Time-Frequency Analysis of Spectrally-Notched Random FM Waveforms

Thomas J. Kramer, Shannon D. Blunt

Radar Systems Lab (RSL), University of Kansas

This work was supported by the Office of Naval Research under Contract # N00014-16-C-2029.

Background

- FM noise (or Random FM) waveforms **[1]** provide enough structure to make them amenable to high-power transmitters while also possessing the high dimensionality and non-repetition of traditional noise waveforms.

- Beginning with **[2, 3]**, a variety of recent new design methods and applications have emerged that employ some form of optimization.

- Generally speaking, these optimization approaches rely on (or in some way preserve) desired shaping of the power spectrum (<u>including roll-off</u>), while retaining the intrinsic randomness and FM structure.

[1] S.D. Blunt, J.K. Jakabosky, C.A. Mohr, P.M. McCormick, et al, "Principles & applications of random FM radar waveform design," to appear in *IEEE Aerospace & Electronic Systems Magazine*.

[2] J Jakabosky, S. D. Blunt, B. Himed, "Waveform design and receive processing for nonrecurrent nonlinear FMCW radar," *IEEE Intl. Radar Conf.*, Washington, DC, May 2015.

[3] J. Jakabosky, S.D. Blunt, B. Himed, "Spectral-shape optimized FM noise radar for pulse agility," *IEEE Radar Conf.*, Philadelphia, PA, May 2016.

Problem Motivation

- A particular class/application of random FM waveforms arises from the inclusion of spectral notches to realize a form of cognitive radar (i.e. reduce the interference imposed on other spectrum users) **[4, 5]**
- The principle of stationary phase **[6, 7]** describes an inverse relationship between an FM signal's chirp rate and spectral density … <u>but does that adequately explain spectral notches with 50+ dB depths?</u> (like in **[4, 5]**)
- Here we examine the structure of these notched waveforms using a variety of time-frequency analysis tools to understand: <u>following spectral shaping optimization, by what physical mechanism are notches actually formed?</u>

[4] B. Ravenscroft, J.W. Owen, J. Jakabosky, S.D. Blunt, A.F. Martone, K.D. Sherbondy, "Experimental demonstration and analysis of cognitive spectrum sensing & notching," *IET Radar, Sonar & Navigation*, vol. 12, no. 12, pp. 1466-1475, Dec. 2018.

[5] C.A. Mohr, S.D. Blunt, "Analytical spectrum representation for physical waveform optimization requiring extreme fidelity," *IEEE Radar Conf.*, Boston, MA, Apr. 2019

[6] E.N. Fowle, "The design of FM pulse compression signals," *IEEE Trans. Information Theory*, vol. 10, no. 1, pp. 61-67, Jan. 1964.

[7] N. Levanon, E. Mozeson, *Radar Signals*, Wiley-IEEE Press, 2004.

FM Waveform Structure

- The baseband representation of an arbitrary FM waveform can be expressed as:

$$s(t) = \exp\left(j2\pi \int_{-\infty}^{t} f(\tau)d\tau \right) = \exp\left(j\theta(t) \right)$$

where $f(\tau)$ is the modulating process (e.g. white noise **[8-11]**) and $\theta(\tau)$ is the subsequent **continuous** phase.

- Given a random initialization, spectral shaping (while retaining the FM structure above) greatly improves autocorrelation performance while also enabling the insertion of deep spectral notches.

[8] T.B. Whiteley, D.J. Adrian, "Random FM autocorrelation fuze system," U.S. Patent #4,220,952, issued Sept. 2, 1980, application filed Feb. 17, 1956.

[9] L. Guosui, G. Hong, Z. Xiaohua, S. Weimin, "The present and future of random signal radars," *IEEE Aerospace & Electronic Systems Mag*., vol. 12, no. 10, pp. 35-40, Oct. 1997.

[10] S.R.J. Axelsson, "Noise radar using random phase and frequency modulation," *IEEE Trans. Geoscience & Remote Sensing*, vol. 42, no. 11, pp. 2370-2384, Nov. 2004.

[11] L. Pralon, B. Pompeo, J.M. Fortes, "Stochastic analysis of random frequency modulated waveforms for noise radar systems," *IEEE Trans. Aerospace & Electronic Systems*, vol. 51, no. 2, pp. 1447-1461, Apr. 2015.

Quick Review: Time-Frequency (TF) Transformations

- Two types: linear and nonlinear.
- Linear TF transforms trade **temporal resolution** for **spectral resolution** **[12]**.
- Nonlinear TF transforms trade between **joint time-frequency resolution** and **interference cross-terms** **[13]**.
- The most commonly used linear TF transform is the Short-Time Fourier Transform (STFT), defined as

$$\mathrm{STFT}_s(t,f) = \int_{-\infty}^{\infty} [s(\tau)\gamma^*(\tau - t)]e^{-j2\pi f\tau}d\tau]$$

where γ is a time-limited window function.

[12] F. Hlawatsch, G.F. Boudreaux-Bartels, "Linear and quadratic time-frequency signal representations," *IEEE Signal Processing Mag.*, vol. 9, no. 2, pp. 21-67, Apr. 1992.

[13] B. Boashash, *Time-Frequency Signal Analysis and Processing*, 2016.

Quick Review: Time-Frequency (TF) Transformations

Common nonlinear TF transforms include

- Wigner-Ville Distribution (WVD): $W_s(t,f) = \underset{\tau \to f}{\mathcal{F}} \left\{ s\left(t+\frac{\tau}{2}\right) s^*\left(t-\frac{\tau}{2}\right) \right\}$

- Spectral Correlation Function (SCF): $C_s(\nu,f) = \underset{t \to \nu}{\mathcal{F}} \{W_s(t,f)\}$

- Fractional Fourier Transform (FrFT):

$$\underset{t \to u}{\mathcal{F}^{\alpha}}\{s(t)\} = \sqrt{\frac{1-j\cot(\alpha)}{2\pi}} \exp\{j0.5\cot(\alpha)\,u^2\} \int_{-\infty}^{\infty} \exp\{j(0.5\cot(\alpha)\,t^2 - \csc(\alpha)\,ut)\} s(t)\,dt$$

- Radon-Wigner Transform (RWT):

$$R_s(\alpha,u) = \underset{t \to u}{\mathcal{F}^{\alpha}}\{s(t)s^*(t)\} \text{ for } 0^\circ \leq \alpha \leq 180^\circ$$

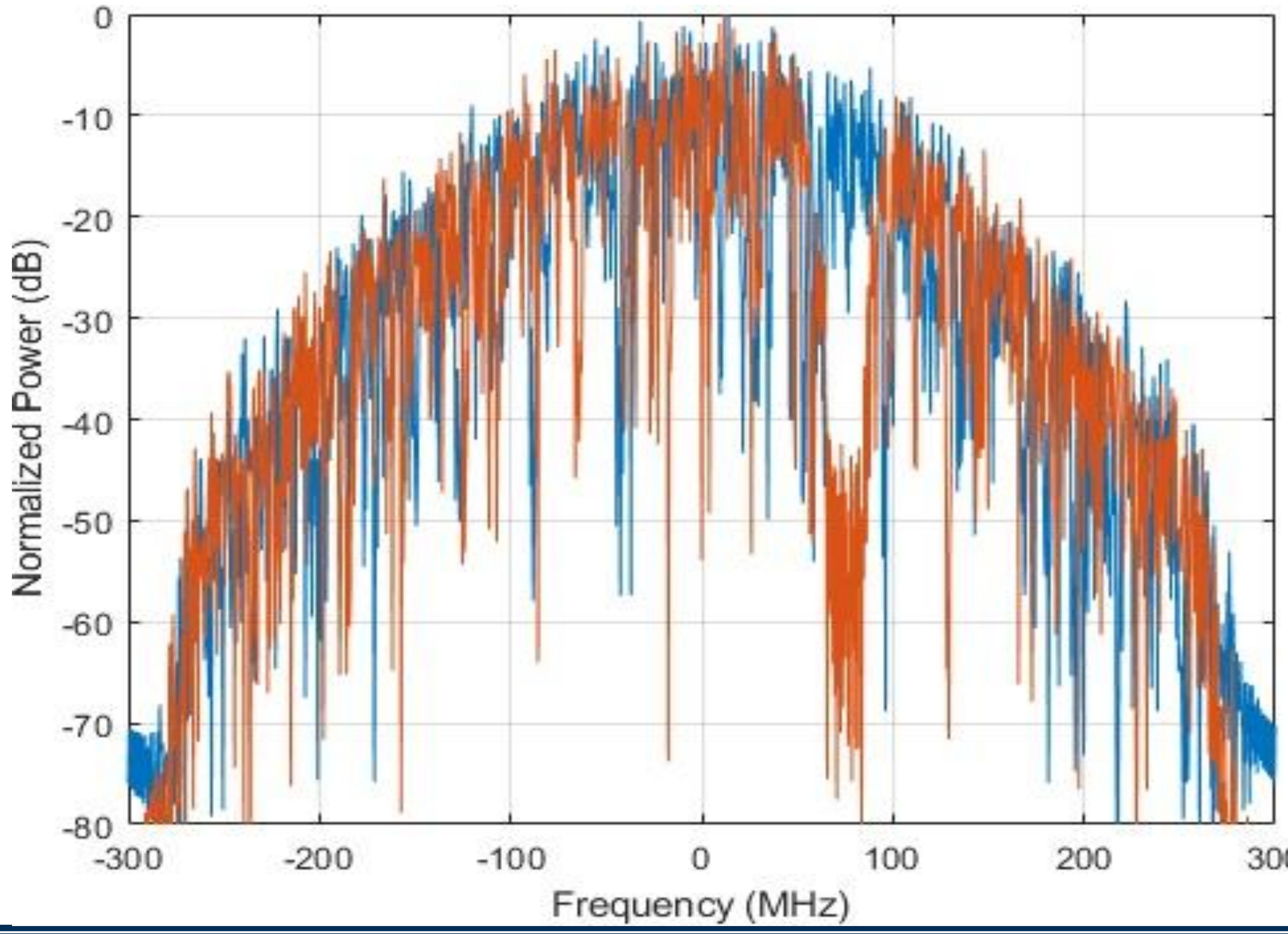
where α and u correspond to the angle and radial variables in the FrFT

Now consider the insights we can glean by applying these various TF transforms to spectrally-notched random FM waveforms …

Power Spectral Density (PSD)

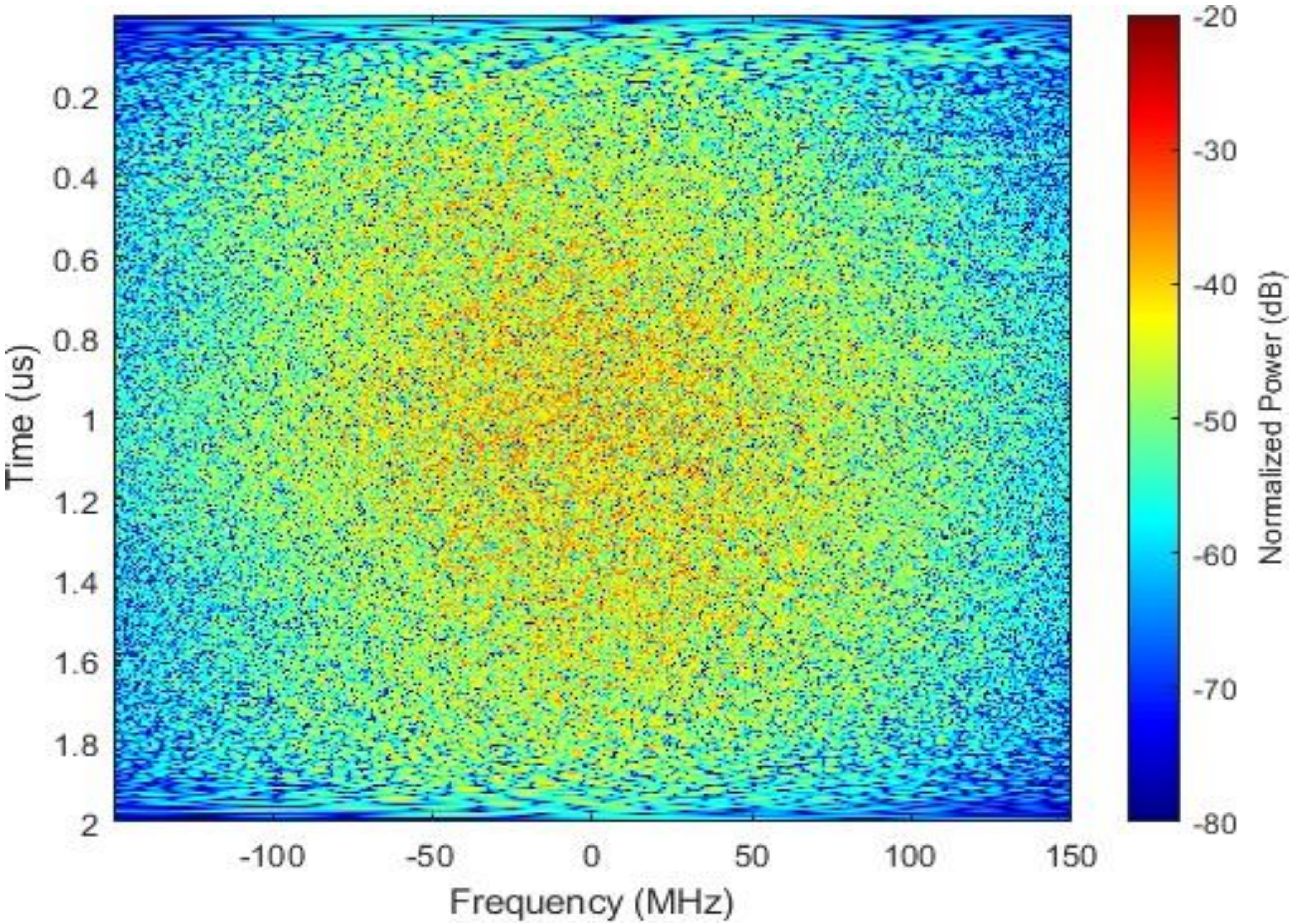
- First compare the measured PSDs of individual random FM waveforms.
- Each is a 2 μs pulse with 3-dB bandwidth of 200 MHz
- In one there is a relatively deep spectral notch clearly evident from 60-90 MHz
- But now let's consider what the TF transforms show.

Baseband PSD of a random FM (blue) and a spectrally-notched random FM (red) measured in loopback



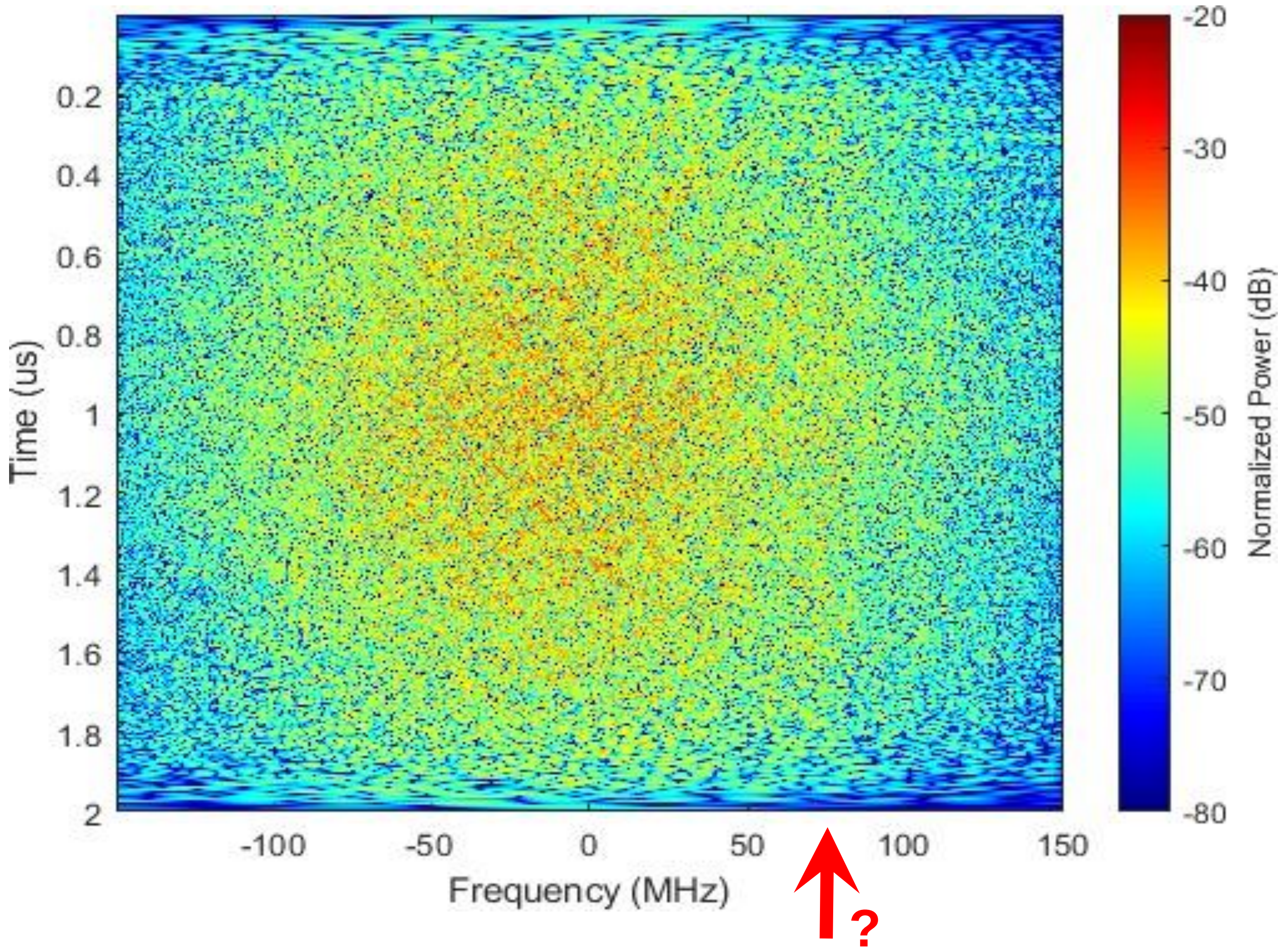
WVD (No Notch)

Let's start with the Wigner-Ville Distribution (WVD)



Single random FM waveform **without** a spectral notch (loopback)

WVD (Notched)

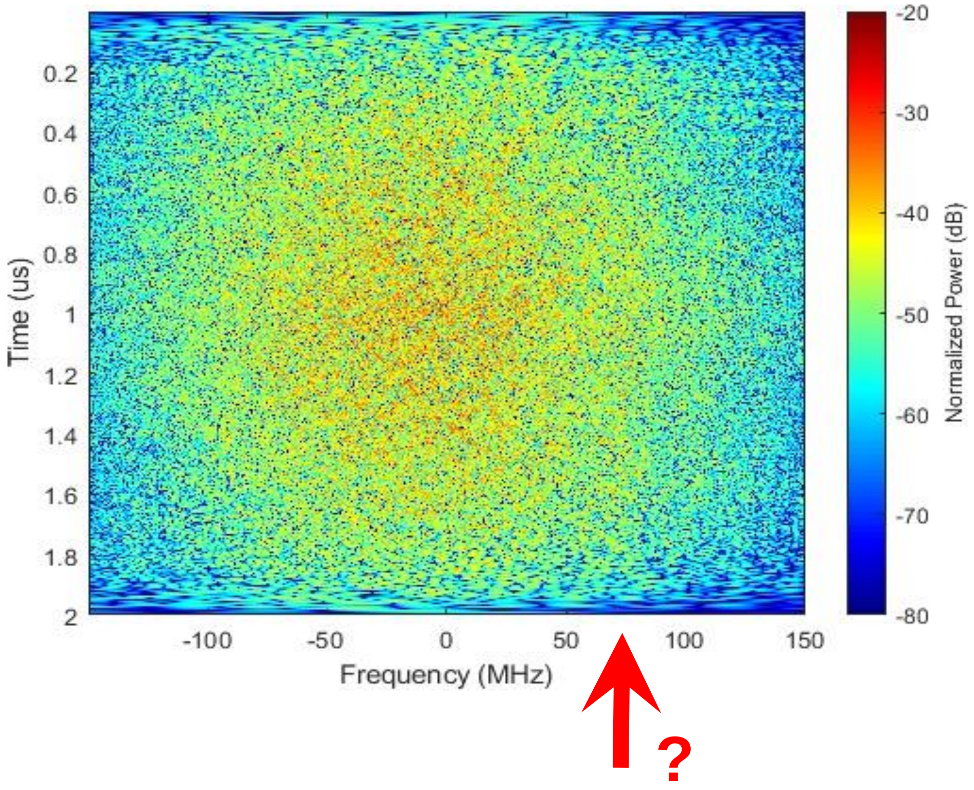


Single random FM waveform **with** a spectral notch (loopback)

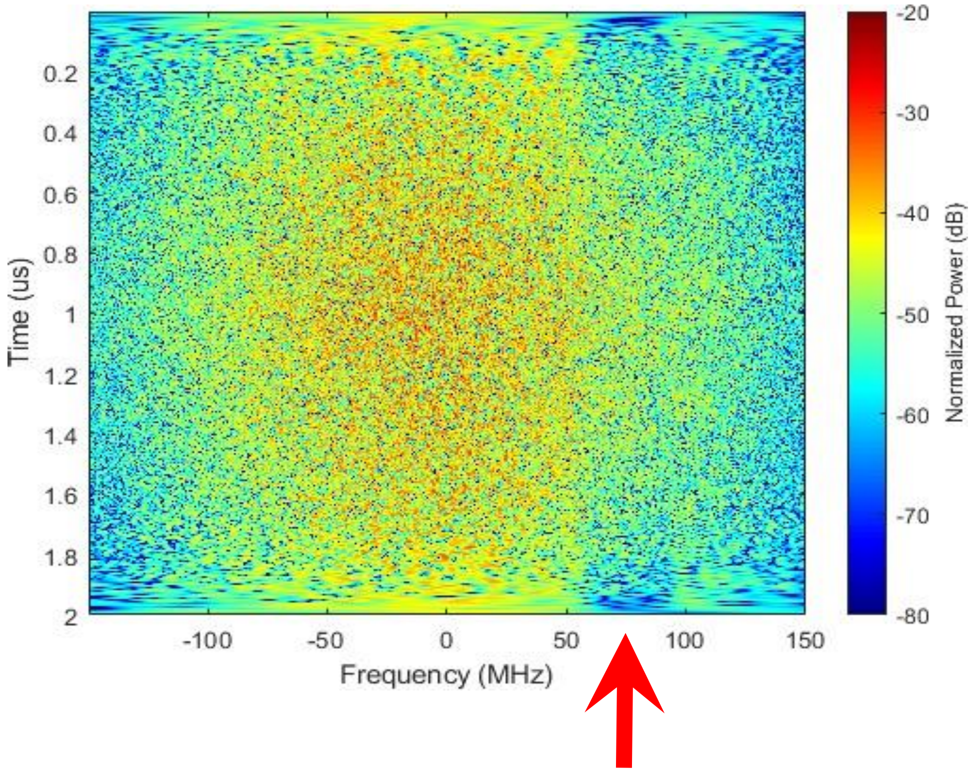
There is no distinguishable difference

WVD (Try averaging them)

Single notched, random FM waveform (loopback)



Average WVD responses over 100 unique random FM waveforms that have the same notch location (loopback)

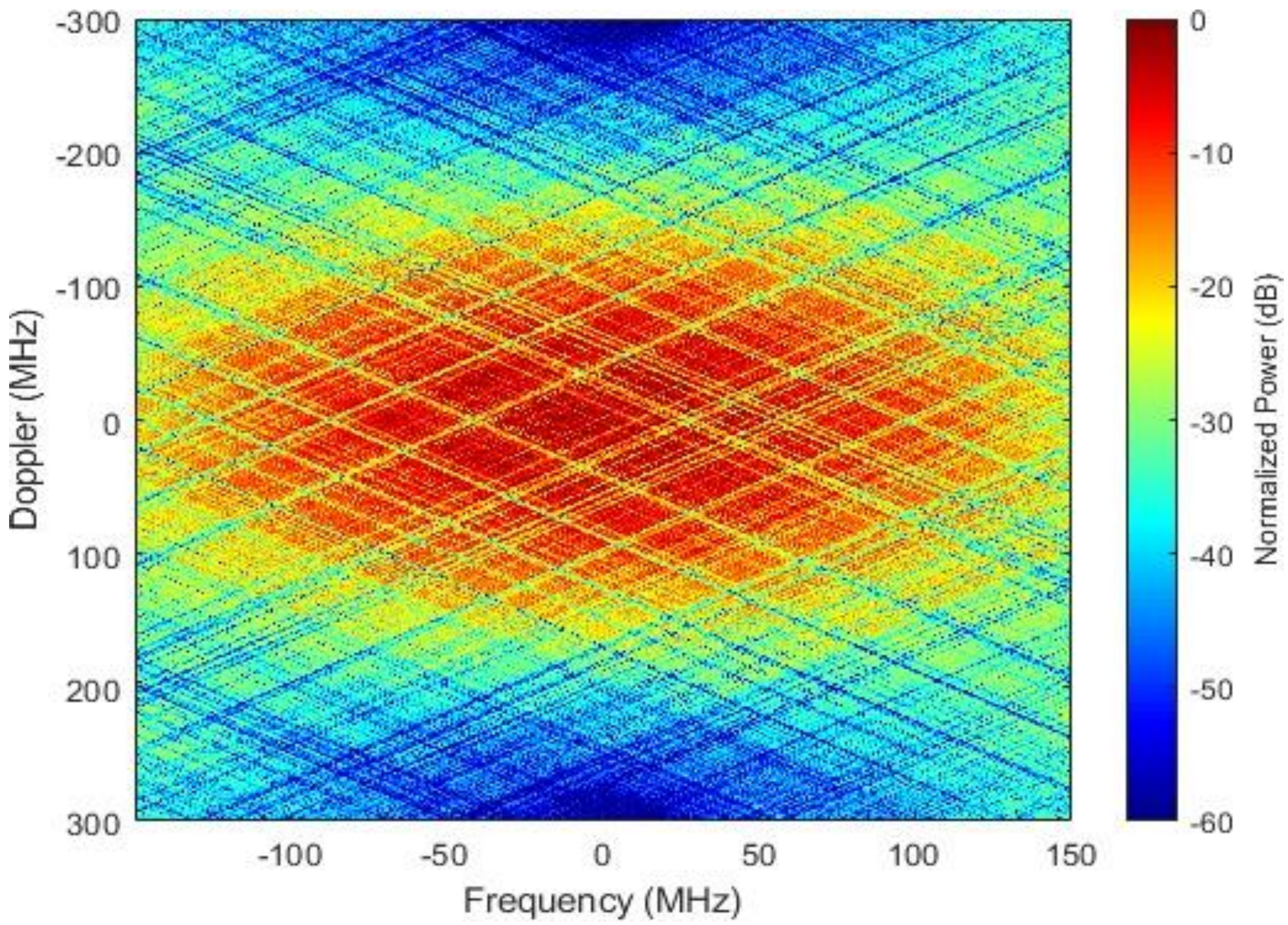


Notch begins to emerge between 60-90 MHz when averaging … but still not very clear.

Observation: Since the WVD measures *instantaneous* spectral content (and notching is not very apparent), suggests notches are based on <u>aggregate spectral content</u>

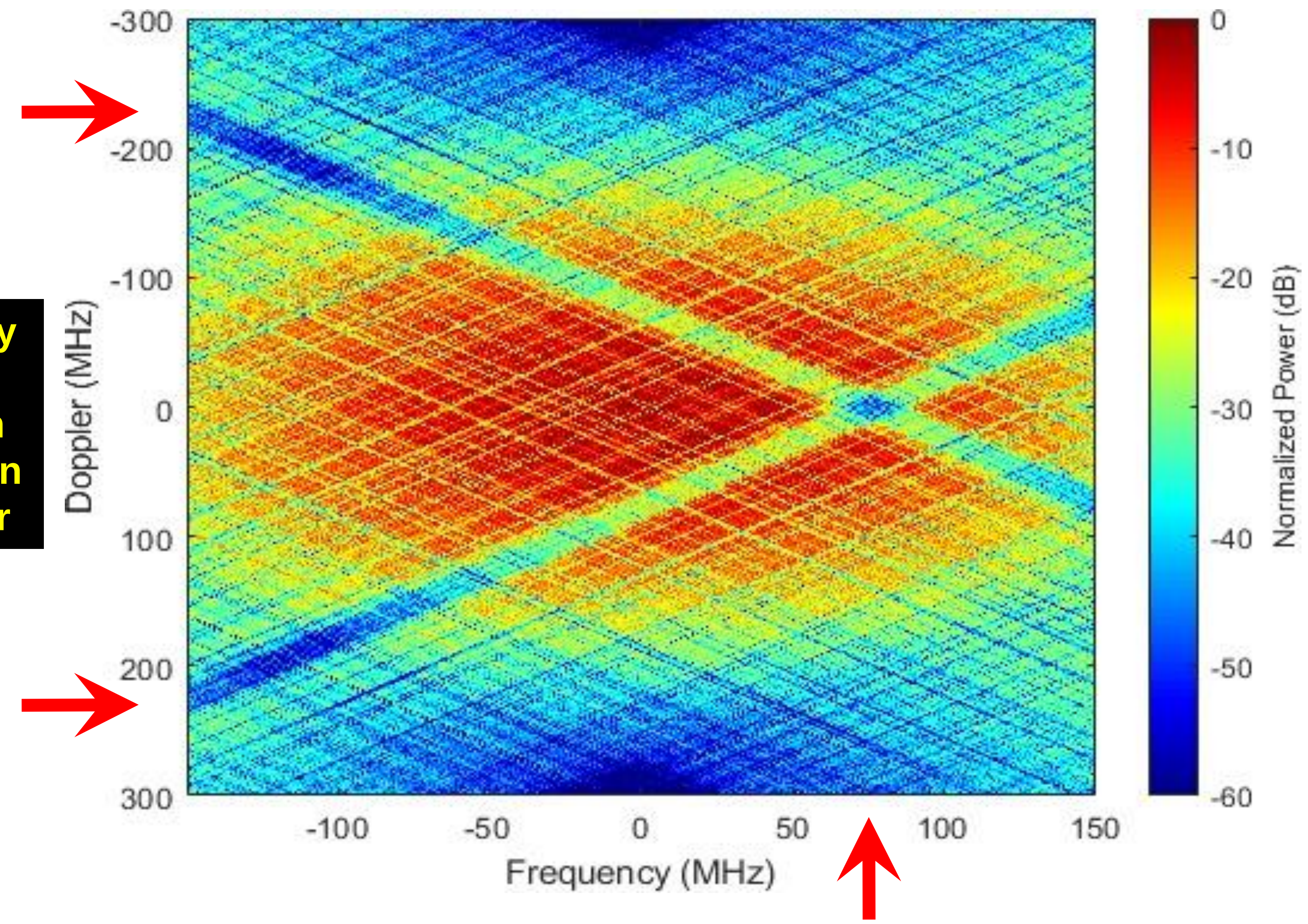
So let's next try the spectral correlation function (SCF), which performs a Fourier transform <u>over the time axis</u>.

SCF (<u>No Notch</u>)



Single random FM waveform **<u>without</u>** a spectral notch (loopback)

SCF (Notched)



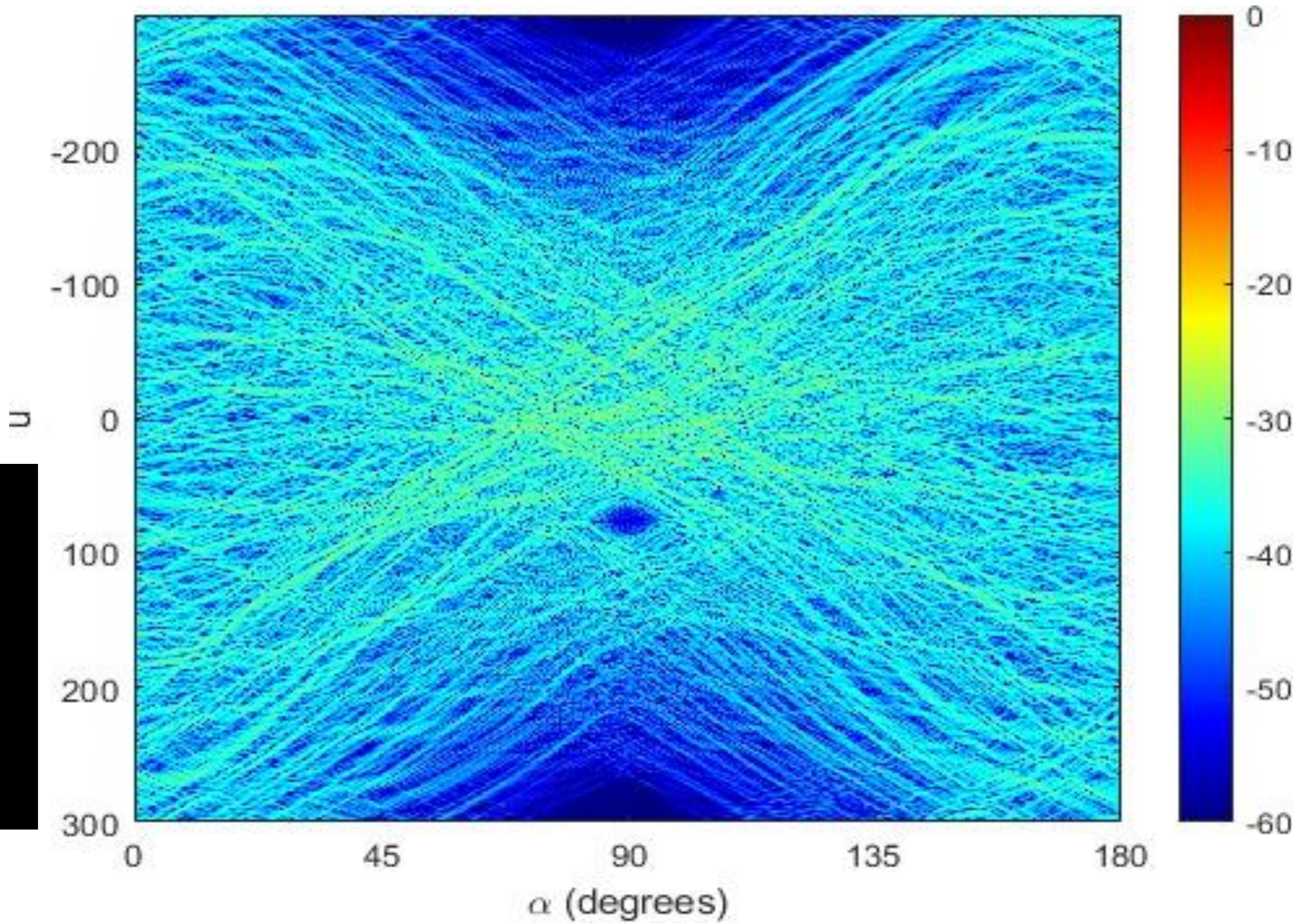
Notch now clearly visible, and spectral location also dependent on fast-time Doppler

Single random FM waveform **with** a spectral notch (loopback)

Observation: Because it considers aggregate spectral content (i.e. in totality over the pulse), the SCF reveals the expected notch … with the expected notch depth only at zero fast-time Doppler

Now examine the Radon-Wigner transform (RWT), which is dependent on the Fraction Fourier transform (FrFT) angle and radial variables α and u

RWT (<u>Notched</u>)



Single random FM waveform **<u>with</u>** a spectral notch (loopback)

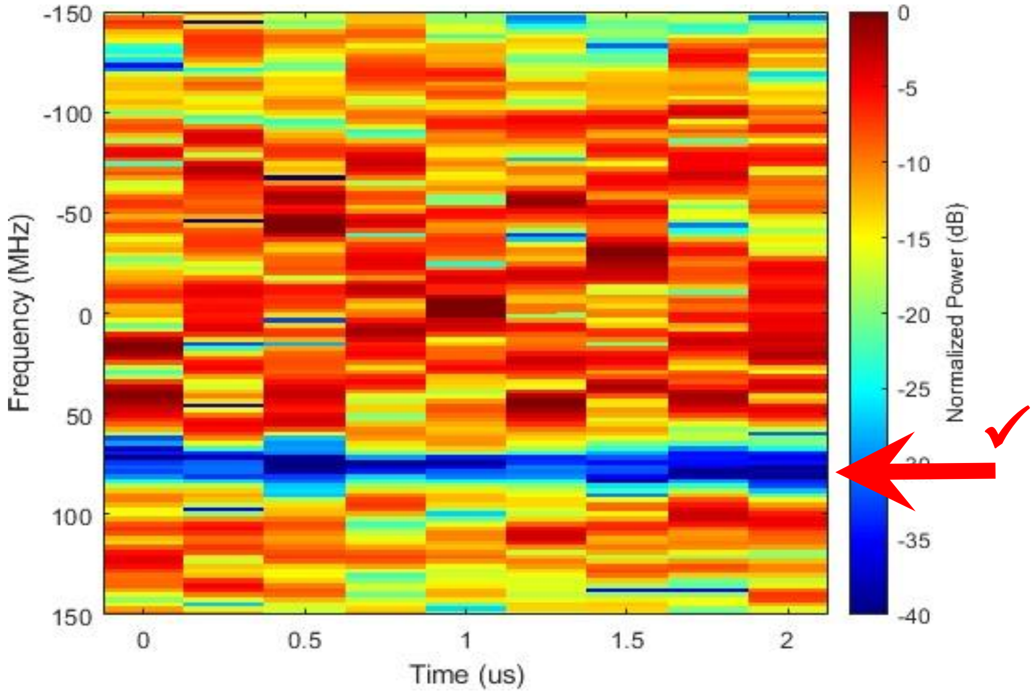
The notch is only visible at α = 90°, which is the special case of FrFT that corresponds to the <u>standard Fourier transform</u>

Observation: The presence of an observable notch is dependent on the time-frequency perspective … in other words, time plays as much a role as frequency.

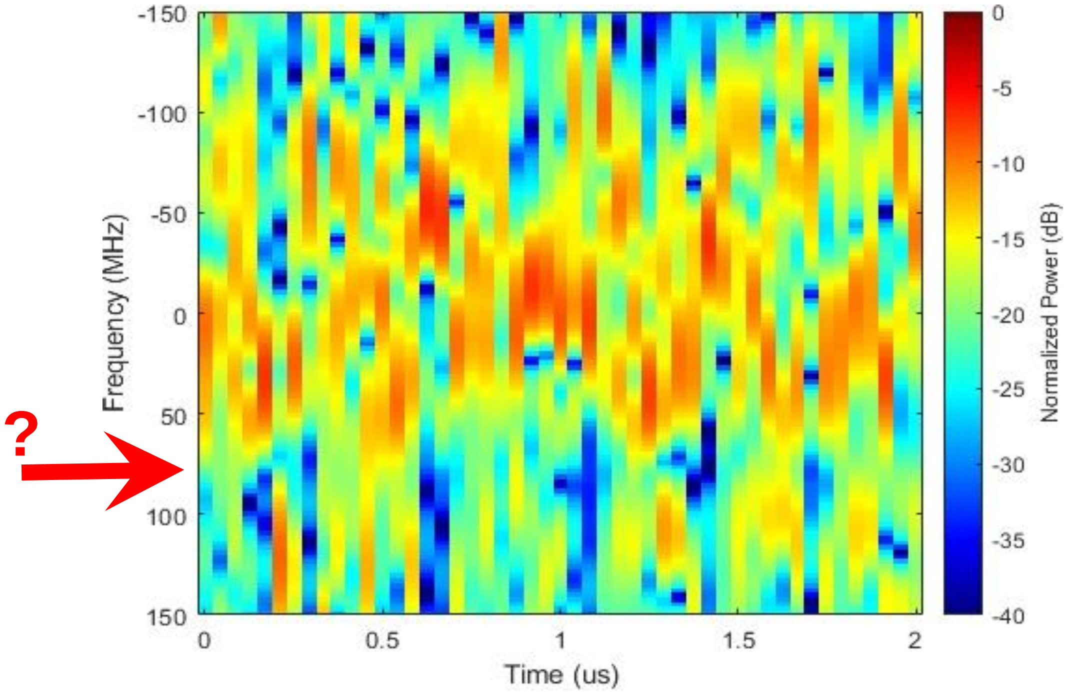
Finally, let's examine the short-time Fourier transform to see how this relationship manifests.

STFT (Notched)

Single random FM waveform **with** a spectral notch (loopback) using 10% time window



Single random FM waveform **with** a spectral notch (loopback) using 2% time window

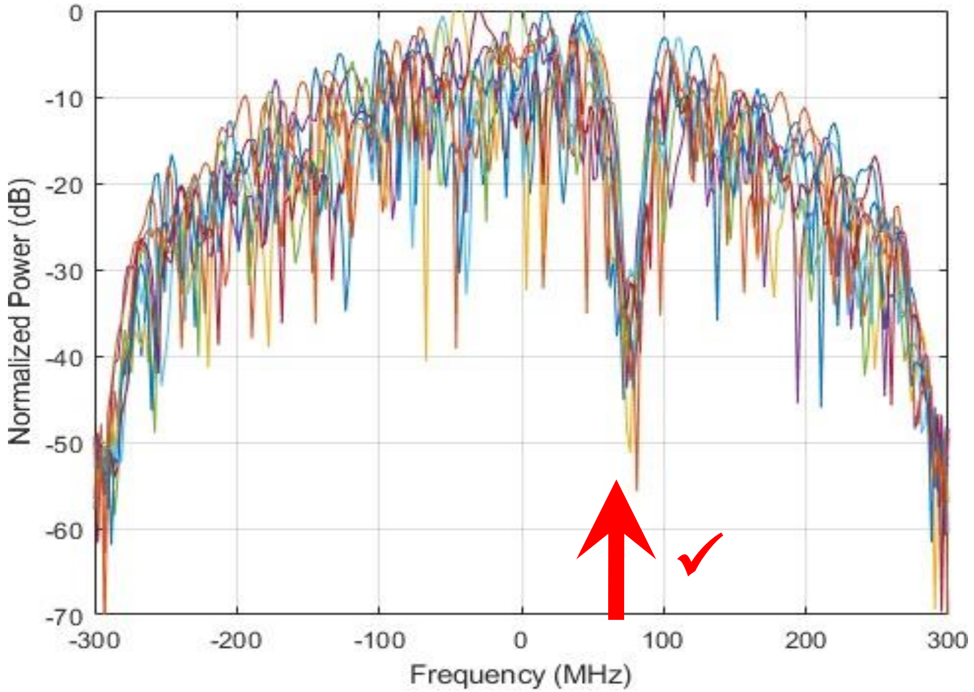


Note: % time window is relative to the 2 μs pulse width

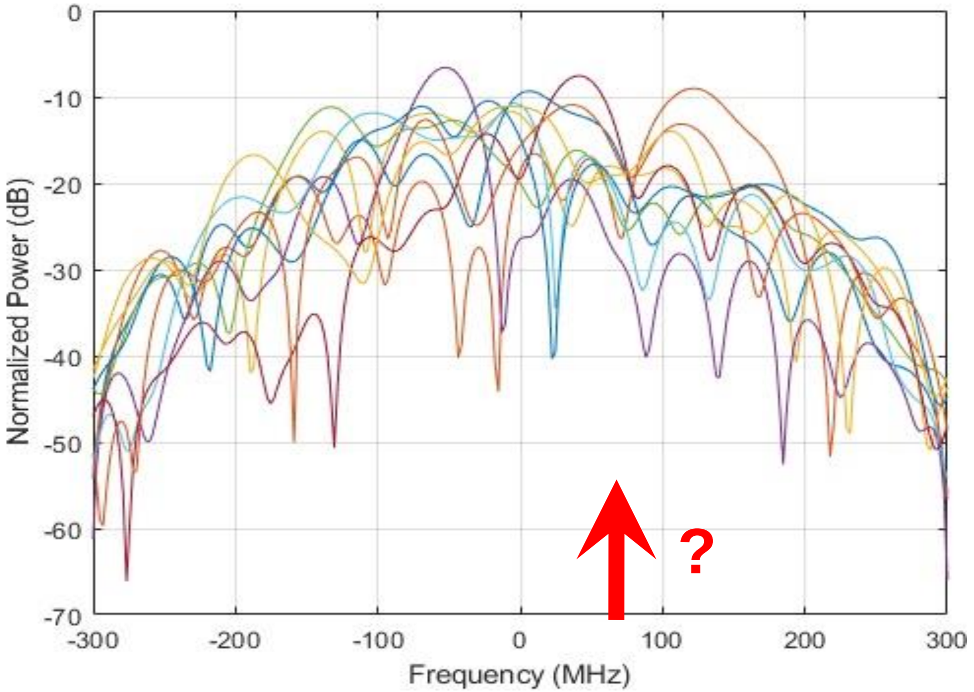
Presence & depth of the notch is highly dependent on the size of the time window over which it is observed.

Cross-section of STFT time-segments

Single random FM waveform **with** a spectral notch (loopback) using 10% time window



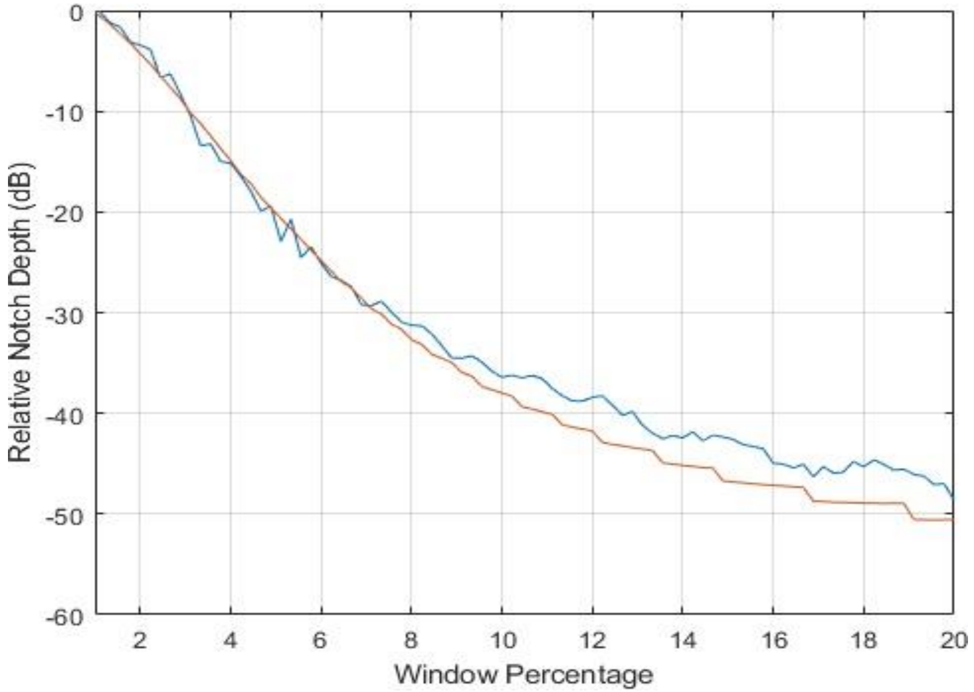
Single random FM waveform **with** a spectral notch (loopback) using 2% time window



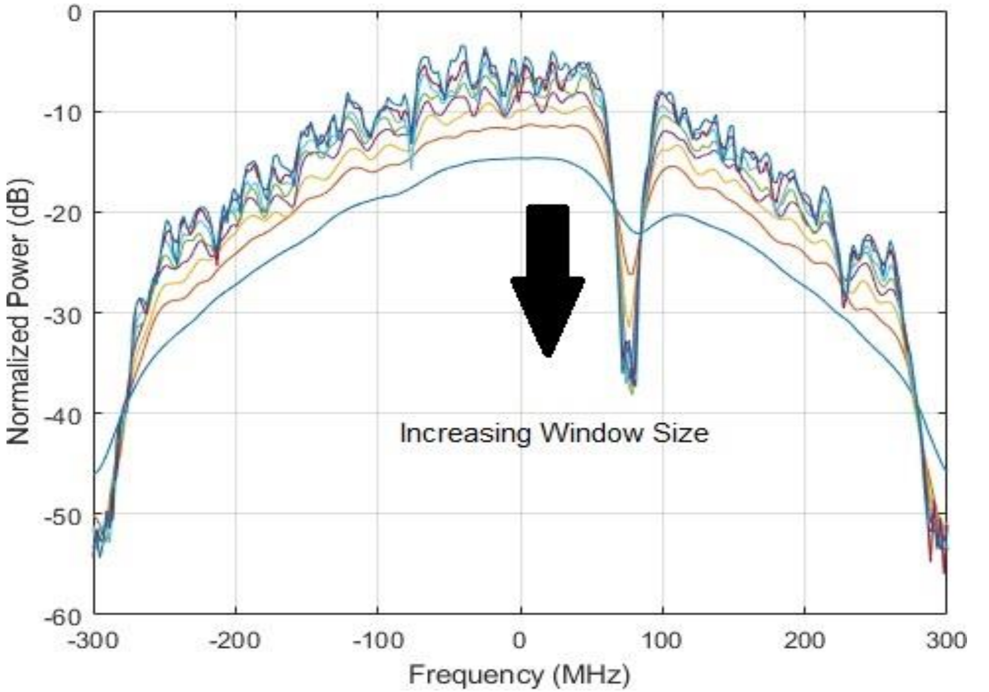
Presence & depth of the notch is highly dependent on the size of the time window over which it is observed.

Notch Depth vs. Window Size

Relative notch depth vs window size as % of pulse width
single waveform (blue trace) and averaged over 1,000 unique waveforms (red trace)



Frequency marginals (sum of STFT over time) as window size increases



Presence & depth of the notch is highly dependent on the size of the time window over which it is observed.

Conclusions

- These observations lead to a departure from the well-known principle of stationary phase (POSP), where the ***monotonic*** instantaneous frequency function is the inverse of the group delay, and there is a one-to-one relationship between them.

- However, random FM waveforms do not have monotonic frequency functions, so the inverse function does not exist.

- Therefore, deep spectral notches are not formed by simply "chirping quickly", but by a cancellation effect over the pulse width.