On the Repeated Use of Random FM Waveforms

Thomas J. Kramer, Matthew B. Heintzelman, Shannon D. Blunt

Radar Systems Lab (RSL), University of Kansas

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Background

- Random FM (RFM) waveforms [1] have constant amplitude and continuous phase, making them amenable to high-power transmitters while also possessing the high dimensionality and non-repetition of traditional noise waveforms.

- While RFM was first proposed in the mid ’50s, it has only been in recent years that spectrum shaping has been incorporated through either optimization or imposed structure.

- The combination of spectrum shaping, uniqueness via randomization, and FM structure provides a physically realizable signal that facilitates new sensing modes:
  - Real-time sense-and-notch cognitive radar,
  - Practical complementary waveform sidelobe cancellation,
  - Intermodulation form of nonlinear radar, and more.

[1] S.D. Blunt, J.K. Jakabosky, C.A. Mohr, P.M. McCormick, et al, “Principles & applications of random FM radar waveform design,” *IEEE Aerospace & Electronic Systems Magazine*, vol. 35, no. 10, pp. 20-28, Oct. 2020.

Problem Motivation

- While RFM generation methods using optimization yield significantly lower range sidelobes than those that do not, they also incur a higher computational cost.

- One could also produce waveforms offline and save them in a library, though doing so also incurs a possibly significant memory requirement.

- Non-repetition is the RFM attribute that provides the high dimensionality that enables new sensing modes.

Problem Motivation

- Given the variety of applications where RFM could be employed, some of which have computational/memory constraints, here we explore the performance trade-space when permitting some degree of repetition throughout the CPI.

FM Waveform Structure

- The baseband representation of an arbitrary FM waveform can be expressed as:

$$s(t) = \exp\left(j2\pi \int_{-\infty}^{t} f(\tau)d\tau \right) = \exp\big(j\theta(t) \big)$$

where $f(\tau)$ is the modulating process (e.g. white noise **[2-5]**) and $\theta(\tau)$ is the subsequent **continuous** phase.

- Given a random initialization, spectrum shaping $s(t)$ can greatly reduce range sidelobes while preserving uniqueness (due to highly nonconvex/nonlinear cost functions).

[2] T.B. Whiteley, D.J. Adrian, "Random FM autocorrelation fuze system," U.S. Patent #4,220,952, issued Sept. 2, 1980, application filed Feb. 17, 1956.

[3] L. Guosui, G. Hong, Z. Xiaohua, S. Weimin, "The present and future of random signal radars," *IEEE Aerospace & Electronic Systems Mag*., vol. 12, no. 10, pp. 35-40, Oct. 1997.

[4] S.R.J. Axelsson, "Noise radar using random phase and frequency modulation," *IEEE Trans. Geoscience & Remote Sensing*, vol. 42, no. 11, pp. 2370-2384, Nov. 2004.

[5] L. Pralon, B. Pompeo, J.M. Fortes, "Stochastic analysis of random frequency modulated waveforms for noise radar systems," *IEEE Trans. Aerospace & Electronic Systems*, vol. 51, no. 2, pp. 1447-1461, Apr. 2015.

Incoherent Sidelobe Averaging

- While spectrum shaping provides lower sidelobes on a per-waveform basis, **slow-time combining** (Doppler or SAR cross-range processing) produces an averaging effect across the sidelobes from unique waveforms, which are incoherent.

- Slow-time combining of M independent RFM waveforms yields a $\mathbf{10\log_{10}(\mathit{M})}$ further reduction in sidelobes.

- In other words, since the sidelobes are random and incoherent, they average in the same manner as **white noise**.

- However, repetition can introduce redundancy, limiting the degree of sidelobe suppression … in some respects.

Analyzing Repeated RFM

matched filter for mth waveform

- Start with the pulse compression response $x_m(\ell) = \mathbf{h}_m^H \mathbf{y}_m(\ell)$

interval of receive scattering induced by mth waveform

- Collect across L range samples and M waveforms to construct matrix $\mathbf{X}$.

- Apply discrete DFT transform ($\boldsymbol{A}^H$) yields matrix

$$\mathbf{Z} = \mathbf{A}^H \mathbf{X}$$

which has individual delay/Doppler samples

$$z(\omega, \ell_o) = \mathbf{a}^H(\omega)\,\mathbf{x}(\ell_o)$$

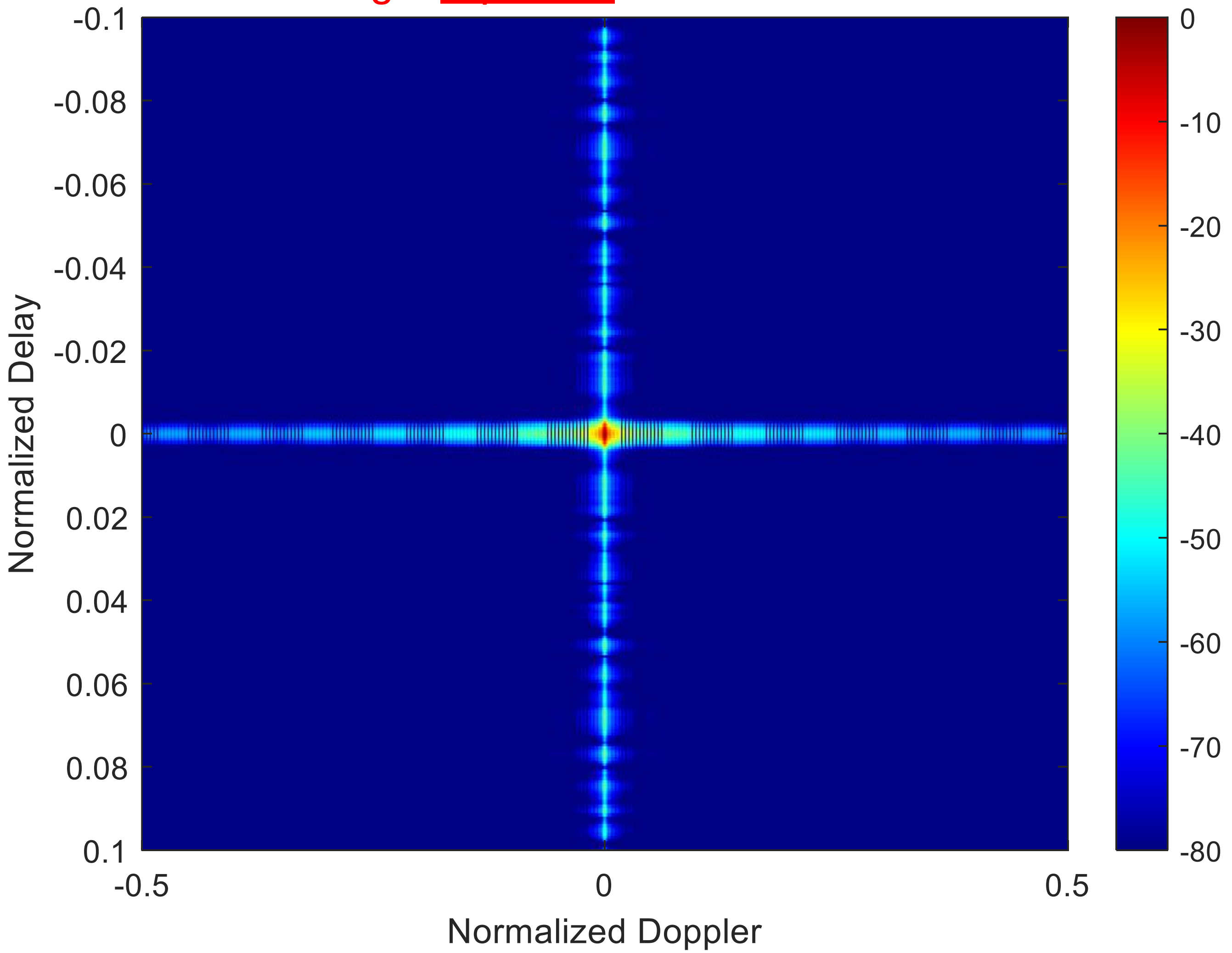
for Doppler steering vector $\mathbf{a}(\omega) = [1 \;\; e^{-j\omega} \;\; e^{-j2\omega} \;\; \cdots \;\; e^{-j(M-1)\omega}]^T$

Single Waveform CPI

- For the special case of a single point scatterer without noise, **Z** contains the **point-spread function** (PSF).

- For a single waveform repeated over the CPI, the PSF has
  - the typical sinc roll-off in Doppler
  - the typical autocorrelation response in delay (zero Doppler axis)

Point-spread function for CPI of M = 500 pulses for single repeated PRO-FM waveform

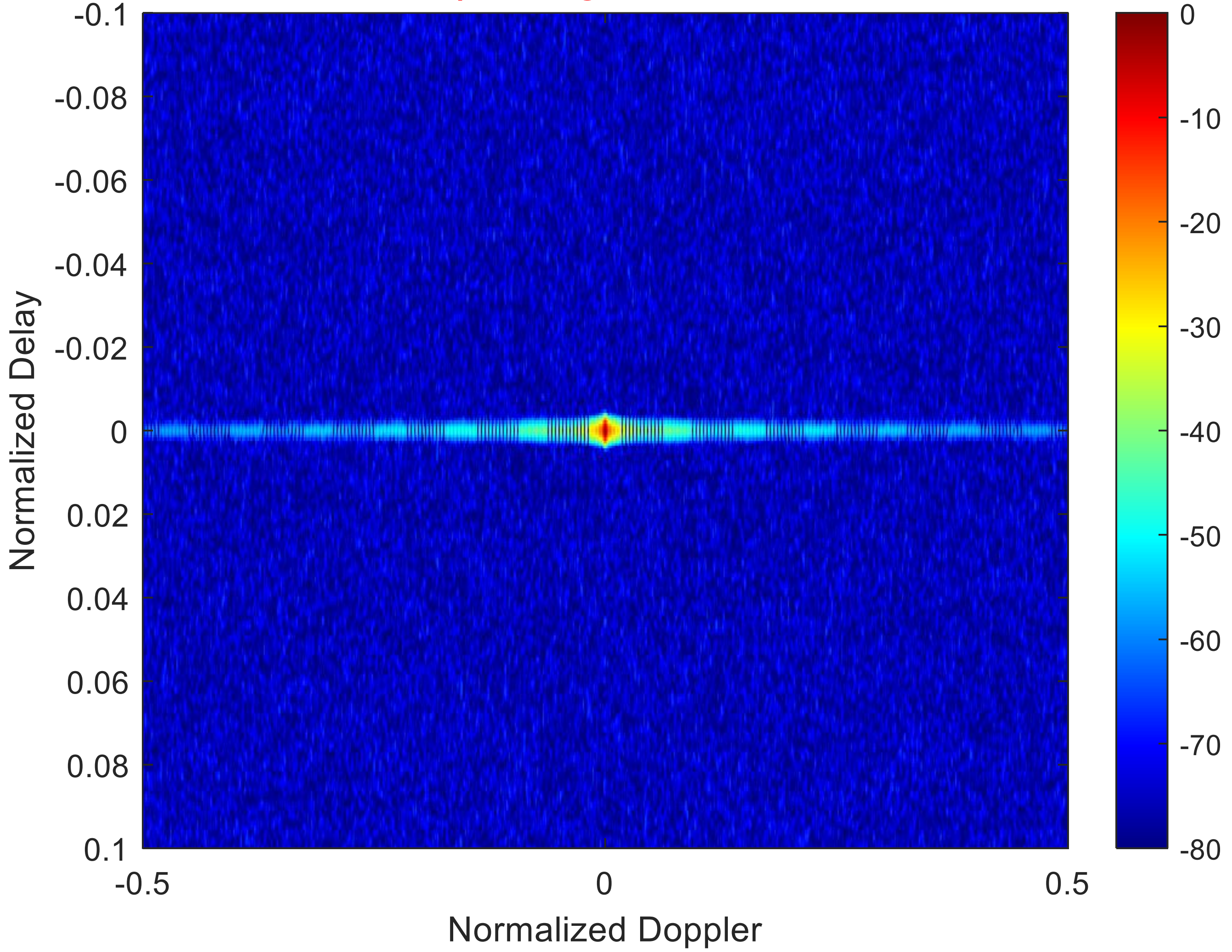


Fully Nonrepeating CPI

- At the other extreme, a CPI could have *M* completely unique waveforms.
- As a result, zero-Doppler sidelobes are reduced by a factor of *M* relative to the mainlobe.
- Sidelobes are now spread across Doppler forming a pedestal due to *range sidelobe modulation* **[1]**.

[1] S.D. Blunt, J.K. Jakabosky, C.A. Mohr, P.M. McCormick, et al, "Principles & applications of random FM radar waveform design," *IEEE Aerospace & Electronic Systems Magazine*, vol. 35, no. 10, pp. 20-28, Oct. 2020.

Point-spread function for CPI of M = 500 pulses with nonrepeating PRO-FM waveforms



Partial Repetition in the CPI

- Between the two extremes, the CPI could use some degree of repetition of K unique waveforms among the set of $M > K$ pulses.

- For the purpose of analysis, consider forming $N = M / K$ contiguous blocks of K pulses each, with each block containing all K waveforms in some order.

- To understand the impact of repetition it is instructive to partition the Doppler steering vector as

$$\begin{aligned}\mathbf{a}(\omega) &= [1 \;\; e^{-j\omega} \;\; e^{-j2\omega} \;\; \cdots \;\; e^{-j(M-1)\omega}]^T \\ &= [\bar{\mathbf{a}}_0^T(\omega) \;\; \bar{\mathbf{a}}_1^T(\omega) \;\; \cdots \;\; \bar{\mathbf{a}}_{N-1}^T(\omega)]^T \;,\end{aligned}$$

where the nth partitioned vector is

$$\bar{\mathbf{a}}_n(\omega) = e^{-j(Kn)\omega}[1 \;\; e^{-j\omega} \;\; e^{-j2\omega} \;\; \cdots \;\; e^{-j(K-1)\omega}]^T$$

Partial Repetition in the CPI

- Consequently, for the pulse compression mainlobe at range ℓ_o the slow-time response for the associated sidelobes at $\ell \neq \ell_o$ can be expressed as

$$
\begin{aligned}
z(\omega,\ell) &= \mathbf{a}^H(\omega)\,\mathbf{x}(\ell) \\
&= \sum_{n=0}^{N-1} \overline{\mathbf{a}}_n^H(\omega)\,\overline{\mathbf{x}}_n(\ell) \\
&= \sum_{n=0}^{N-1} e^{-j(Kn)\omega}\,\overline{\mathbf{a}}_0^H(\omega)\,\overline{\mathbf{x}}_n(\ell) \\
&= \sum_{n=0}^{N-1} e^{-j(Kn)\omega}\,\rho_n(\omega,\ell)
\end{aligned}
$$

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$$

factor of K sidelobe suppression due to incoherent averaging

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factor of K sidelobe suppression due to incoherent averaging

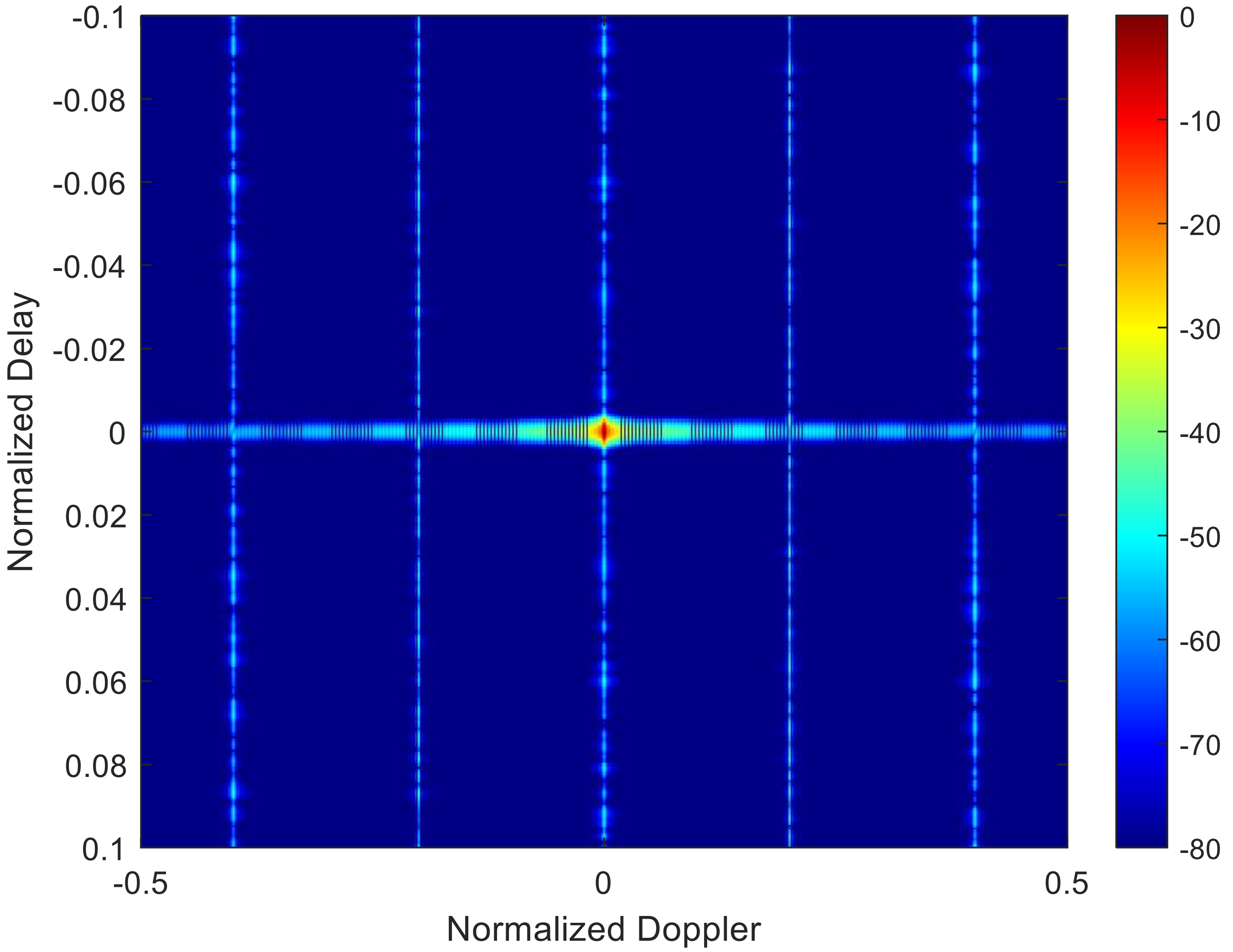
$$\underbrace{= \sum_{n=0}^{N-1} e^{-j(Kn)\omega}\, \rho_n(\omega,\ell)}$$

Subsequent weighted summation by exponential term is impacted by relative ordering of waveforms across blocks

Identically Repeated Waveform Blocks

- First consider the identical repetition of each block of K unique waveforms
- Now a periodic sinc function repeats on a Doppler interval of $\pm$ PRF/K.
  - Note: not Doppler aliasing.
- Repetition translates into a degree of sidelobe coherency, producing this repeated concentration of ambiguity.

Point-spread function for CPI of M = 500 pulses with K = 10 unique PRO-FM waveforms repeated N = 50 times



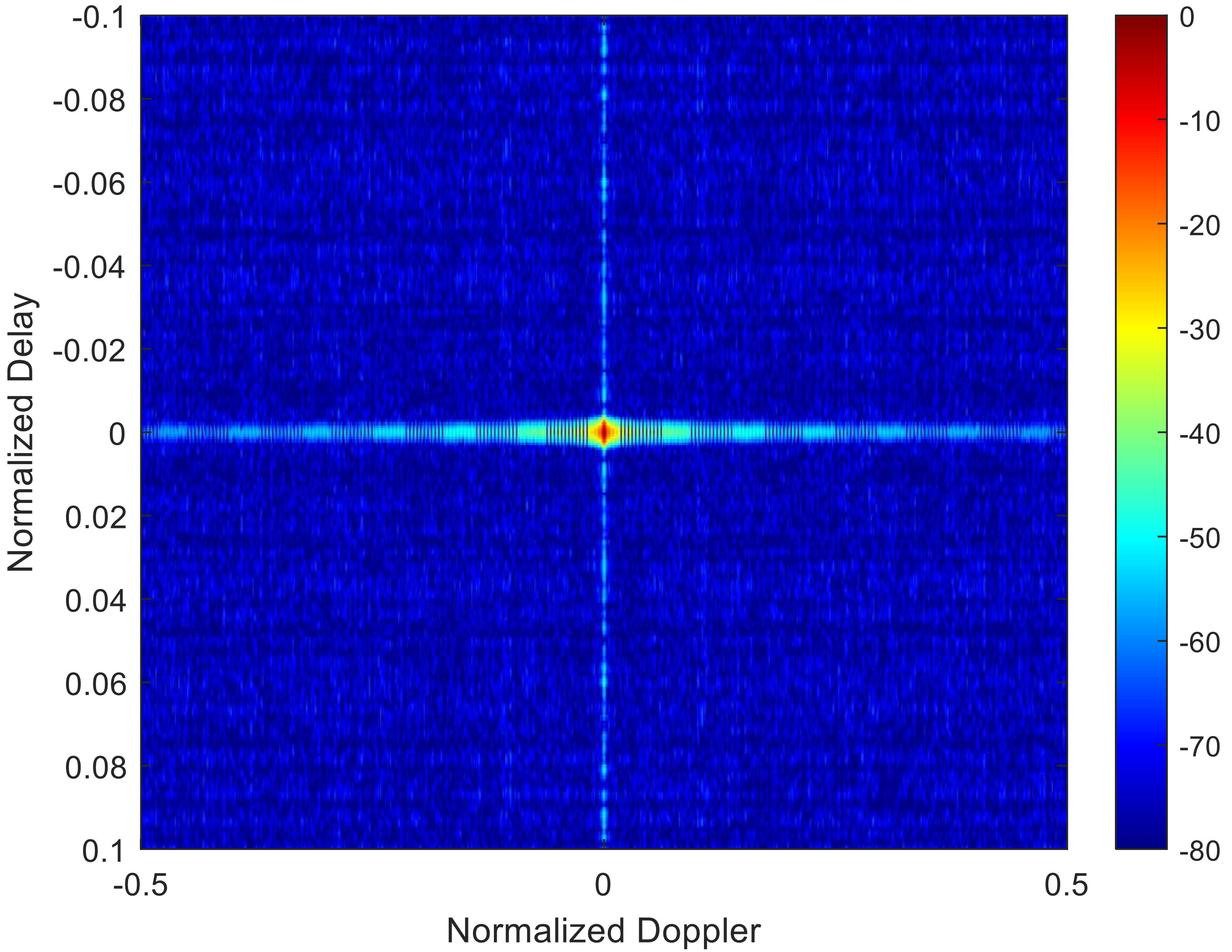
Randomized Waveform Blocks

- However, when the K waveforms are randomly permuted in each of the N blocks, the repeated sinc structure decoheres.

- Since ambiguity is conserved, it now spreads more evenly across the PSF pedestal.

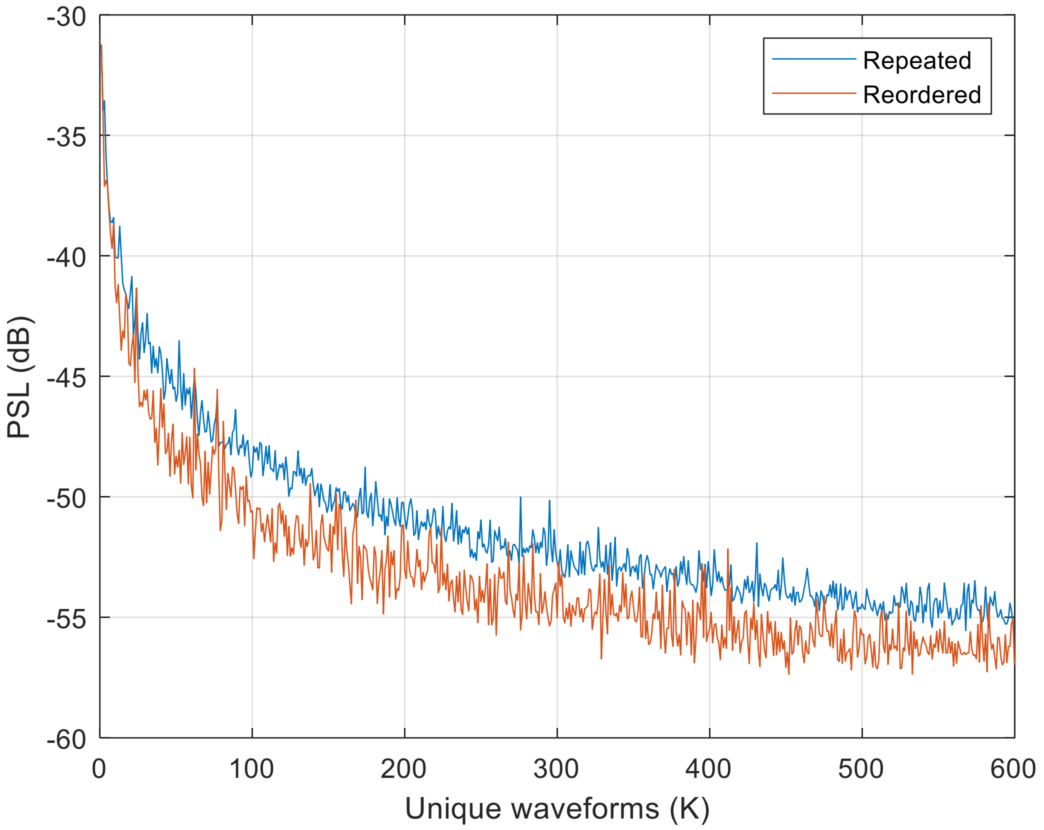
- Note: the zero-Doppler cut is unchanged by waveform ordering (repeated vs. random)
  - still a factor of K sidelobe reduction compared to fully repeated case

Point-spread function for CPI of M = 500 pulses with K = 10 unique PRO-FM waveforms randomly permuted N = 50 times

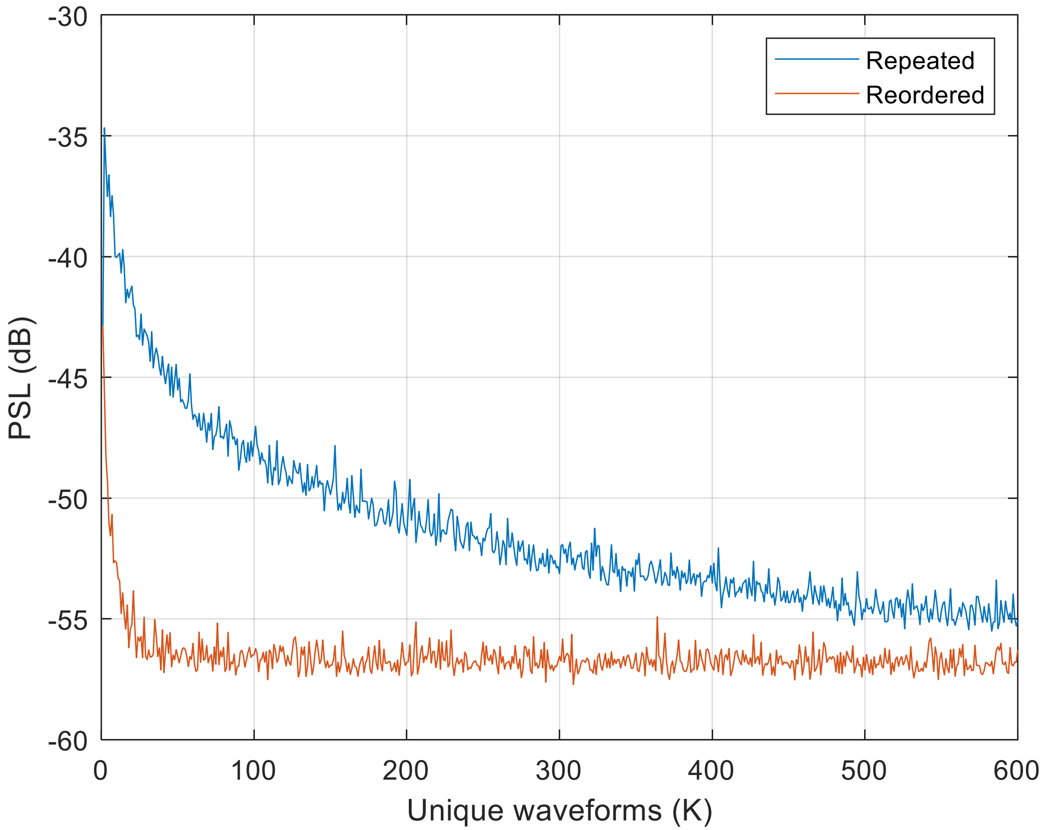


Peak Sidelobe Level (PSL) Comparisons

Monte Carlo (1000 trials) comparison of PSL for repeated and randomly reordered waveform sets (<u>including</u> zero Doppler sidelobes)



Monte Carlo (1000 trials) comparison of PSL for repeated and randomly reordered waveform sets (<u>excluding</u> zero Doppler sidelobes)



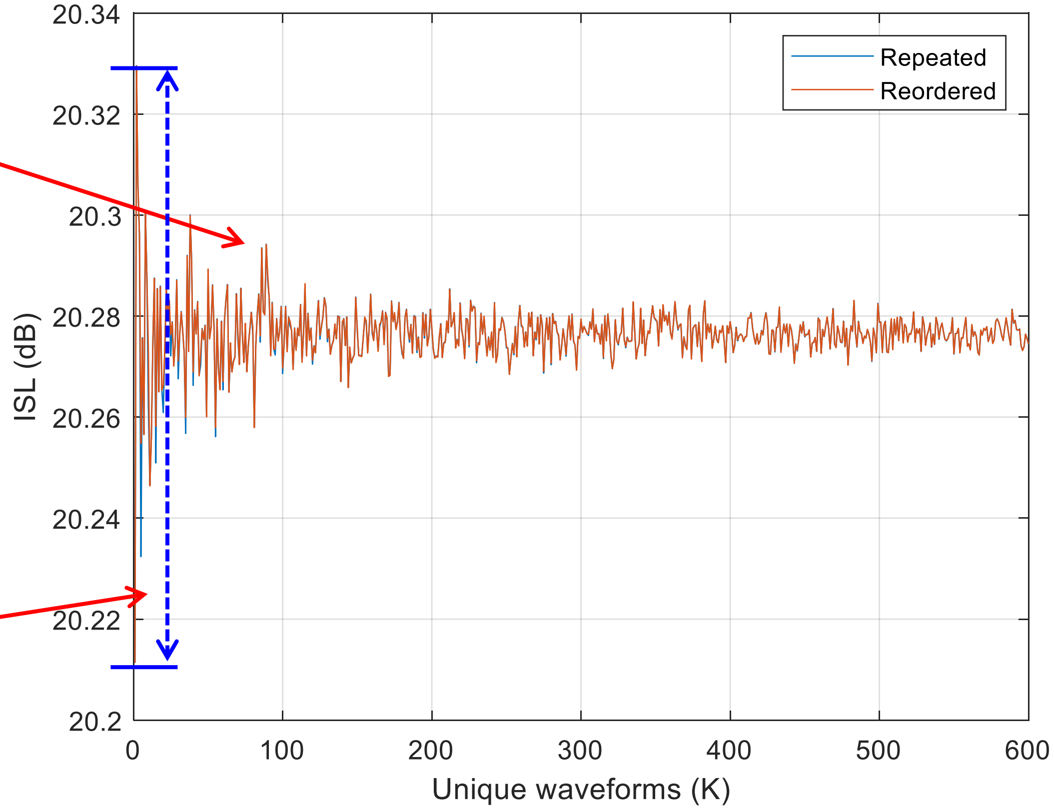
PSL outside zero Doppler is consistent across *K* when waveform blocks are randomly reordered, but decreases linearly with *K* when blocks are identically repeated.

Integrated Sidelobe Level (ISL) Comparison

Monte Carlo (1000 trials) comparison of ISL for repeated and randomly reordered waveform sets (excluding zero Doppler sidelobes)

Results are essentially identical

… and virtually unchanged vs. *K* (largest difference is 0.22 dB)



Numerical effects notwithstanding, ambiguity is conserved.

Experimental Setup

Free-space measurements taken from rooftop of Nichols Hall at the University of Kansas

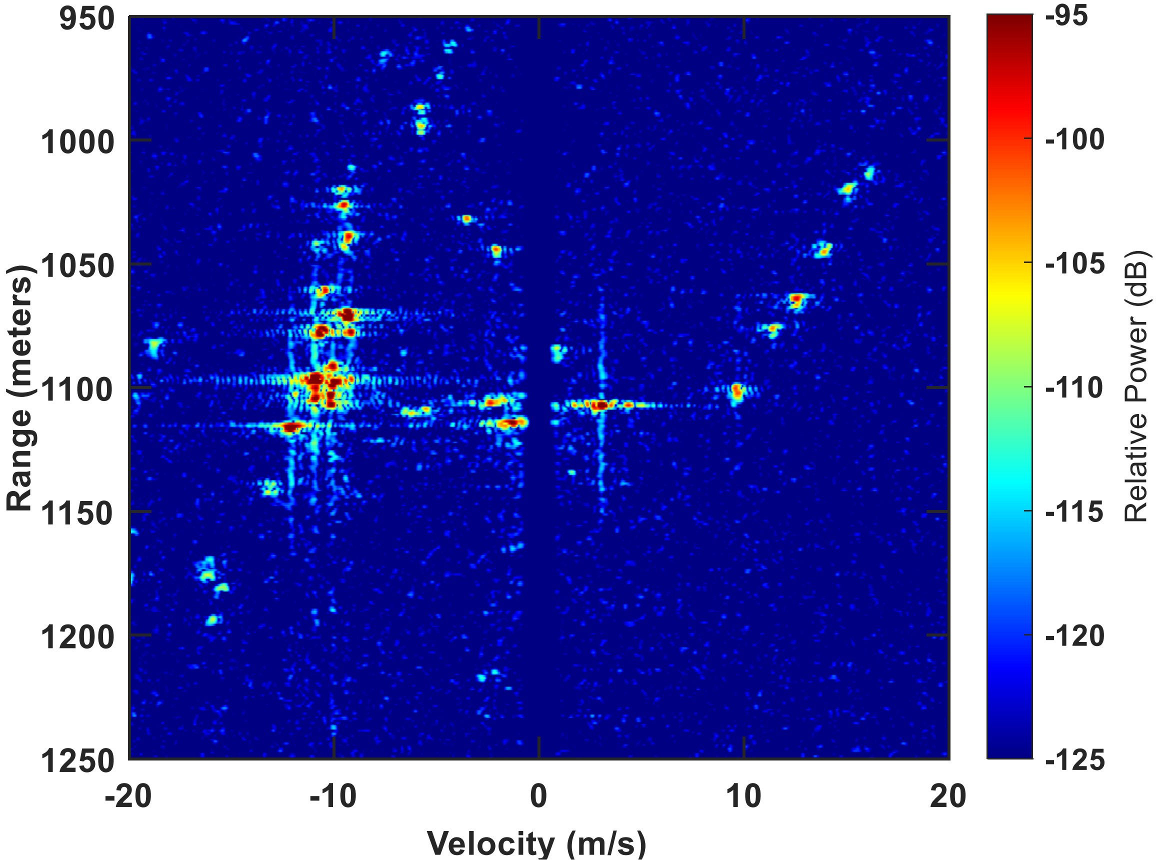
Moving vehicles traverse the intersection of 23rd and Iowa streets. Trees and buildings also in view.

Three cases collected back-to-back (consistent set of movers for comparison)

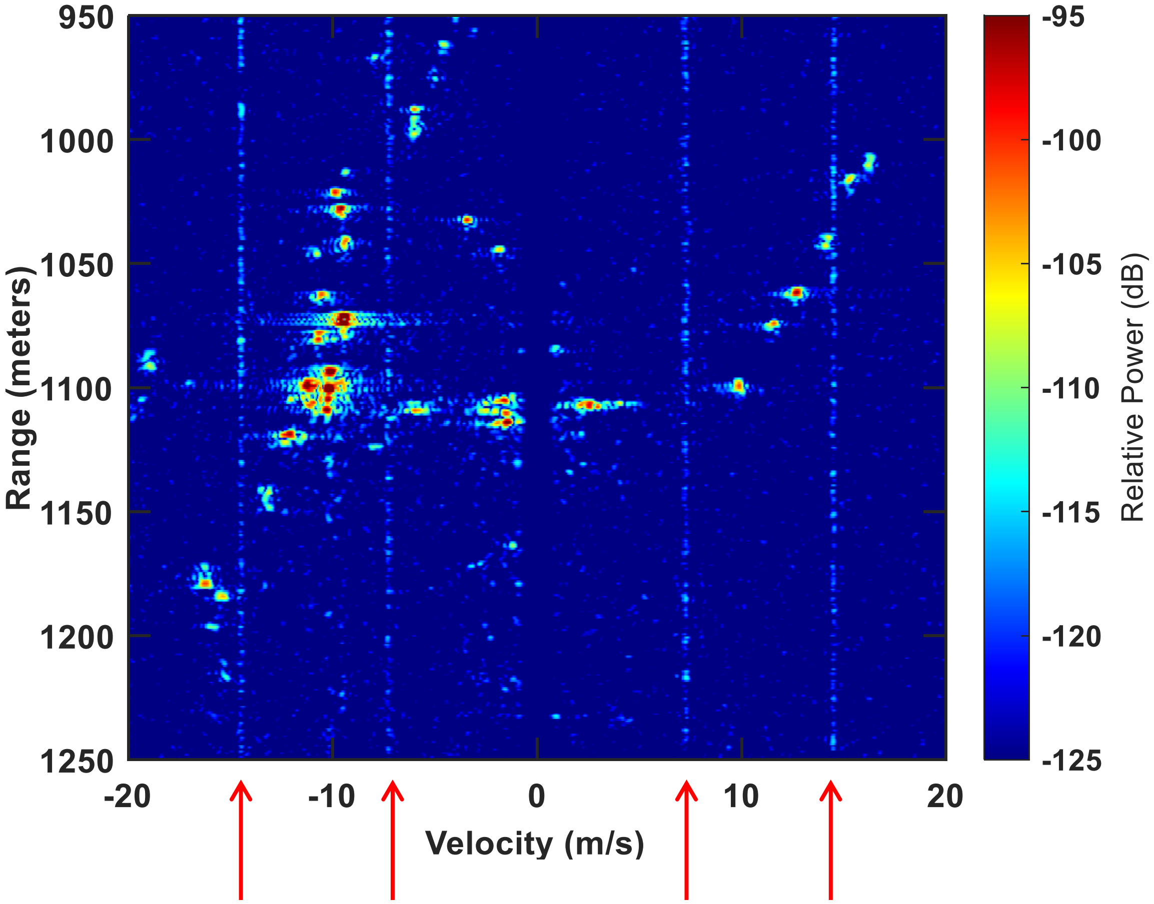
- M = 4000 unique PRO-FM waveforms
- M = 4000 pulses, K = 150 unique PRO-FM waveforms in repeated blocks
- M = 4000 pulses, each waveform randomly selected from the K = 150 unique set

Identically Repeated Waveform Blocks

Range-Doppler response for M = 4,000 nonrepeating waveforms & zero-Doppler clutter cancellation



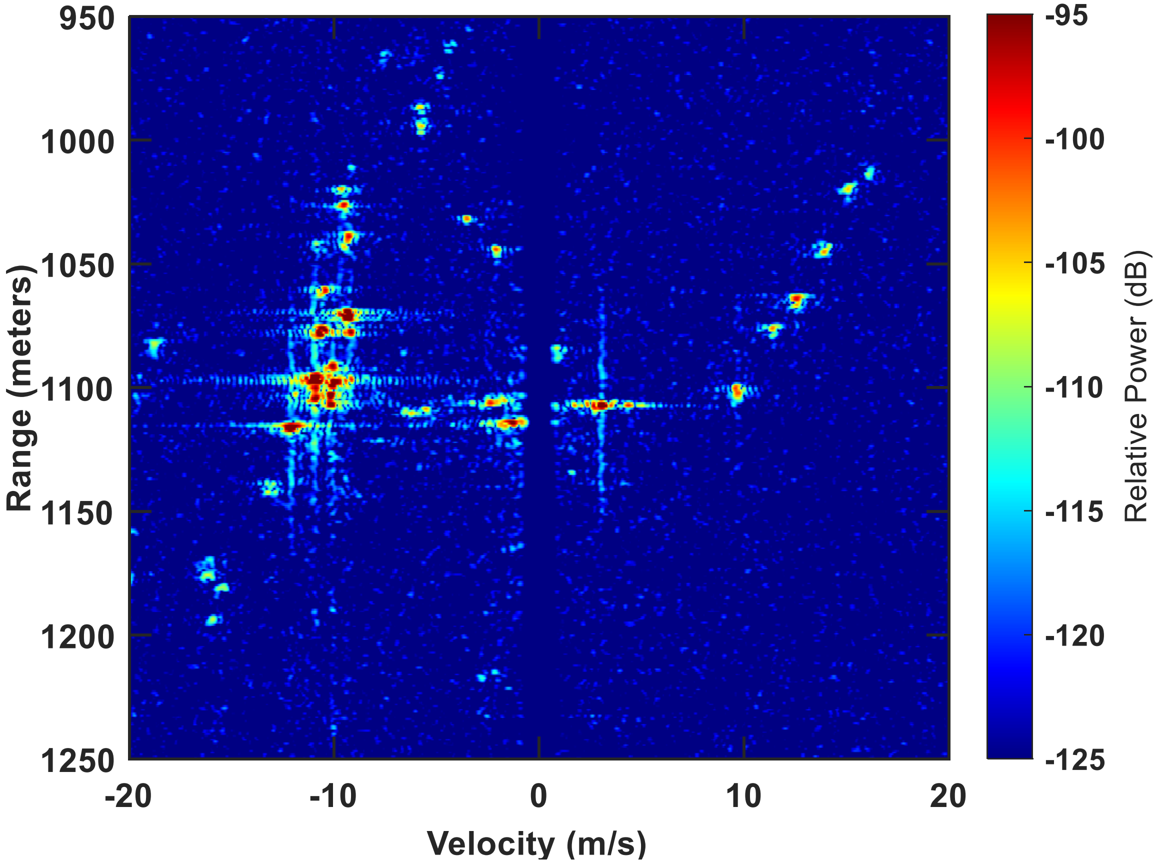
Range-Doppler response for M = 4,000 pulses with identically repeated blocks (K = 150 waveforms) & zero-Doppler clutter cancellation



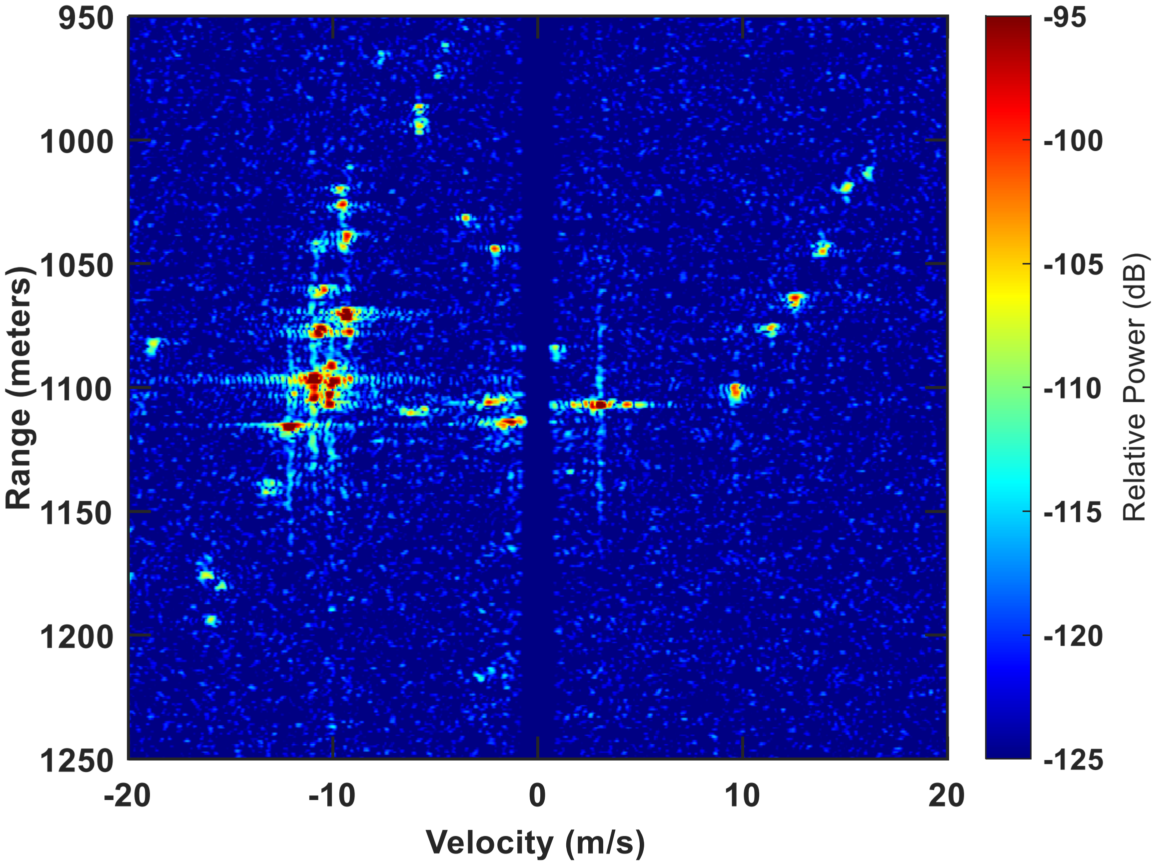
Repeated sinc visible at multiples of ± 7 m/s

Randomized Waveform Selection

Range-Doppler response for *M* = 4,000 nonrepeating waveforms & zero-Doppler clutter cancellation



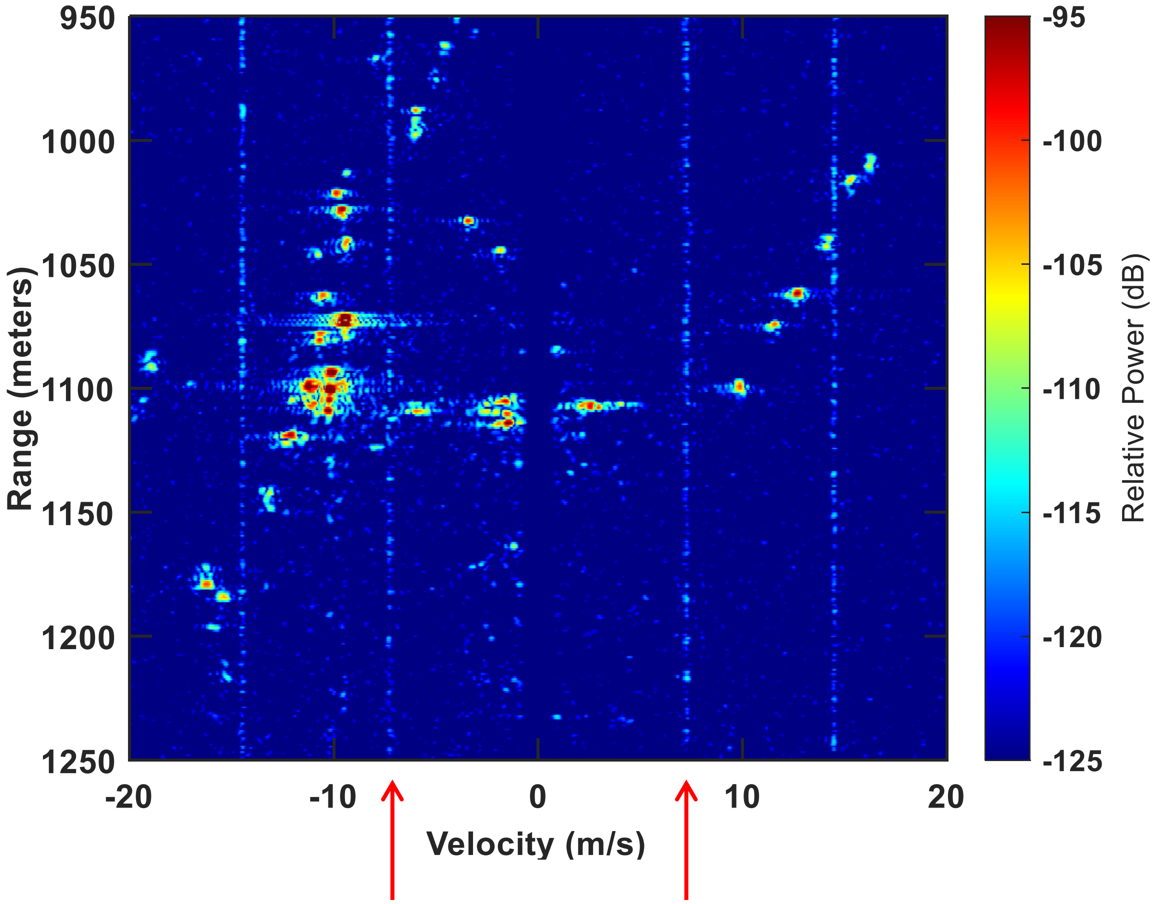
Range-Doppler response for *M* = 4,000 pulses with random selection from *K* = 150 waveforms & zero-Doppler clutter cancellation



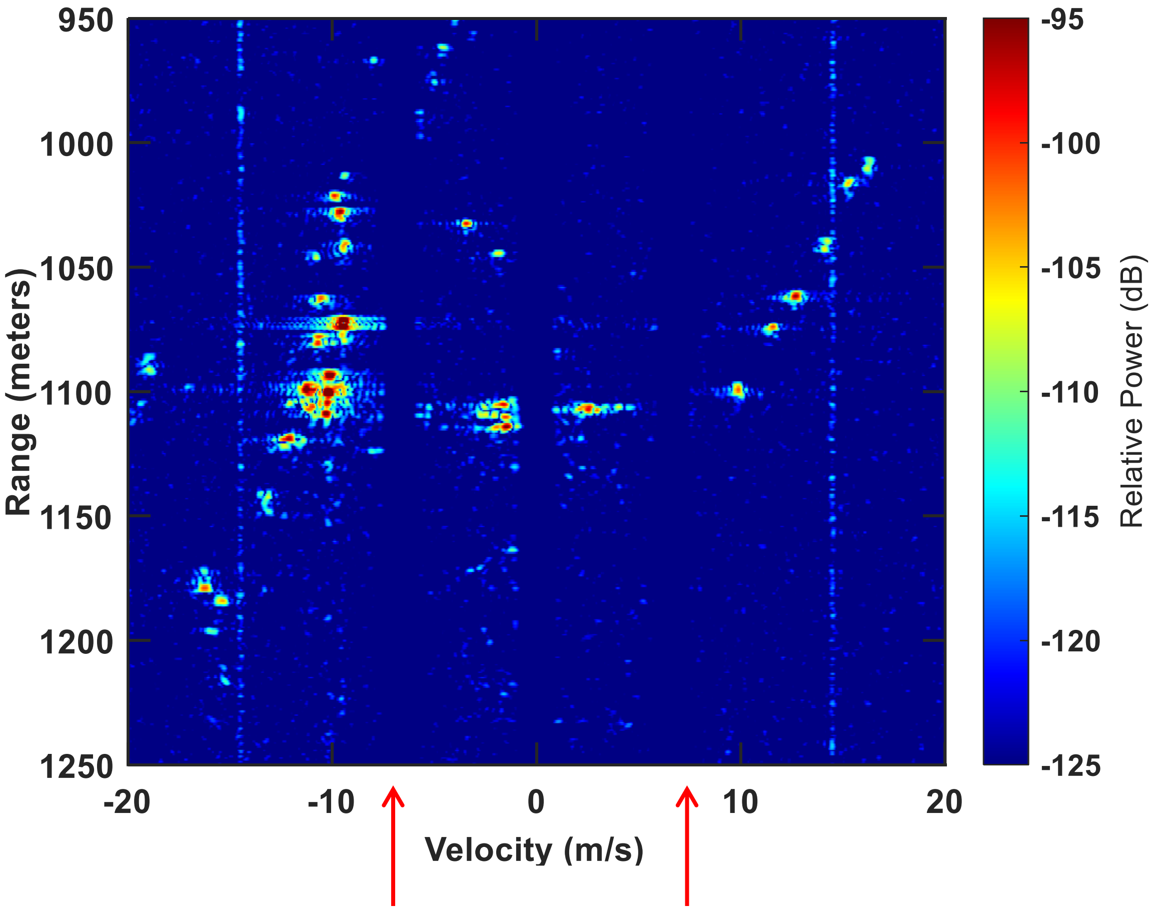
Repeated sinc replaced by slightly higher background (due to PSF pedestal)

Suppressing the Repeated Sinc

Range-Doppler response for *M* = 4,000 pulses with identically repeated blocks (*K* = 150 waveforms) & zero-Doppler clutter cancellation



Range-Doppler response for *M* = 4,000 pulses with identically repeated blocks (*K* = 150 waveforms) & clutter cancellation @ zero-Doppler and **±PRF/*K***



Produces additional blind Dopplers … but introduces a new design trade-space.

Conclusions

- Employing a degree of repetition for nonrepeating random FM waveforms provides a way to address potential limitations on processing and memory.
- Repetition introduces a trade-space involving the concentration vs. distribution of ambiguity caused by range sidelobe modulation (RSM).
- Simulation and free-space measurements demonstrate these trade-offs, and show how these effects could be controlled based on the application.