Experimental Evaluation of Super-Gaussian-Shaped Random FM Waveforms

Matthew B. Heintzelman, Thomas J. Kramer, Shannon D. Blunt
Radar Systems Lab (RSL), University of Kansas, Lawrence KS

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Motivation

- Random FM (RFM) waveforms based on spectrum shaping have some useful attributes:
  - Fourier relationship between spectral density and autocorrelation means "goodness" can be achieved through spectrum shaping
  - Degree of transmitter distortion impacted by out-of-band roll-off
  - Readily incorporate spectral notching to reduce mutual interference between radar and other spectrum users
- However, reducing range sidelobes and improving spectral containment are competing goals.
  - Gaussian-shaped spectral density → Gaussian-shaped autocorrelation, meaning no sidelobes (theoretically)
  - Much tighter roll-off than a sinc function (associated with phase codes), but still broader than LFM

Motivation

- While traditional noise radar can achieve wideband operation, long-range and high-power wideband applications are typically reserved for LFM
  - FM minimizes spectral regrowth due to nonlinear transmitter distortion (constant amplitude, continuous phase, and no instantaneous intermodulation)
  - Compact LFM spectrum allows for nominal receive sampling
- RFM structure can provide the extremely high dimensionality of noise radar while enabling high-power operation
  - But Gaussian roll-off requires "over-sampling" relative to 3-dB bandwidth to ensure waveform fidelity is retained

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Here we experimentally evaluate a recent approach to improve the spectral compactness of RFM waveforms

Gaussian Design Template

- Previous RFM spectral shaping approaches [1-3] have relied on a Gaussian template
  - For pulsed structure, with pulse width T and 3-dB bandwidth B, optimized RFM waveforms can realize approaching 20 $\log_{10}(TB)$ dB on a per-pulse basis
  - A further 10 $\log_{10}(M)$ dB in sidelobe suppression is achieved via slow-time processing due to incoherent sidelobe averaging (mainlobe remains coherent)
- However, while the need for 2× to 4× oversampling, relative to B, is fine for10s of MHz, wideband operation in the 100s of MHz to GHz may not be feasible
- Consequently, it was recently shown that use of a super-Gaussian template provides a useful trade-space to reduce this sampling overhead

[1] J. Jakabosky, S.D. Blunt, B. Himed, "Spectral-shape optimized FM noise radar for pulse agility," *IEEE Radar Conf.*, Philadelphia, PA, May 2016.

[2] C.A. Mohr, S.D. Blunt, "FM noise waveforms optimized according to a temporal template error (TTE) metric," *IEEE Radar Conf.*, Boston, MA, Apr. 2019.

[3] C.A. Mohr, P.M. McCormick, S.D. Blunt, C. Mott, "Spectrally-efficient FM noise radar waveforms optimized in the logarithmic domain," *IEEE Radar Conf.*, Oklahoma City, OK, Apr. 2018.

Super-Gaussian Design Template

- The super-Gaussian template takes the general form

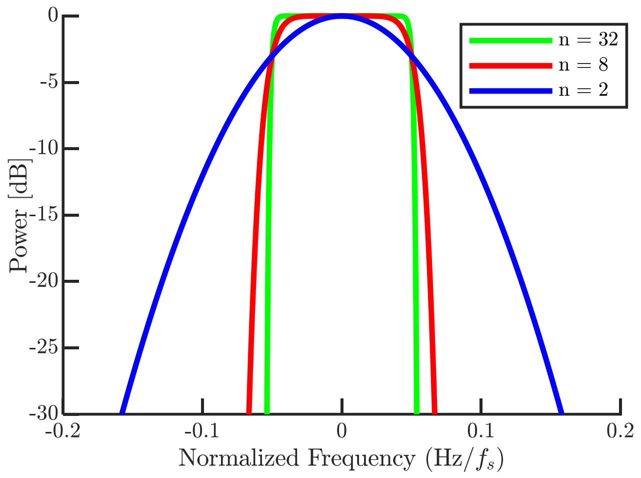
$$f(x) = A \exp\left(-\frac{1}{2}\left|\frac{x-\gamma}{\sigma}\right|^{n}\right)$$

  where exponent n controls the trade-off between lower autocorrelation sidelobes and better spectral containment
  - $n = 2$ corresponds to Gaussian, while $n \to \infty$ corresponds to a rectangular template
- Templates compared here are generated to have identical 3-dB bandwidths using

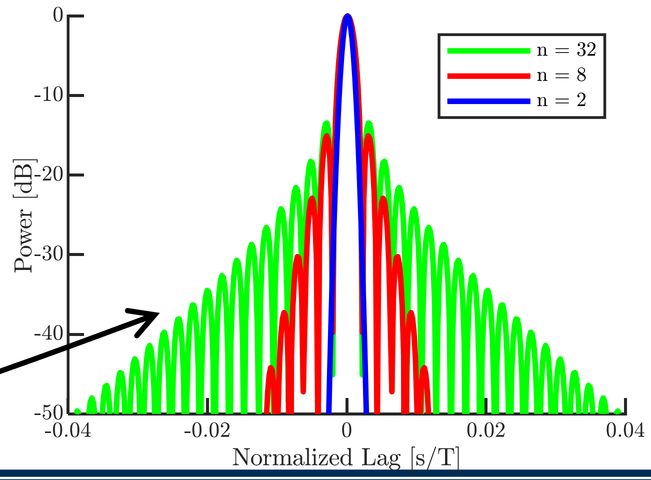
$$\sigma = \frac{|x_0 - \gamma|}{(2\ln(2A))^{1/n}} = \frac{(B/2)}{(2\ln(2))^{1/n}}$$

- Corresponding autocorrelations (via inverse Fourier transform) reveal higher "persistent" sidelobes as the spectrum becomes more compact

Comparison of super-Gaussian spectral templates



Comparison of super-Gaussian autocorrelations (inverse Fourier transform of spectral templates)



To assess in practice, we need to select an RFM waveform design approach …

PRO-FM Optimization

- For the purpose of demonstration, consider the PRO-FM method [1] for spectrally shaping RFM waveforms

- For a set of M waveforms, PRO-FM seeks to match the spectral template $|G(f)|^2$ after initializing the mth waveform with randomly generated FM signal $p_{0,m}(t)$ via K alternating projections of

$$r_{k+1,m}(t) = \boldsymbol{\mathcal{F}}^{-1}\{|G(f)| \exp\left(j\angle\left\{\boldsymbol{\mathcal{F}}\{p_{k,m}(t)\}\right\}\right)\}$$

and

$$p_{k+1,m}(t) = u(t) \exp\left(j\angle\{r_{k+1,m}(t)\}\right)$$

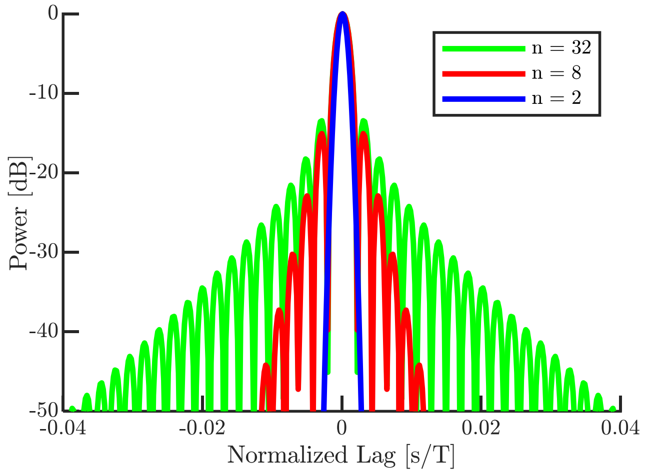
where u(t) is a rectangular pulse with support on [0, T]

[1] J. Jakabosky, S.D. Blunt, B. Himed, "Spectral-shape optimized FM noise radar for pulse agility," *IEEE Radar Conf.*, Philadelphia, PA, May 2016.

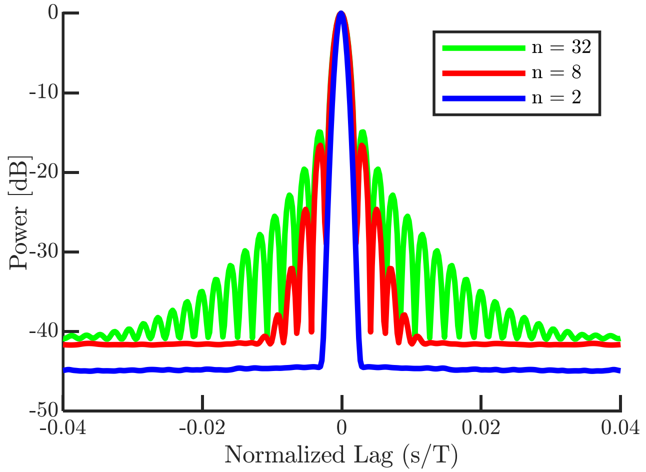
Super-Gaussian PRO-FM

- For n = 2, 8, and 32 super-Gaussian spectral templates, 5000 unique PRO-FM waveforms were generated for each case
  - Based on TB = 472 and 10× oversampling relative to 3-dB bandwidth
- Compare the idealized autocorrelations (inverse Fourier transform of spectral templates) with the simulated RMS combination of 5000 PRO-FM waveforms
- Perfectly matching the spectral template is not possible given the constraint on rectangular pulse shape
- Higher values of n (tighter spectrum) produces:
  - Higher & broader extent of persistent sidelobes (<u>same</u> as ideal)
  - Somewhat raised sidelobe floor (<u>different</u> from ideal)

Comparison of super-Gaussian autocorrelations (inverse Fourier transform of spectral templates)



Comparison of super-Gaussian autocorrelations (RMS combination over 5000 PRO-FM each)



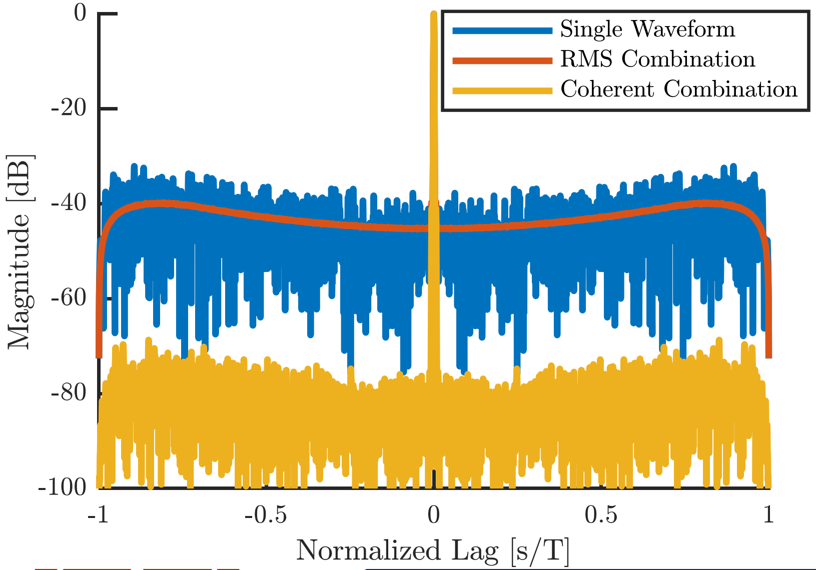
Experimental Setup – Loopback

- Use the same sets of 5000 PRO-FM waveforms for $n = 2$, 8, and 32, implemented on an arbitrary waveform generator (AWG) and captured in loopback via a spectrum analyzer
  - Loopback capture allows for characterization of hardware-induced distortion
- Here, $T = 6.67$ μs and $B = 70.7$ MHz (hence $TB = 472$), PRF = 50 kHz, and the center frequency $f_c = 3.55$ GHz
- Though this arrangement is still narrowband, it permits examination of the sidelobe vs spectral containment trade-space
- We use the loopback-captured versions to evaluate autocorrelations, cross-correlations, and spectral densities
  - Computed on a per-waveform basis, ensemble RMS, and slow-time processing

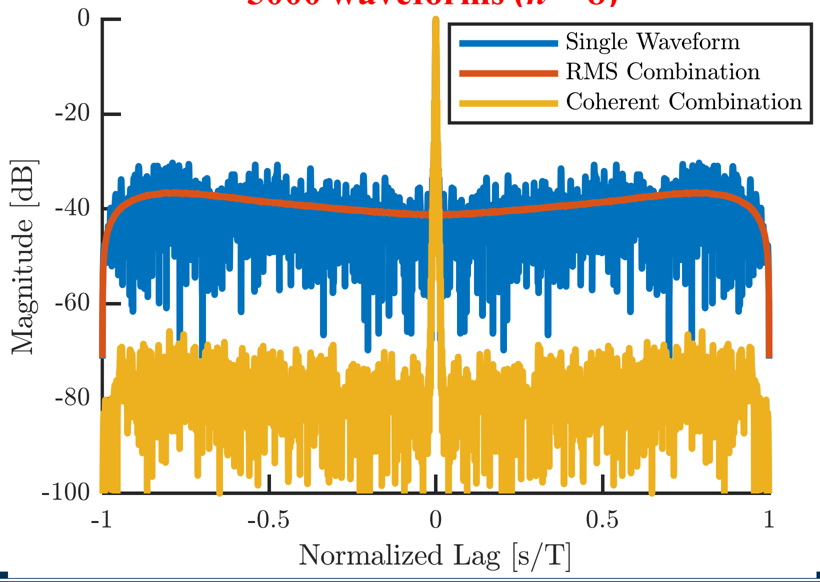
Loopback Autocorrelations

- As with simulation, we see slight elevation in RMS sidelobe floor with increasing *n*
- Likewise for higher / broader sidelobe extent for higher *n*
- Near-in "shoulder" lobes also appear due to hardware effects

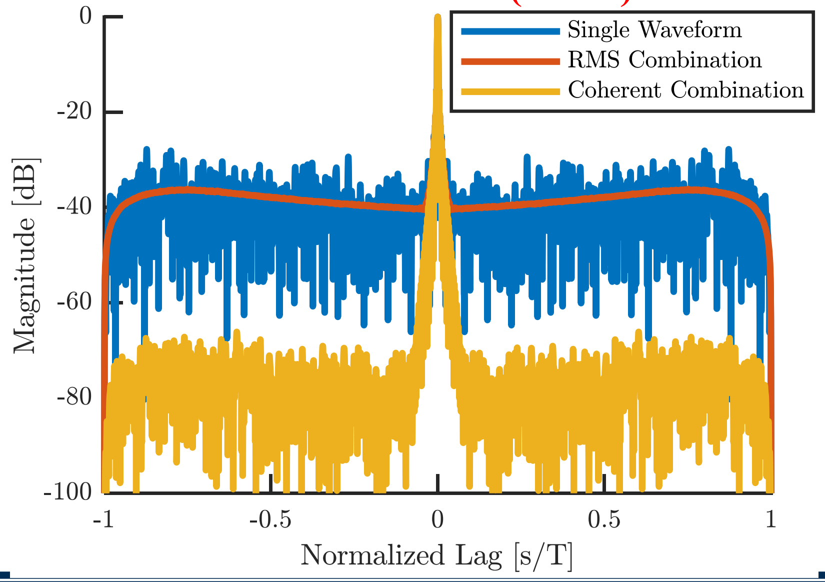
Loopback autocorrelation for 5000 waveforms (*n* = 2)



Loopback autocorrelation for 5000 waveforms (*n* = 8)



Loopback autocorrelation for 5000 waveforms (*n* = 32)

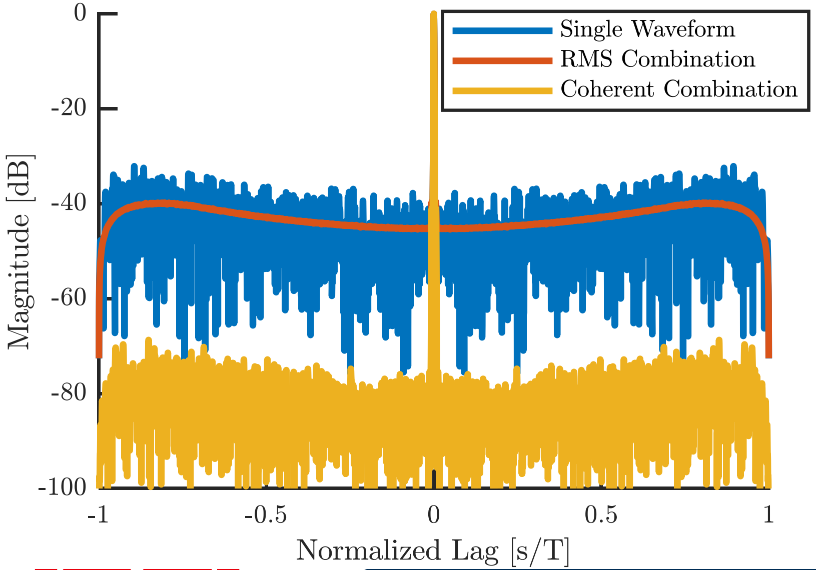


Loopback Autocorrelations

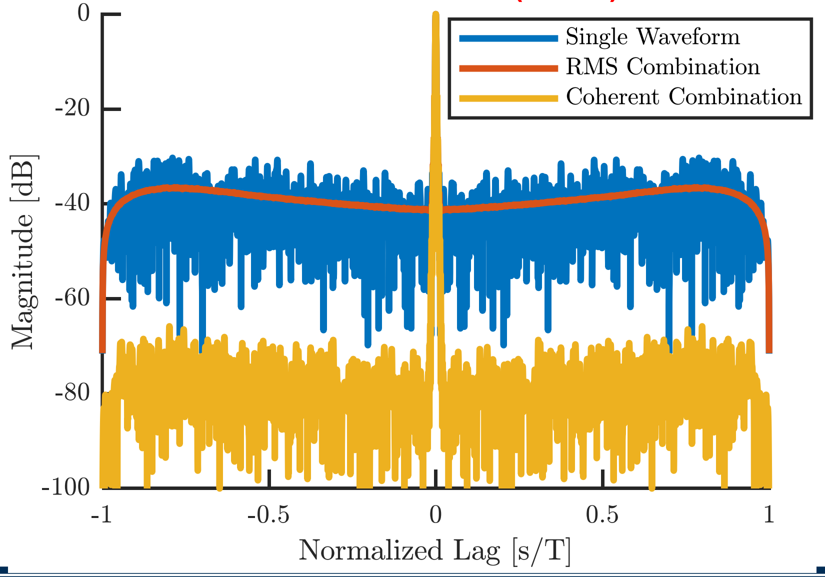
- As with simulation, we see slight elevation in RMS sidelobe floor with increasing *n*
- Likewise for higher / broader sidelobe extent for higher *n*
- Near-in “shoulder” lobes also appear due to hardware effects
- Note the PSL determination with vs. without these effects

	RMS PSL (including persistent/shoulder sidelobes)	RMS PSL (excluding persistent/shoulder sidelobes)
n = 2	–32.6 dB	–39.8 dB
n = 8	–17.1 dB	–36.5 dB
n = 32	–15.4 dB	–36.2 dB
	quite different	fairly consistent

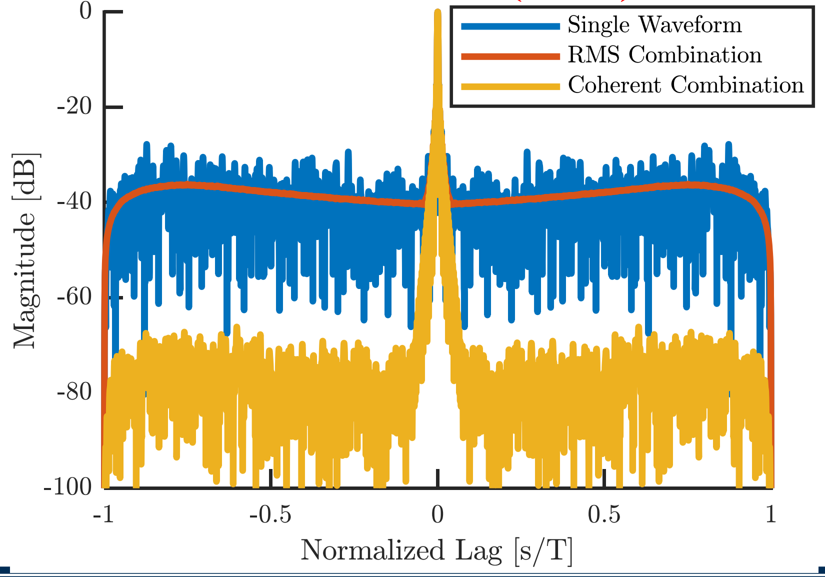
Loopback autocorrelation for 5000 waveforms (*n* = 2)



Loopback autocorrelation for 5000 waveforms (*n* = 8)

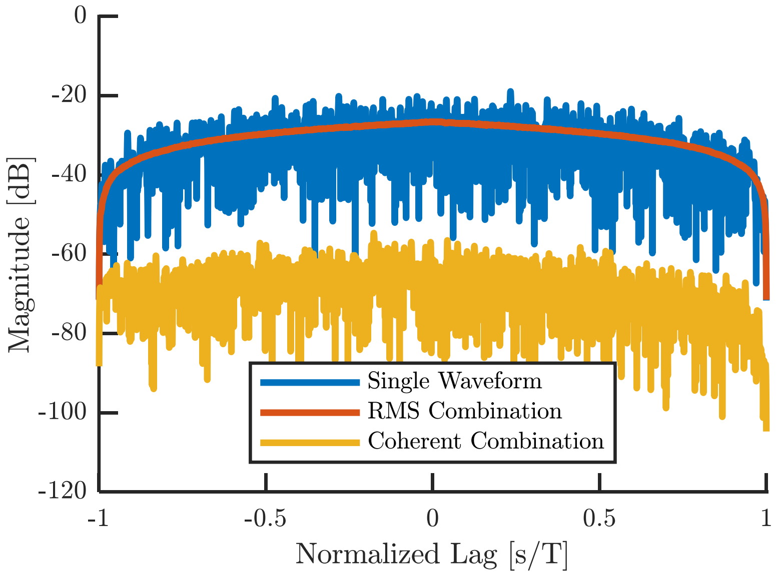


Loopback autocorrelation for 5000 waveforms (*n* = 32)



Loopback Cross-Correlations

- For cross-correlation, the first waveform in each set was cross-correlated with the other 4999, followed by RMS combining or slow-time combining (incoherent in this case), where the latter yields 10 $\log_{10}(4999) = 37$ dB further cross-correlation suppression
- The three sets of results are practically identical, so only the $n = 8$ case is shown



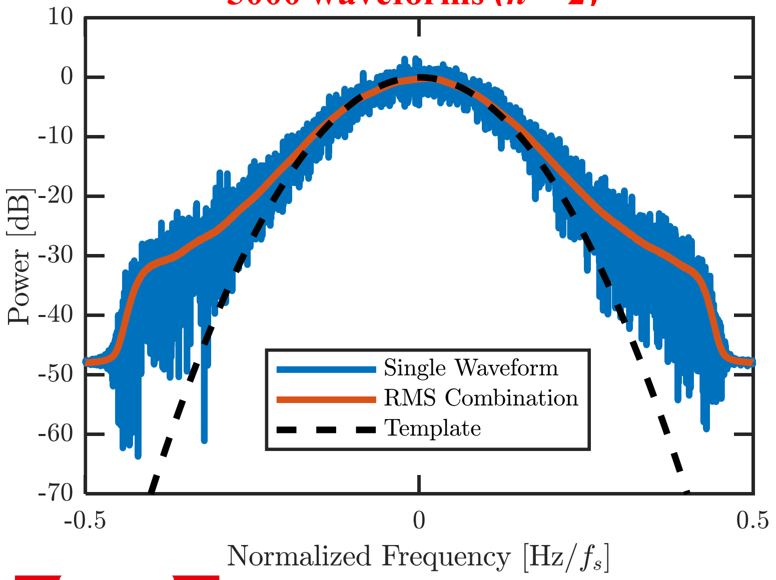
Loopback cross-correlation results for 5000 waveforms (*n* = 8)

- Increasing *n* does produce slight degradation of RMS cross-correlation floor
  - For $n = 2$, the floor is 1.8 dB lower, while for $n = 32$, the floor is 0.5 dB higher

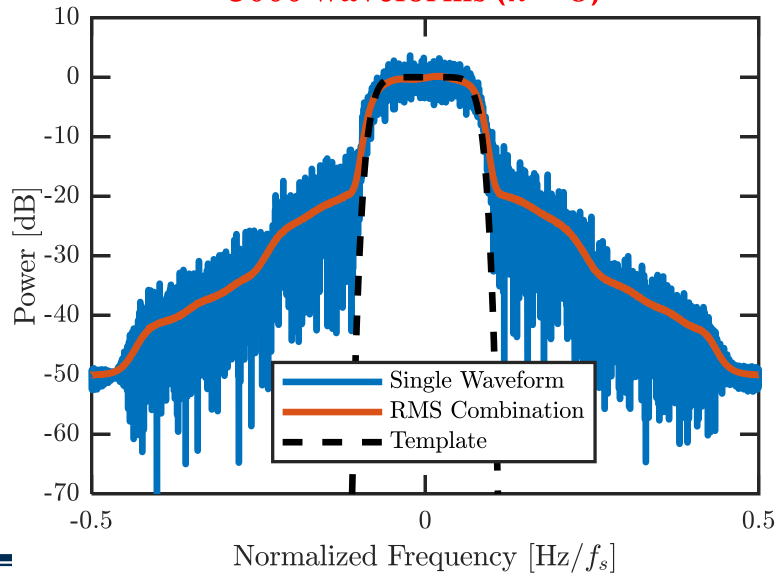
Loopback Spectra

- Templates & resulting waveforms all have approximately the same 3-dB bandwidth
- The percentage of "in-band power" is 75.1%, 97.5%, and 98.5% for *n* = 2, 8 and 32, respectively
  - Thus higher *n* provides greater concentration of spectral content
- For *n* = 2, 8, and 32, the RMS spectra deviate from the template by 5% of 3-dB bandwidth at –8.3 dB, –14.3 dB, and –18.0 dB, respectively
  - Increasing *n* appears to enable better template matching

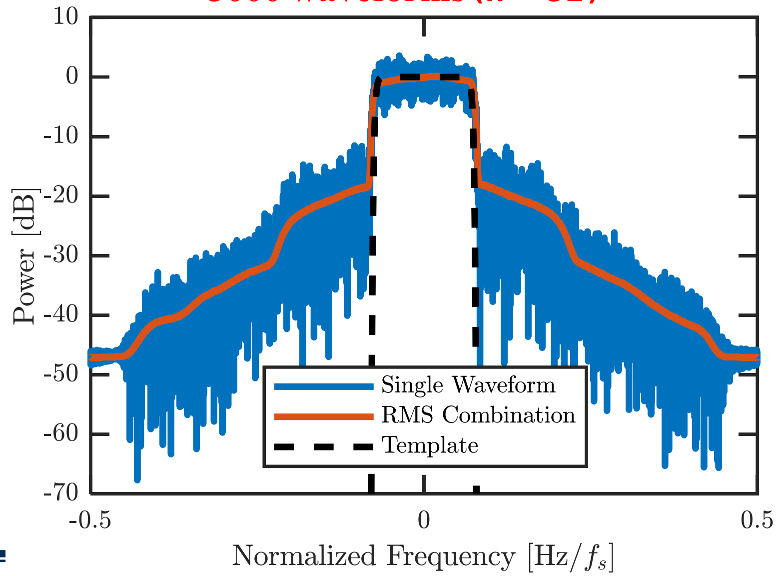
Loopback spectral content for 5000 waveforms (*n* = 2)



Loopback spectral content for 5000 waveforms (*n* = 8)

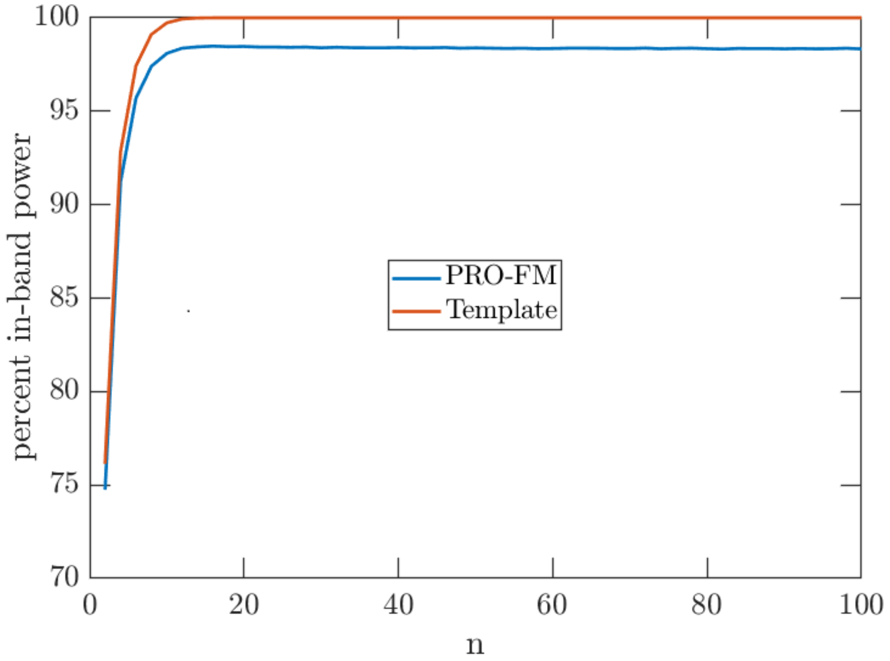


Loopback spectral content for 5000 waveforms (*n* = 32)



In-Band Power (Simulation)

- Based on experimental observations for spectral power concentration, the percent of in-band (3 dB) power was simulated as a function of n
- In the limit as $n \to \infty$, the template approaches a rectangle, where 100% of the spectral content is contained within the 3-dB bandwidth. This condition is essentially attained when $n > 15$
- For PRO-FM, power concentration saturates at 98.5% for $n > 15$. The residue is due to enforcement of the rectangular pulse shape (perfect band-limiting is impossible)

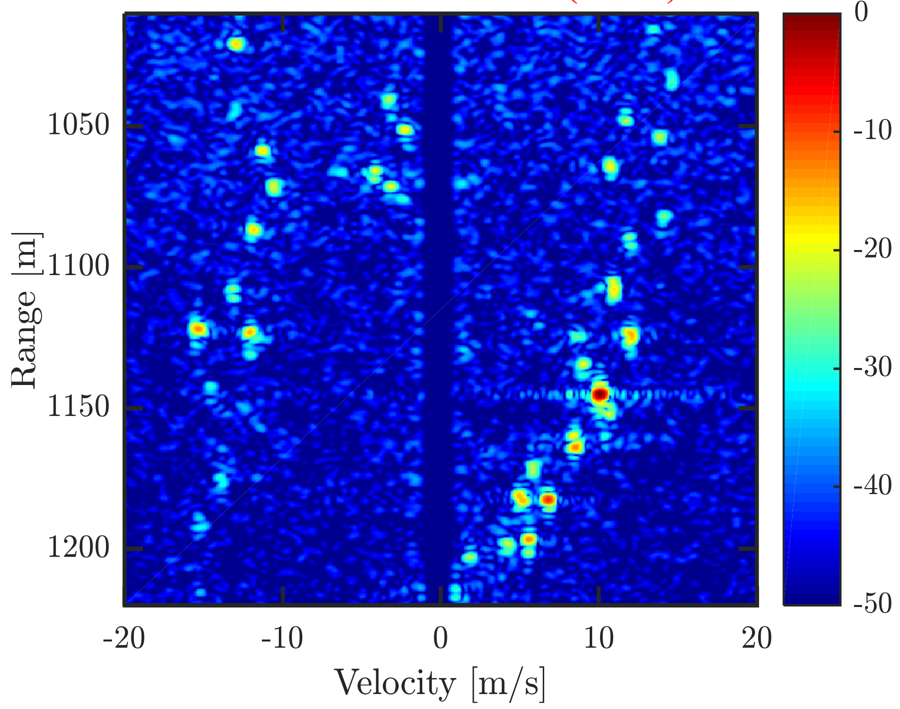


Simulation of percent in-band power within 3-dB bandwidth vs. n

Open-Air Measurements

- Open-air MTI measurements were collected for the 5000 waveforms in the $n = 8$ case, illuminating the 23rd & Iowa traffic intersection in Lawrence, KS from the roof of Nichols Hall on the KU campus
- Results include a -40 dB Taylor window (for Doppler sidelobes) and a simple zero-Doppler projection for clutter cancellation
- Multiple movers are clearly visible and results are consistent with other RFM waveform approaches
- Confirms that improved spectral containment via super-Gaussian shaping is viable for practical RFM waveform design

Open-air range-Doppler response for 5000 waveforms ($n = 8$)



Conclusions

- The super-Gaussian function has been experimentally demonstrated to provide a practical spectral design template for random FM waveforms
- Increasing the exponential shape parameter n greater than 2 (Gaussian) yields increasingly tighter spectral containment
  - Greater signal power density within signal bandwidth
  - Necessary for extension to wideband operation where oversampling is less feasible
- The trade-offs incurred for better containment include
  - emergence of persistent range sidelobes close to the mainlobe, which could be viewed as a broadened mainlobe
  - Marginal increase in the RMS autocorrelation sidelobe floor
  - Similar small increase in the cross-correlation floor.