Experimental Evaluation of Adaptive Doppler Estimation for PRI-Staggered Radar

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Motivation

- **Doppler processing** is instrumental for discrimination between movers and clutter for moving target indication (**MTI**) radar based on radial motion relative to the platform

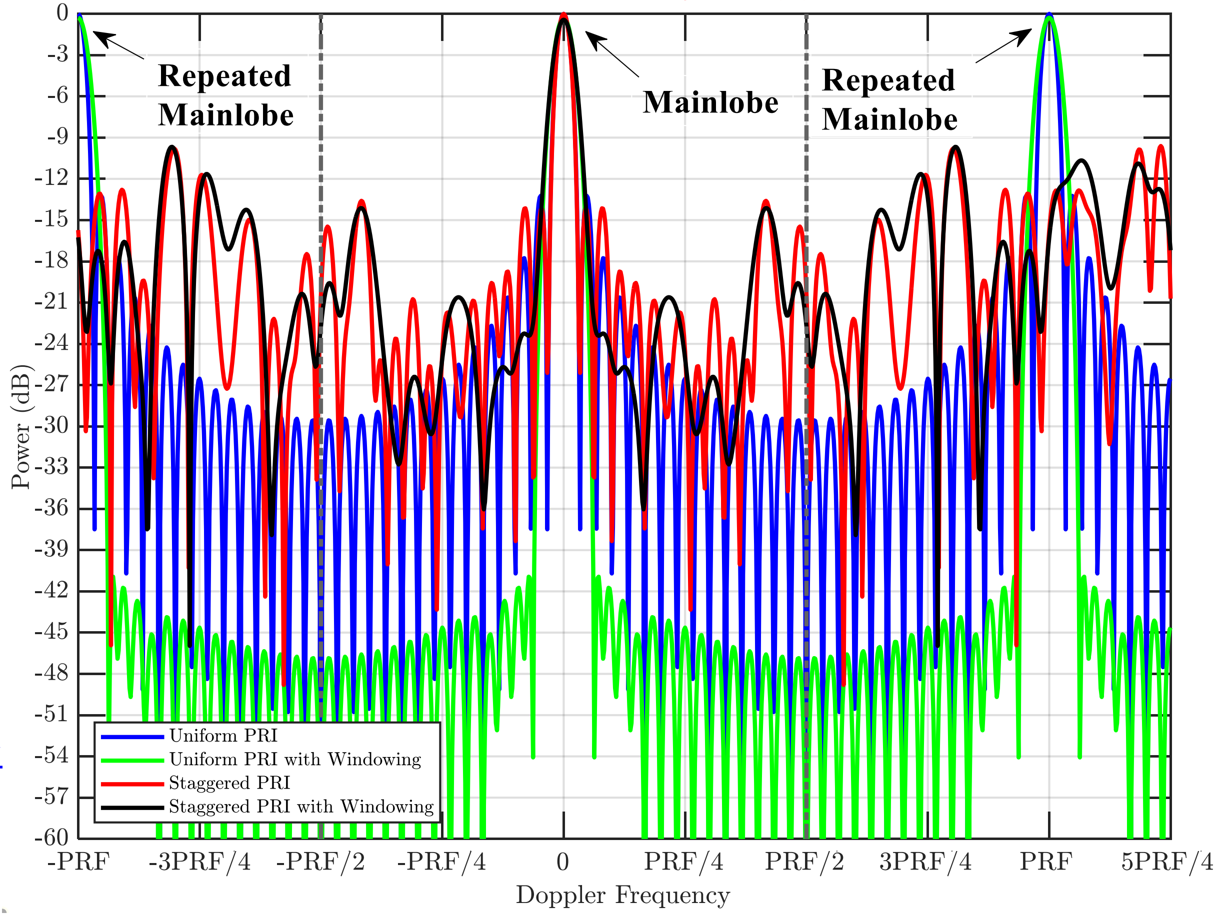
- **Uniform pulse repetition interval (PRI)** *maintains a constant repetition interval* over the coherent processing interval (CPI)
  - Introduces a trade-space between unambiguous range and Doppler (velocity) that is dictated by the PRI and pulse repetition frequency (PRF), respectively
  - Doppler frequencies outside the unambiguous interval of [−PRF/2, +PRF/2] are aliased back into this interval due to PRI periodicity
  - Realizes uniform slow-time sampling that permits the use of the fast Fourier transform (FFT)

- **Random PRI staggering** *varies the PRI extent* on a pulse-to-pulse basis over the CPI
  - Avoids trade-space restriction of unambiguous Doppler to avoid aliasing of movers
  - Imposes non-uniform slow-time sampling
  - Creates some additional complications …

Motivation

- Doppler Processing of **uniform PRI using FFT** is well known to produce a periodic-sinc response
  - Windowing is often used to reduce sidelobes, with degraded resolution and signal-to-noise ratio (SNR) loss also occurring
- Doppler processing of **random PRI Staggering** deviates from periodic-sinc response, causing higher and somewhat flatter sidelobes
  - **Windowing provides no benefit for PRI staggering**
- Here we use **re-iterative superresolution (RISR)** to reduce Doppler sidelobes of random PRI staggering in the context of clutter cancellation
- This approach is demonstrated experimentally with free-space measurements

Standard and tapered Doppler processing for uniform and randomly staggered PRI



PRI-Staggered Radar Signal Model

- Consider a radar receiving M PRIs in a CPI. Each PRI contains a low duty cycle pulse with the **same pulse duration across the CPI**. The received response from the illuminated scatterers and noise for the mth PRI can be expressed as

waveform, range profile, Doppler frequency, noise

$$y(m,t) = \sum_{f_D} [s(t) * x(t; f_D)] e^{j\pi f_D T_{acc}(m)} + n(m,t) \quad \text{defined over: } 0 \le t \le T_m$$

- Pulse compression is performed using matched (or mismatched) filter $h(t)$ as

$$z(m,t) = h(t) * y(m,t)$$

- Following pulse compression and IQ sampling, the $M \times 1$ collection of slow-time samples at the ℓth range bin can be represented as

$$\mathbf{z}(\ell) = \sum_{f_D} \tilde{x}(\ell; f_D)\mathbf{v}(f_D) + \tilde{\mathbf{n}}(\ell) \cong \mathbf{V}\tilde{\mathbf{x}}(\ell) + \tilde{\mathbf{n}}(\ell)$$

pulse compressed scattering, steering vector, noise, steering vectors in columns, scattering

mth PRI

$$\underset{\substack{\uparrow \\ \text{average PRI}}}{T_m = T_{avg} + \Delta T_m}$$

mth deviation

$$\Delta T_m \sim U(-\delta, +\delta)$$

fixed value

mth accumulation

$$T_{acc}(m) = \sum_{q=0}^{m-1} T_q$$

Doppler Filter Bank

- Each $M \times 1$ **Doppler steering vector** is a function of Doppler frequency

$$\mathbf{v}(f_{\mathrm{D}}) = \begin{bmatrix} 1 & e^{j\pi f_{\mathrm{D}} T_{\mathrm{acc}}(2)} & e^{j\pi f_{\mathrm{D}} T_{\mathrm{acc}}(3)} & \cdots & e^{j\pi f_{\mathrm{D}} T_{\mathrm{acc}}(M)} \end{bmatrix}^T$$

- A $M \times N$ **Doppler filter bank** is formed from Doppler steering vectors

$$\mathbf{V} = \begin{bmatrix} \mathbf{v}(-\beta f_{\mathrm{avg}}/2) & \mathbf{v}(-\beta f_{\mathrm{avg}}/2+\Delta f) & \mathbf{v}(-\beta f_{\mathrm{avg}}/2+2\Delta f) & \cdots & \mathbf{v}(+\beta f_{\mathrm{avg}}/2) \end{bmatrix}$$

oversampling factor

Spacing between frequencies points: $\Delta f = \beta f_{\mathrm{avg}}/N$ Number of columns: $N = \beta K M$

- $K > 1$ provides better visibility and reduces Doppler straddling
- $\beta = 1$ for *uniform PRI* $\rightarrow$ Doppler extent $[-f_{\mathrm{avg}}/2, +f_{\mathrm{avg}}/2]$
- $\beta > 1$ for *PRI staggering* $\rightarrow$ Doppler extent $[-\beta f_{\mathrm{avg}}/2, +\beta f_{\mathrm{avg}}/2]$ which could be quite large for some staggering sequences
- Can replace β with β_{mov} to include *all expected movers*

extent of unambiguous Doppler

$$\beta = T_{\mathrm{avg}} \mathrm{LCM}\underbrace{\{f_1, f_2, \cdots, f_M\}}_{\textit{PRFs}}$$

average PRF

$$f_{\mathrm{avg}} = 1/T_{\mathrm{avg}}$$

Standard Doppler Processing

- The $N \times 1$ standard Doppler response can be expressed as

$$\hat{\mathbf{x}}_{\mathrm{DP}}(\ell) = \mathbf{W}_{\mathrm{DP}}^{H} \mathbf{R}_{\mathrm{canc}}^{-1}(\ell) \mathbf{z}(\ell)$$

where cancellation is performed using $M \times M$ cancellation matrix

$$\mathbf{R}_{\mathrm{canc}}(\ell) = \mathbf{R}_{\mathrm{clut}}(\ell) + \mathbf{R}_{\mathrm{int}}(\ell) + \mathbf{R}_{\mathrm{nse}}(\ell)$$

comprised of the covariance matrices for *clutter* $\mathbf{R}_{\mathrm{clut}}(\ell)$, *interference* $\mathbf{R}_{\mathrm{int}}(\ell)$ and *noise* $\mathbf{R}_{\mathrm{nse}}(\ell)$ and matrix standard Doppler processing is performed using normalized filter bank

$$\mathbf{W}_{\mathrm{DP}} = (1/M)\mathbf{V}$$

- We will use a *reiterative minimum mean-square error* (RMMSE) transform to replace $\mathbf{W}_{\mathrm{DP}}$ to:
  - Reduce Doppler sidelobes relative to those produced by $\mathbf{W}_{\mathrm{DP}}$, and
  - Achieve superresolution with minimal SNR loss

Re-iterative Superresolution

- **Re-iterative superresolution (RISR)** is a form of RMMSE estimation developed for robust adaptive beamforming [1]

- A partial gain constraint was developed for RISR to enhance practical performance [2]
  - Incorporates unity gain constraint, then introduces trade-off between constrained/unconstrained
  - For high fidelity (mitigate Doppler straddling), oversampling relative to nominal resolution is required

- In [3], RISR was expanded for use with clutter cancellation in both **joint** and **sequential cancel-then-estimate procedures**, which involve replacing the Fourier transform with a RISR transform

- Here it is shown that the sequential procedure can likewise be employed for Doppler processing when PRI-staggering is used

[1] S.D. Blunt, T. Chan, K. Gerlach, "Robust DOA estimation: the reiterative superresolution (RISR) algorithm," *IEEE Trans. Aerospace and Electronic Systems*, vol. 47, no. 1, pp. 332-346, Jan. 2011.
[2] E. Hornberger, S.D. Blunt, T. Higgins, "Partially constrained adaptive beamforming for super-resolution at low SNR," *IEEE Intl. Workshop Computational Advances in Multi-Sensor Adaptive Processing*, Cancun, Mexico, Dec. 2015
[3] C.C. Jones, L.A. Harnett, C.A. Mohr, S.D. Blunt, C.T. Allen, "Structure-based adaptive radar processing for joint clutter cancellation and moving target estimation," *IEEE Intl. Radar Conf.*, Washington, DC, Apr. 2020.

Re-iterative Superresolution

- RISR is obtained using **the MSE cost function**:

$$J_{\text{MSE}}(\ell, f_{\text{D}}) = E\left[|\, \tilde{x}(\ell; f_{\text{D}}) - \mathbf{w}^H(\ell, f_{\text{D}})\mathbf{z}(\ell)\,|^2 \right]$$

- Unconstrained (U) Filter [1]:

$$\mathbf{w}_{\text{U}}(\ell, f_{\text{D}}) = \left(E[\mathbf{z}(\ell)\mathbf{z}^H(\ell)] \right)^{-1} E[\tilde{x}^*(\ell; f_{\text{D}})\mathbf{z}(\ell)]$$
$$= \mathbf{D}^{-1}(\ell)\mathbf{v}(f_{\text{D}})\rho(\ell, f_{\text{D}})$$

- Gain-Constrained (GC) Filter:

$$\mathbf{w}_{\text{GC}}(\ell, f_{\text{D}}) = \frac{1}{\mathbf{v}^H(f_{\text{D}})\mathbf{D}_i^{-1}(\ell)\mathbf{v}(f_{\text{D}})}\mathbf{D}_i^{-1}(\ell)\mathbf{v}(f_{\text{D}})$$

- **Partially Constrained (PC) Filter:**

$$\mathbf{w}_{\text{PC}}(\ell, f_{\text{D}}) = \frac{(\rho(\ell, f_{\text{D}}))^{1-\alpha}}{\left(\mathbf{v}^H(f_{\text{D}})\mathbf{D}^{-1}(\ell)\mathbf{v}(f_{\text{D}})\right)^{\alpha}}\mathbf{D}^{-1}(\ell)\mathbf{v}(f_{\text{D}})$$

expected power in a given range/Doppler cell

$$\rho(\ell, f_{\text{D}}) = E\left[|\, \tilde{x}(\ell; f_{\text{D}})\,|^2 \right]$$

structured covariance matrix

$$\mathbf{D}(\ell) = \mathbf{V}\mathbf{P}(\ell)\mathbf{V}^H + \mathbf{R}_{\text{n}}$$

expected power matrix

$$\mathbf{P}(\ell) = [\hat{\mathbf{x}}(\ell)\hat{\mathbf{x}}^H(\ell)] \odot \mathbf{I}_{N\times N}$$

noise covariance

$$\mathbf{R}_{\text{n}} = \sigma_{\text{n}}^2 \mathbf{I}_{M\times M}$$

noise power

constraint parameter

(U) $0 \le \alpha \le 1$ (GC)

[1] S.D. Blunt, T. Chan, K. Gerlach, "Robust DOA estimation: the reiterative superresolution (RISR) algorithm," *IEEE Trans. Aerospace and Electronic Systems*, vol. 47, no. 1, pp. 332-346, Jan. 2011.

Re-iterative Superresolution

- **The unconstrained RISR** produces significant superresolution enhancement, but has the tendency to suppress lower SNR signals and provides no meaningful noise floor for subsequent detection processing

- The **constrained RISR** produces modest superresolution enhancement, and preserves lower SNR signals while providing a meaningful noise floor for detection processing

- The resulting **partially constrained [2] filter provides a useful middle ground**

- The collection of partially constrained Doppler filters likewise form a $M \times N$ filter bank

$$\mathbf{W}_{\mathrm{PC}}(\ell) = \left[\mathbf{w}_{\mathrm{PC}}(\ell, -\beta f_{\mathrm{avg}}/2) \quad \cdots \quad \mathbf{w}_{\mathrm{PC}}(\ell, +\beta f_{\mathrm{avg}}/2) \right]$$

- Thus, the **MMSE Doppler response** is obtained by simply applying the filter bank

$$\hat{\mathbf{x}}_{\mathrm{PC}}(\ell) = \mathbf{W}_{\mathrm{PC}}^{H}(\ell)\mathbf{R}_{\mathrm{canc}}^{-1}(\ell)\mathbf{z}(\ell)$$

- **The expected power** $\rho(\ell, f_{\mathrm{D}})$ needed for filter formulation is not known *a priori*. Instead, **a recursive procedure is used to estimate this quantity**

[2] E. Hornberger, S.D. Blunt, T. Higgins, "Partially constrained adaptive beamforming for super-resolution at low SNR," *IEEE Intl. Workshop Computational Advances in Multi-Sensor Adaptive Processing*, Cancun, Mexico, Dec. 2015

Adaptive Doppler Estimation

Receive Processing Flow Diagram

$$\hat{\mathbf{x}}_{\text{DP}}(\ell) = \mathbf{W}_{\text{DP}}^{H}(\ell)\mathbf{R}_{\text{canc}}^{-1}(\ell)\mathbf{z}(\ell)$$

Initial estimate

$$\hat{\mathbf{x}}_{i=0}(\ell) = \hat{\mathbf{x}}_{\text{DP}}(\ell)$$

Compute expected power

$$\mathbf{P}_i(\ell) = [\hat{\mathbf{x}}_{i-1}(\ell)\hat{\mathbf{x}}_{i-1}^{H}(\ell)] \odot \mathbf{I}_{N\times N}$$

Form adaptive filters

$$\mathbf{w}_i(\ell, f_{\text{D}}) = \frac{(\rho(\ell, f_{\text{D}}))^{1-\alpha}}{(\mathbf{v}^{H}(f_{\text{D}})\mathbf{D}_i^{-1}(\ell)\mathbf{v}(f_{\text{D}}))^{\alpha}}\mathbf{D}_i^{-1}(\ell)\mathbf{v}(f_{\text{D}})$$

$$\mathbf{w}_i(\ell, -\beta f_{\text{avg}}/2) \quad \mathbf{w}_i(\ell, -\beta f_{\text{avg}}/2 + \Delta f) \quad \cdots \quad \mathbf{w}_i(\ell, +\beta f_{\text{avg}}/2)$$

Collect into filter bank

$$\mathbf{W}_i(\ell) = \left[\mathbf{w}_i(\ell, -\beta f_{\text{avg}}/2) \quad \cdots \quad \mathbf{w}_i(\ell, +\beta f_{\text{avg}}/2)\right]$$

Apply filter bank

$$\hat{\mathbf{x}}_i(\ell) = \mathbf{W}_i^{H}(\ell)\mathbf{R}_{\text{canc}}^{-1}(\ell)\mathbf{z}(\ell)$$

ith iteration

Open-Air Measurements

Transmit Parameters

waveform
Linear FM

carrier frequency
3.55 GHz

time-bandwidth product
150

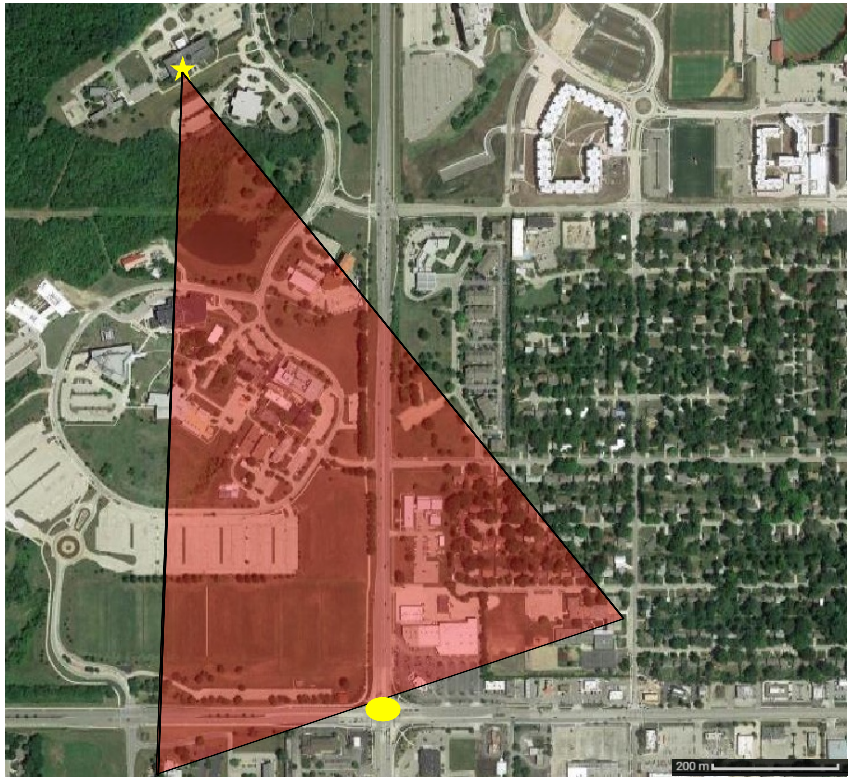
sub-CPI
30 sub-CPIs

pulses per sub-CPI
40 pulses

effective average PRF
3.2 kHz → 80 Hz

First 4 pulses in each sub-CPI were pre-summed. Other 36 were discarded.

Annotated field of view for measured results



Courtesy of Google Maps

Hardware instrumentation setup

Experimental Results - Uniform

Receive Processing Parameters

eff. pulses in CPI

$M = 30$

number of points

$N = 2400$

Doppler extent

$\beta_{\text{mov}} = 16$

oversampling

$K = 5$

iterations

$I_{\text{iter}} = 10$

constraint parameter

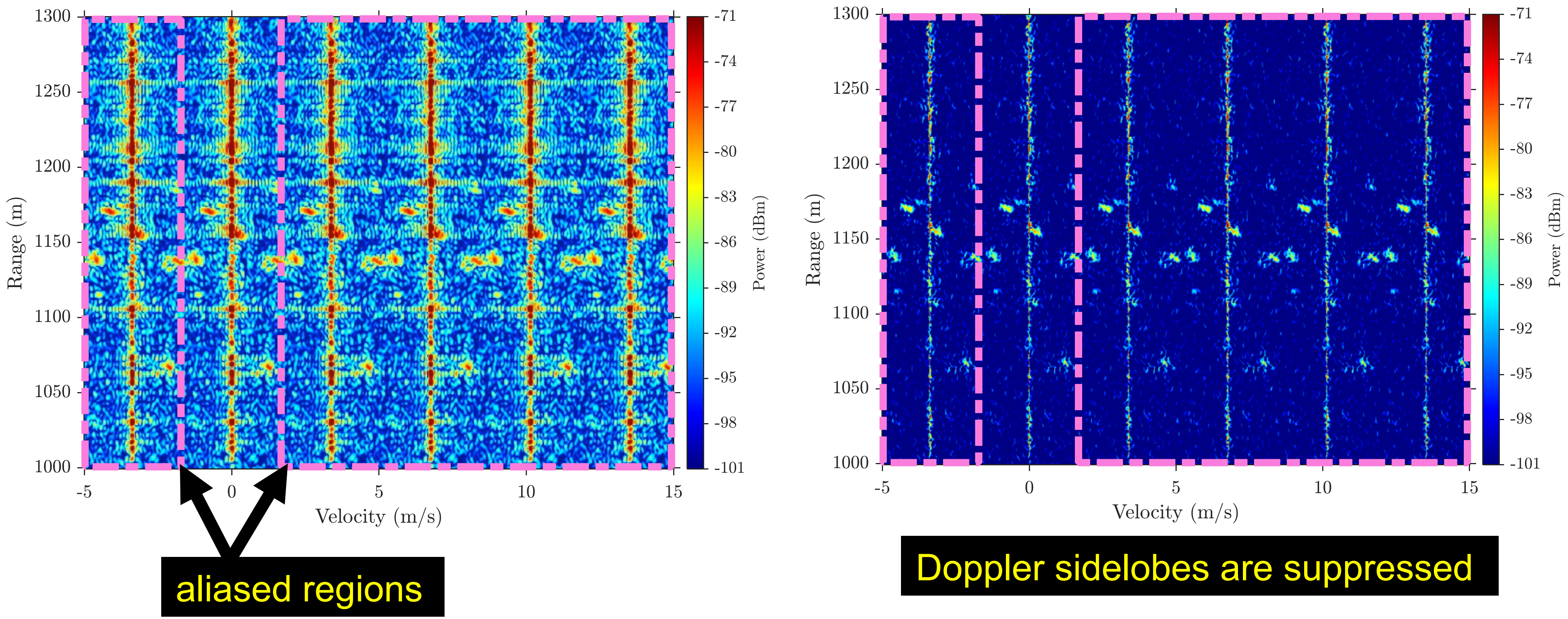
$\alpha = 0.8$

noise power

$\sigma_n^2 = -75$ dBm

Uniform PRIs -- w/o Cancellation

Standard Doppler Processing | **RISR Adaptive Doppler Estimation**



- Pre-summing used to reduce a transmit PRF of 3.2 kHz to effective PRF of 80 Hz
- Standard Doppler processing performed via FFT (uniform) and compensated DFT (staggered)

Experimental Results - Uniform

Receive Processing Parameters

eff. pulses in CPI

$M = 30$

number of points

$N = 2400$

Doppler extent

$\beta_{\text{mov}} = 16$

oversampling

$K = 5$

iterations

$I_{\text{iter}} = 10$

constraint parameter

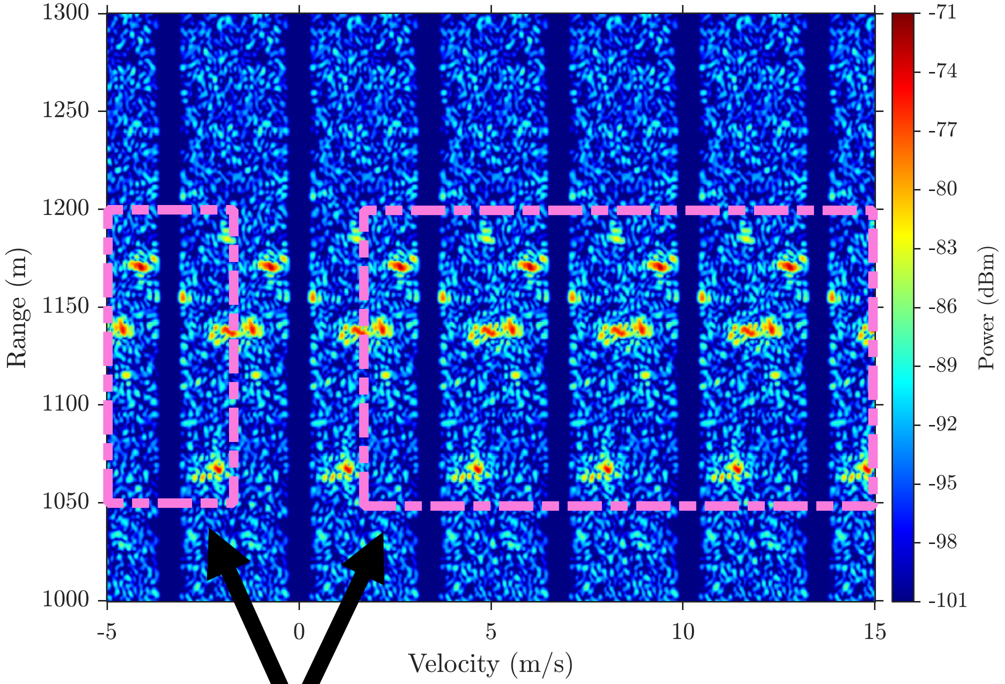
$\alpha = 0.8$

noise power

$\sigma_n^2 = -75$ dBm

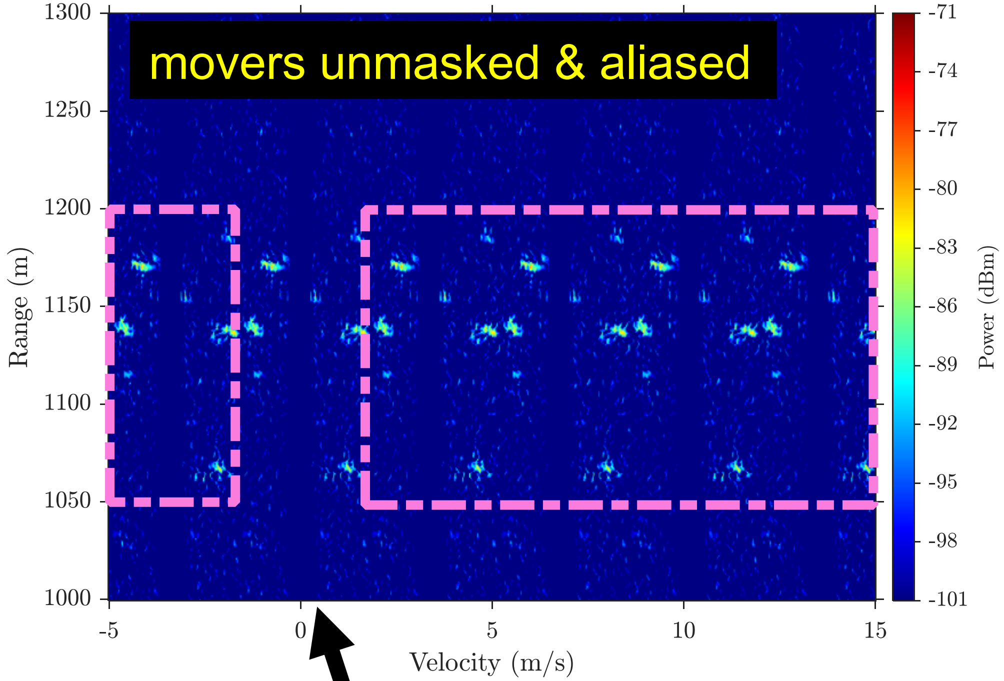
Uniform PRIs – with Cancellation

Standard Doppler Processing



aliased regions

RISR Adaptive Doppler Estimation



clutter removed

- Pre-summing used to reduce a transmit PRF of 3.2 kHz to effective PRF of 80 Hz
- Standard Doppler processing performed via FFT (uniform) and compensated DFT (staggered)

Experimental Results - Staggered

Receive Processing Parameters

eff. pulses in CPI

$M = 30$

number of points

$N = 2400$

Doppler extent

$\beta_{\text{mov}} = 16$

oversampling

$K = 5$

iterations

$I_{\text{iter}} = 10$

constraint parameter

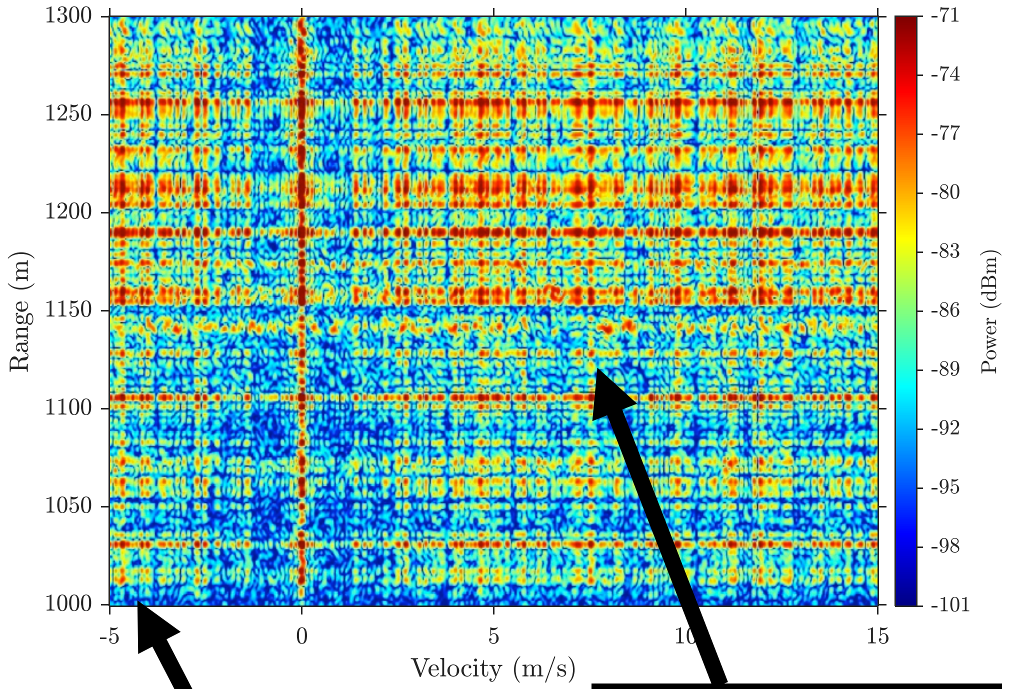
$\alpha = 0.8$

noise power

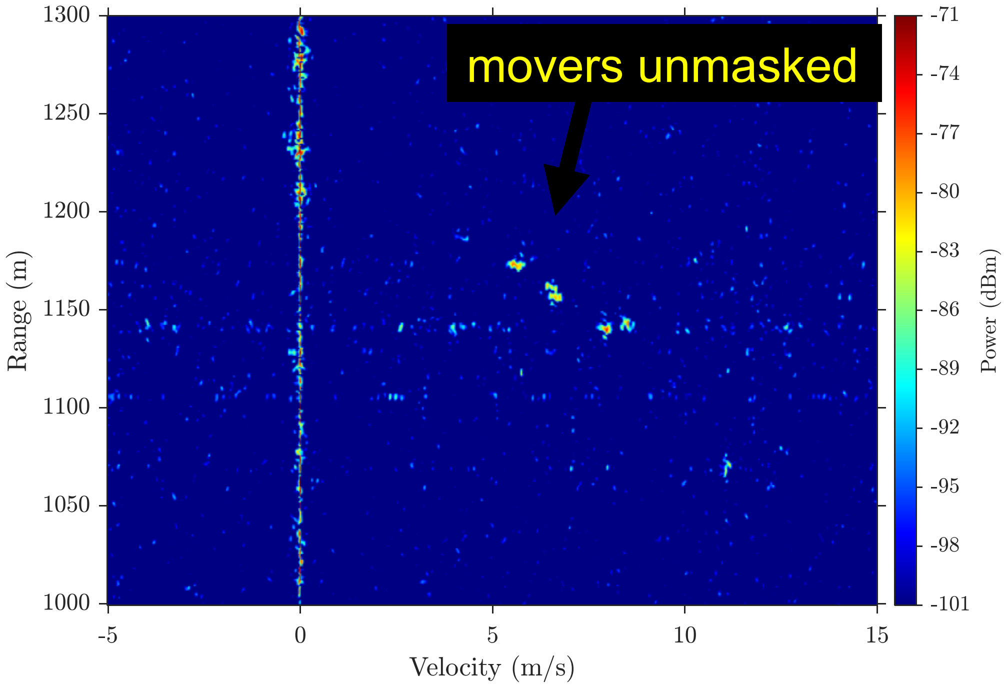
$\sigma_n^2 = -75$ dBm

Randomly Staggered PRIs -- w/o Cancellation

Standard Doppler Processing



RISR Adaptive Doppler Estimation



aliasing avoided

movers obscured

Doppler sidelobes are suppressed

- Pre-summing used to reduce a transmit PRF of 3.2 kHz to effective PRF of 80 Hz
- Standard Doppler processing performed via FFT (uniform) and compensated DFT (staggered)

Experimental Results - Staggered

Receive Processing Parameters

eff. pulses in CPI

$M = 30$

number of points

$N = 2400$

Doppler extent

$\beta_{mov} = 16$

oversampling

$K = 5$

iterations

$I_{iter} = 10$

constraint parameter

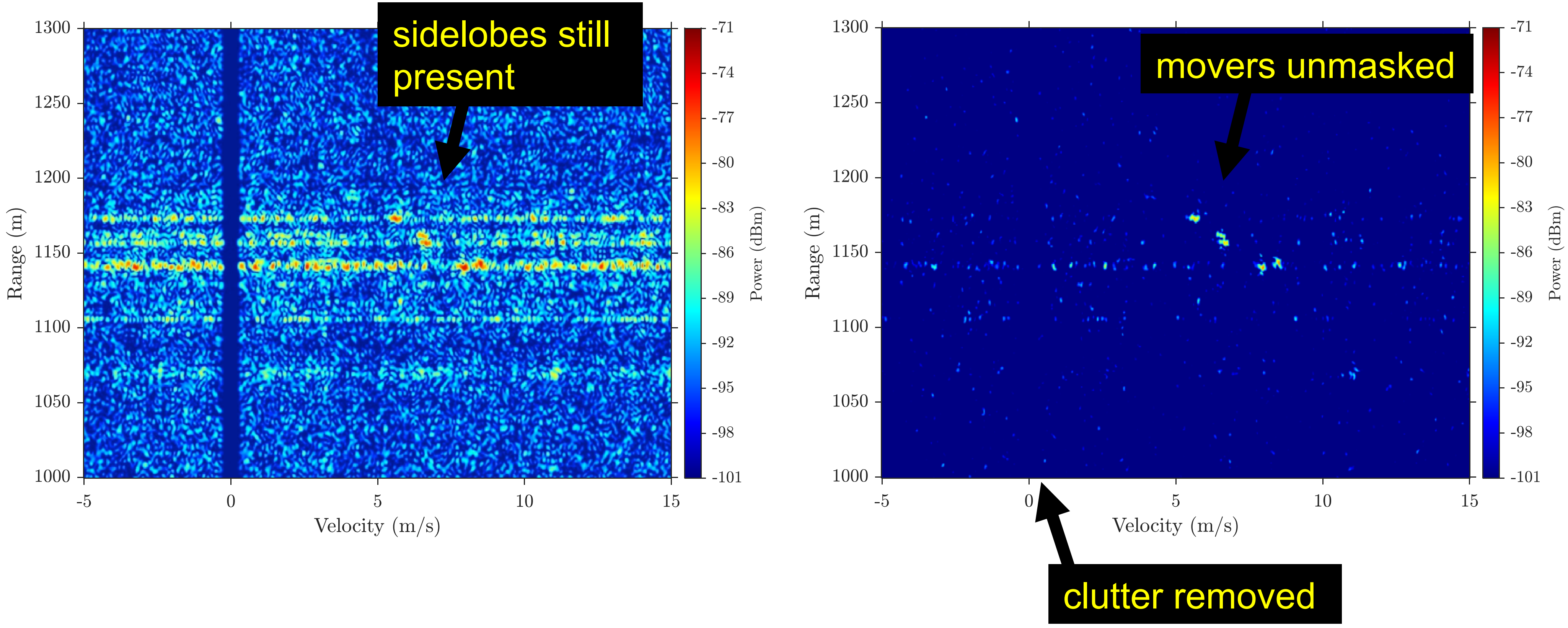
$\alpha = 0.8$

noise power

$\sigma_n^2 = -75$ dBm

Randomly Staggered PRIs -- with Cancellation

Standard Doppler Processing | **RISR Adaptive Doppler Estimation**



- Pre-summing used to reduce a transmit PRF of 3.2 kHz to effective PRF of 80 Hz
- Standard Doppler processing performed via FFT (uniform) and compensated DFT (staggered)

Conclusions

- **Staggering facilitates the extension of unambiguous Doppler as a trade-off for higher and flatter sidelobes**

- **Re-iterative superresolution (RISR), a form of reiterative MMSE (RMMSE) developed for beamforming, has been applied for adaptive Doppler estimation**
  - applied to uniform and staggered PRI pulse arrangements, without and with clutter cancellation
  - **compensates for high Doppler sidelobes and provides Doppler super-resolution**

- **Open-air measurements demonstrate the prospect of enhanced discernibility of movers and detection performance benefits**