• Here the “oversampling” in $s$ becomes problematic and can produce a super-resolution effect causing severe mismatch loss.

• To compensate, the $A$ matrix is beam-spoiled to become $\tilde{A}$.

• This ensures the mismatch filter has the same nominal resolution as the matched filter.

– Beam-spoiling is performed by zeroing out the $K-1$ rows above & below the $m$th row, for oversampling factor $K$ [3].

\[
A = \begin{bmatrix}
s_1 & 0 & \cdots & 0 \\
\vdots & s_1 & \ddots & \vdots \\
s_N & \vdots & \ddots & 0 \\
0 & s_N & \cdots & s_1 \\
\vdots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & s_N 
\end{bmatrix}
\]

Accounting for FM

• Using this beam-spoiled matrix as well as a diagonal loading term to improve stability and control mismatch loss, the solution becomes [3]

\[ h = (\tilde{A}^H \tilde{A} + \delta I)^{-1} \tilde{A}^H e_m \]

where

– \( \delta \) is a diagonal loading factor and \( I \) is an \( MN \times MN \) identity matrix,
– \( \tilde{A} \) is the beam-spoiled \( A \) matrix with \( K-1 \) rows above and below the \( m \)th row replaced with zeros

Mismatch Complementary-on-Receive Filtering

• Given a contiguous subset of $Q$ diverse FM waveforms, MiCRFt is formulated by expanding the LS problem as

$$\sum_{q=1}^{Q} \tilde{A}_q h_q = Q e_m$$

where the scaling by $Q$ accounts for the gain when pre-summing the $Q$ MMF responses.

• Then rearrange into the single matrix-vector equation

$$\tilde{B}\tilde{h} = Q e_m$$

in which $\tilde{h} = [h_1^T \ h_2^T \ \cdots \ h_Q^T]^T$ is an $MNQ \times 1$ composite of MMFs and the combined matrix $\tilde{B} = [\tilde{A}_1 \ \tilde{A}_2 \ \cdots \ \tilde{A}_Q]$ has dimensionality $((M+1)N - 1) \times MNQ$
Accounting for Range Straddling

- Unavoidable aliasing – reduced but not eliminated by “oversampling” – leads to range straddling [3-5] => loss in SNR and hinders sidelobe suppression

- MiCRFt can further compensate for range straddling by introducing $L$ equally-spaced delay offsets $\ell T_s / L$, for $\ell = 0, 1, \ldots, L - 1$, when discretizing $s(t)$.

- The subsequent beam-spoiled, Toeplitz matrices constructed from these delay-offset versions can then be used to formulate the MiCRFt LS problem as

$$\sum_{q=1}^{Q} \tilde{A}_{q,\ell} h_q = Q e_m$$

for the $\ell$th delay offset. Note that $h_q$ does not vary with $\ell$ (i.e. desire an invariant filter response).

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• Then collect the $L$ delay-offset matrices for each of the $Q$ waveforms into the $((M +1)N −1)L \times MNQ$ matrix

\[
\mathbf{C} = \begin{bmatrix}
\mathbf{A}_{1,0} & \mathbf{A}_{2,0} & \cdots & \mathbf{A}_{Q,0} \\
\mathbf{A}_{1,1} & \mathbf{A}_{2,1} & \cdots & \mathbf{A}_{Q,1} \\
\vdots & \vdots & \ddots & \vdots \\
\mathbf{A}_{1,L−1} & \mathbf{A}_{2,L−1} & \cdots & \mathbf{A}_{Q,L−1}
\end{bmatrix}
\]

so that the complete LS problem becomes

\[
\mathbf{C}\mathbf{h} = Q \mathbf{e}_m
\]

where $\mathbf{e}_m = [\mathbf{e}_m^T \mathbf{e}_m^T \cdots \mathbf{e}_m^T]^T$ is a length $((M +1)N −1)L$ vector that is a concatenation of $L$ replicas of the elementary vector.

• The solution to which is

\[
\mathbf{h} = Q (\mathbf{C}^H \mathbf{C} + \delta \mathbf{I})^{-1} \mathbf{C}^H \mathbf{e}_m
\]

for $\mathbf{I}$ an identity $MNQ \times MNQ$ matrix.
• The set of $Q$ MMF filters of length-$MN$ realized by

$$\mathbf{\bar{h}} = Q(\mathbf{\bar{C}}^H\mathbf{\bar{C}} + \delta \mathbf{I})^{-1}\mathbf{\bar{C}}^H\mathbf{\bar{e}}_m$$

provide complementary sidelobe cancellation when applied to the echoes induced by their associated waveform and then pre-summed in slow-time.

Comments:
• The high dimensionality above presently precludes real-time operation
• Pre-summing incurs a trade-off that reduces the Doppler space
• Two random FM waveforms based on [6] with $BT = 300$ and $K = 3$ were generated.

• To ensure a fair and consistent comparison with previous LS MMF, all MMFs use the same number of beam-spoiled rows ($K$ above and below), as well as $M = 4$ and $\delta = 1$.

• For MF, LS MMF, and MiCRFt ($L = 1$), the two filters responses were pre-summed to provide a direct comparison.

• The same filters were used on the maximally straddled version of the waveforms \((0.5T_S)\) to show worst-case degradation.

• Clearly, the MiCRFt peak sidelobe level is much higher (about 16 dB) ... though it is still below –60 dB.
Simulation – Worst-Case w/ Compensation

- The worst-case straddled \((0.5T_S)\) version of each waveform was added to MiCRFt \((L = 2)\) and the new worst-case straddled response (now \(0.25T_S\)) examined

\[
\mathbf{h} = Q(\tilde{\mathbf{C}}^H \tilde{\mathbf{C}} + \delta \mathbf{I})^{-1} \tilde{\mathbf{C}}^H \bar{\mathbf{e}}_m
\]

\[
\tilde{\mathbf{C}} = \begin{bmatrix}
\tilde{\mathbf{A}}_{1,0} & \tilde{\mathbf{A}}_{2,0} & \cdots & \tilde{\mathbf{A}}_{Q,0} \\
\tilde{\mathbf{A}}_{1,1} & \tilde{\mathbf{A}}_{2,1} & \cdots & \tilde{\mathbf{A}}_{Q,1} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{\mathbf{A}}_{1,L-1} & \tilde{\mathbf{A}}_{2,L-1} & \cdots & \tilde{\mathbf{A}}_{Q,L-1}
\end{bmatrix}
\]

- Over 7 dB in peak sidelobe suppression is regained (down to almost \(-70\) dB)
• Typically the performance of a filter degrades as the range straddling increases (increasing mismatch)

• However, for MiCRFt $L = 2$ the mismatch loss is rather flat as a function of the amount of range straddling

• In other words, the MiCRFt filters can be made relatively invariant to straddling

<table>
<thead>
<tr>
<th>Filter Type</th>
<th>Total Mismatch Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>offset = $0T_s$</td>
</tr>
<tr>
<td>MF</td>
<td>0 dB</td>
</tr>
<tr>
<td>LS MMF</td>
<td>1.0 dB</td>
</tr>
<tr>
<td>MiCRFt $(L = 1)$</td>
<td>0.2 dB</td>
</tr>
<tr>
<td>MiCRFt $(L = 2)$</td>
<td>0.5 dB</td>
</tr>
</tbody>
</table>
Open-Air Experimental results
Open-Air Results: Zero-Doppler Slice

- **Two** random FM waveforms based on [6] with $BT = 150$ and $K = 3$ were generated.
- MMFs generated using the same parameters used previously, and $L = 2$.
- After pre-summing, LS MMF response suppresses sidelobes relative to MF ... but also extends them.
- MiCRFt also has extended sidelobes, but they are much lower due to complementary cancellation.

Range sidelobe modulation (RSM) occurs when sidelobes change over the CPI, causing a smearing in Doppler.
After projection-based clutter cancelation (stationary platform)

Separate transmit and receive antennas were used, so there is a direct path leakage that dominates.
Open-Air Results – Range Doppler Response (1000 waveforms)

After projection-based clutter cancelation (stationary platform)

MF

LS MMF

MiCRFt

Direct path response
Open-Air Results – Range Doppler Response (1000 waveforms)

After projection-based clutter cancelation (stationary platform)

Intersection with moving cars/trucks
Open-Air Results – Range Doppler Response (1000 waveforms)

After projection-based clutter cancelation (stationary platform)

Close-up of the traffic intersection: MiCRFt has significantly enhanced visibility of all moving targets
Conclusions

• Mismatch Complementary-on-Receive filtering (MiCRFt) exploits the increased degrees of freedom provided by arbitrary random FM waveforms.

• These added degrees of freedom offer a significant reduction in range sidelobes and range sidelobe modulation (RSM) of clutter relative to the LS MMF and the matched filter.

• The trade-offs for this enhanced sensitivity are high computational complexity (reduction currently being investigated) and reduction in the maximum unambiguous Doppler.