# Zero-Order Reconstruction Optimization of Waveforms (ZOROW) for Modest DAC Rates

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**This work was supported in part by the Office of Naval Research under Contract #N00014-16-C-2029, by the Army Research Office under Contract #W911NF-15-2-0063, and by a subcontract with Matrix Research, Inc. for the Air Force Research Laboratory under Prime Contract #FA8650-14-D-1722.**
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# Motivation

- A given radar waveform can only be as good as the system used to produce it.
- Compared to high-performance laboratory equipment, software-defined radios (SDRs) have much lower digital-to-analog converter (DAC) rates and are more susceptible to non-ideal hardware effects.
- For radar waveforms that require a high degree of fidelity (i.e. spectrally notched waveforms), these effects can result in significant performance loss.
- To account for these limitations, the Zero-Order Reconstruction Optimization of Waveforms (ZOROW) design approach explicitly accounts for DAC implementations where the signal's 3dB bandwidth is on the order of the DAC rate.

# DAC Operation and Waveform Model

Most DACs generate analog signals as a contiguous sequence of rectangular structures weighted by the values of their input digital samples.

This is a *zeroth-order hold* implementation.

For a pulsed signal, the "ideal" DAC output is:

$$s(t) = \begin{cases} \displaystyle\sum_{n=1}^{N} d_n \mathrm{rect}\left(\frac{t - T_s(n-1/2)}{T}\right) & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

$$\mathrm{rect}(t) = \begin{cases} 1 & -1/2 \le t \le 1/2 \\ 0 & \text{otherwise} \end{cases}$$

digital sample value

rect(•) function duration $T_s = 1/f_s$

signal duration $T = NT_s$

| | |
|---|---|
| $f_s$: | DAC rate |
| $N$: | Number of digital samples |

To generate complex baseband signals, separate I and Q channels are needed. Both channels can be represented by assuming each $d_n$ value is complex.

# DAC Operation and Waveform Model

The spectrum of this signal model is a superposition of complex exponentials weighted by a $\mathrm{sinc}(\cdot)$ function via

$$S(f) = \frac{\sin(\pi f T_s)}{\pi f} \sum_{n=1}^{N} d_n \exp(-j2\pi f(n - 1/2)T_s)$$

The spectral intervals on $[(m - 1/2)f_s, (m + 1/2)f_s], \quad m \in \mathbb{Z}$

are copies or "images" of the fundamental interval $[-f_s/2, +f_s/2]$

These intervals are weighted by the $\mathrm{sinc}(\cdot)$ envelope, meaning the DAC power spectrum is *only guaranteed* to roll-off as quickly as a $\mathrm{sinc}^2(\cdot)$ function … which a quite poor

# DAC Spectral Manipulation

If the bandwidth of the signal (by any meaningful measure of bandwidth) is on the order of the DAC rate $f_s$, significant energy will reside in the spectral images.

DACs address for this in several ways:

Digital interpolation: Prior to analog implementation, the signal is digitally upsampled and filtered (i.e. if the input rate is $\bar{f}_s$, it is upsampled to the DAC rate of $f_s$). This creates more separation between the baseband spectrum and its images.

Inverse sinc filter: Digitally pre-distort the spectrum such that the DAC produces the desired spectrum.

Reconstruction filter: After the initial first-order-hold analog operation, the lowpass analog reconstruction filter suppresses the images. (Digital interpolation also helps here)

From a radar perspective, the digital filters in particular result in deleterious effects.

# DACs and radar signals

- Radar signals tend to have relatively wide bandwidths. A modest input DAC rate on the order of $\bar{f}_s \sim 100$ MHz incentivizes using as much of $\bar{\bar{f}}_s$ as possible for signal content.
  - But doing so magnifies the distortion effects of the DAC digital and analog filters.
- Radar signals tend to be high in power, which incentivizes constant amplitude waveforms AND operating at the full-scale output of the DAC.

Thus DAC filtering inevitably induces unwanted amplitude modulation, which leads to reduced power efficiency and, even worse, amplitude clipping (nonlinear distortion).

# The ZOROW signal model

ZOROW time-domain signal:

$$s(t;\boldsymbol{\phi}) = \begin{cases} \sum_{n=1}^{N} \exp(j\phi_n)\ \mathrm{rect}\left(\dfrac{t-\bar{T}_s(n-1/2)}{T}\right) & 0 \le t \le T \\ 0 & \text{otherwise} \end{cases}$$

signal is constant amplitude

input DAC interval $\bar{T}_s = 1/\bar{f}_s$

digital phase sequence

$$\boldsymbol{\phi} = [\phi_1 \ \ \phi_2 \ \ \cdots \ \ \phi_N]^T$$

ZOROW spectrum:

$$S(f;\boldsymbol{\phi}) = \frac{\sin(\pi f \bar{T}_s)}{\pi f} \sum_{n=1}^{N} \exp(-j(2\pi f(n-1/2)\bar{T}_s) + \phi_n)$$

model accounts for sinc envelope

The ZOROW model explicitly accounts for the DAC input rate and sinc spectral envelope

# The ZOROW signal model

To facilitate representation of the ZOROW spectrum on a computer, it needs to be discretized as

$$S(f_m;\boldsymbol{\phi}) = \frac{\sin(\pi f_m \bar{T}_s)}{\pi f_m} \sum_{n=1}^{N} \exp(-j(2\pi f_m (n-1/2)\bar{T}_s) + \phi_n) \quad \text{for} \quad f_m = m\Delta f$$

$$\Delta f \leq \frac{1}{2T}$$

Since the Nyquist sampling theory is reversible **[1]**, this condition guarantees the ZOROW time domain signal can be perfectly reconstructed from its sampled spectrum

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Further, since all frequency intervals beyond the fundamental interval $-\bar{f}_s/2 \leq f \leq +\bar{f}_s/2$ are scaled images …

… the analytical ZOROW spectrum can be unambiguously represented by discretizing only the fundamental interval, and thus the required number of samples is $M = 2T\bar{f}_s - 1$.


[1] C.A. Mohr, S.D. Blunt, 'Analytical spectrum representation for physical waveform optimization requiring extreme fidelity," *2019 IEEE Radar Conf.*, Boston, MA, Apr. 2019.

# ZORQW Spectral Notching

The signals most sensitive to distortion are those requiring the most precision, such as when achieving extremely low range sidelobes or deep spectral notches.

To realize deep spectral notches, the cost function

$$J = \sum_{m} |S(f_m; \boldsymbol{\phi})|^2$$

can be defined in which $f_m$ , for particular values of $m$, span some interval(s) $[f_{\min}\ f_{\max}]$ where spectral notch(es) are desired.

$J$ sums the power in these intervals, so by minimizing $J$ the power in the spectral notch(es) is minimized.

So how to minimize $J$?

# Gradient-Based Optimization

- Because the cost function is a continuous function of $\boldsymbol{\phi}$, a gradient can be calculated.

- Here a Heavy Ball gradient-descent method is used to minimize the cost function
  - A good compromise between convergence rate, algorithmic complexity, and stability
  - Search direction can be reset to steepest descent direction if found to be an ascent direction

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**Gradient-descent structure**

$$\boldsymbol{\phi}_{i+1} = \boldsymbol{\phi}_i + \mu_i \mathbf{p}_i$$

$$\mathbf{p}_i = \begin{cases} -\nabla_{\boldsymbol{\phi}_i} J & \text{when } i = 0 \\ -\nabla_{\boldsymbol{\phi}_i} J + \beta \mathbf{p}_{i-1} & \text{otherwise} \end{cases}$$

| | |
|---|---|
| $\boldsymbol{\phi}_i$ | parameters to be optimized at the $i^{\text{th}}$ iteration |
| $\mu_i$ | step-size at the $i^{\text{th}}$ iteration |
| $\nabla_{\boldsymbol{\phi}_i} J$ | gradient at the $i^{\text{th}}$ iteration |
| $\beta$ | Heavy Ball parameter, where $0 \leq \beta < 1$ |

# ZOROW Spectral Notching Gradient

The gradient with respect to the phase sequence $\boldsymbol{\phi}$ is:

$$\nabla_{\boldsymbol{\phi}} J = 2\Im\left\{\sum_{m}\left(\nabla_{\boldsymbol{\phi}} S(f_m;\boldsymbol{\phi})\right)^{*} S(f_m;\boldsymbol{\phi})\right\}$$

$$\nabla_{\boldsymbol{\phi}} S(f_m;\boldsymbol{\phi}) = \left[\frac{\partial S(f_m;\boldsymbol{\phi})}{\partial \phi_1} \quad \cdots \quad \frac{\partial S(f_m;\boldsymbol{\phi})}{\partial \phi_N}\right]^{\boldsymbol{T}}$$

$$\frac{\partial S(f_m;\boldsymbol{\phi})}{\partial \phi_n} = j\,\frac{\sin(\pi f_m \bar{T}_{\mathrm{s}})}{\pi f_m}\exp\left(j\left(\phi_n - 2\pi f_m (n-1/2)\bar{T}_{\mathrm{s}}\right)\right)$$

The companion paper **[2]** demonstrates a rather efficient implementation of ZOROW for real-time cognitive interference avoidance.

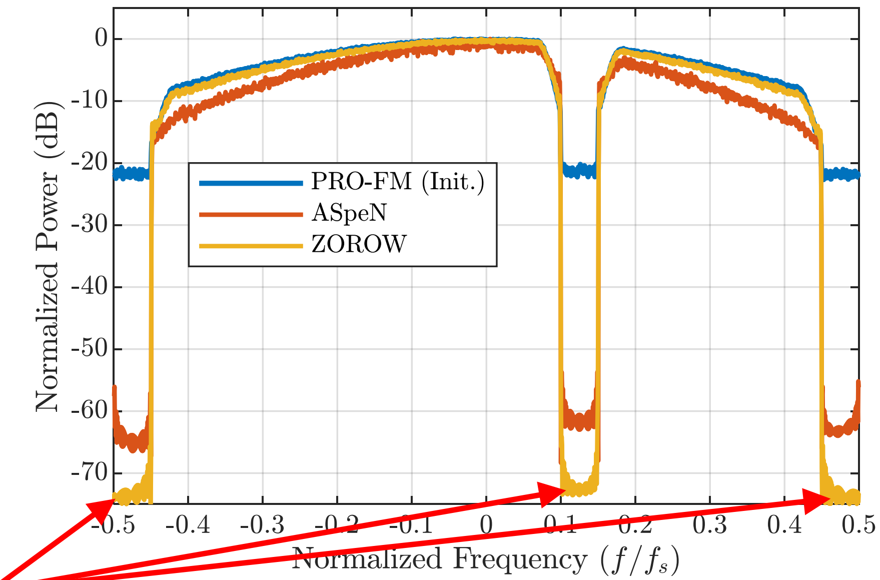

[2] J.W. Owen, C.A. Mohr, B.H. Kirk, S.D. Blunt, A.F. Martone, K.D. Sherbondy, "Demonstration of real-time cognitive radar using spectrally-notched random FM waveforms," *2020 IEEE International Radar Conf.*, Washington, DC, Apr. 2020.

# Optimization Results

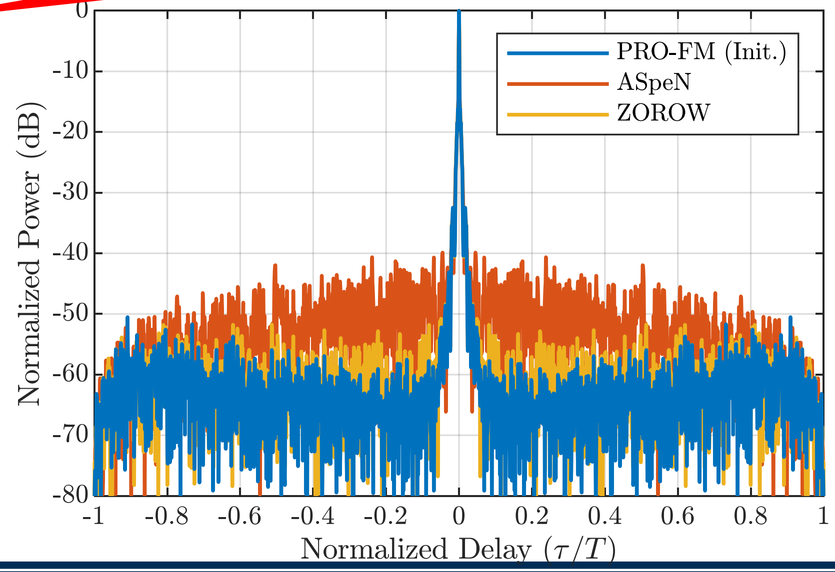
Since ZOROW only creates spectral notches it is necessary to initialize with existing waveforms

- PRO-FM (Pseudo-Random Optimized) random FM [3] waveforms can only achieve shallow spectral notches. They are used to initialize ASpeN and ZOROW and
- ASpeN (Analytical Spectral Notching) [1] is the predecessor to ZOROW has been experimentally shown to produce notches as deep as –57 dB on high DAC rate test equipment (in loopback)
- ZOROW can achieve –73 dB notch depth (in simulation)

Mean spectra and autocorrelation of 1000 waveforms for each case




Simulated (Ideal) Spectral Notches



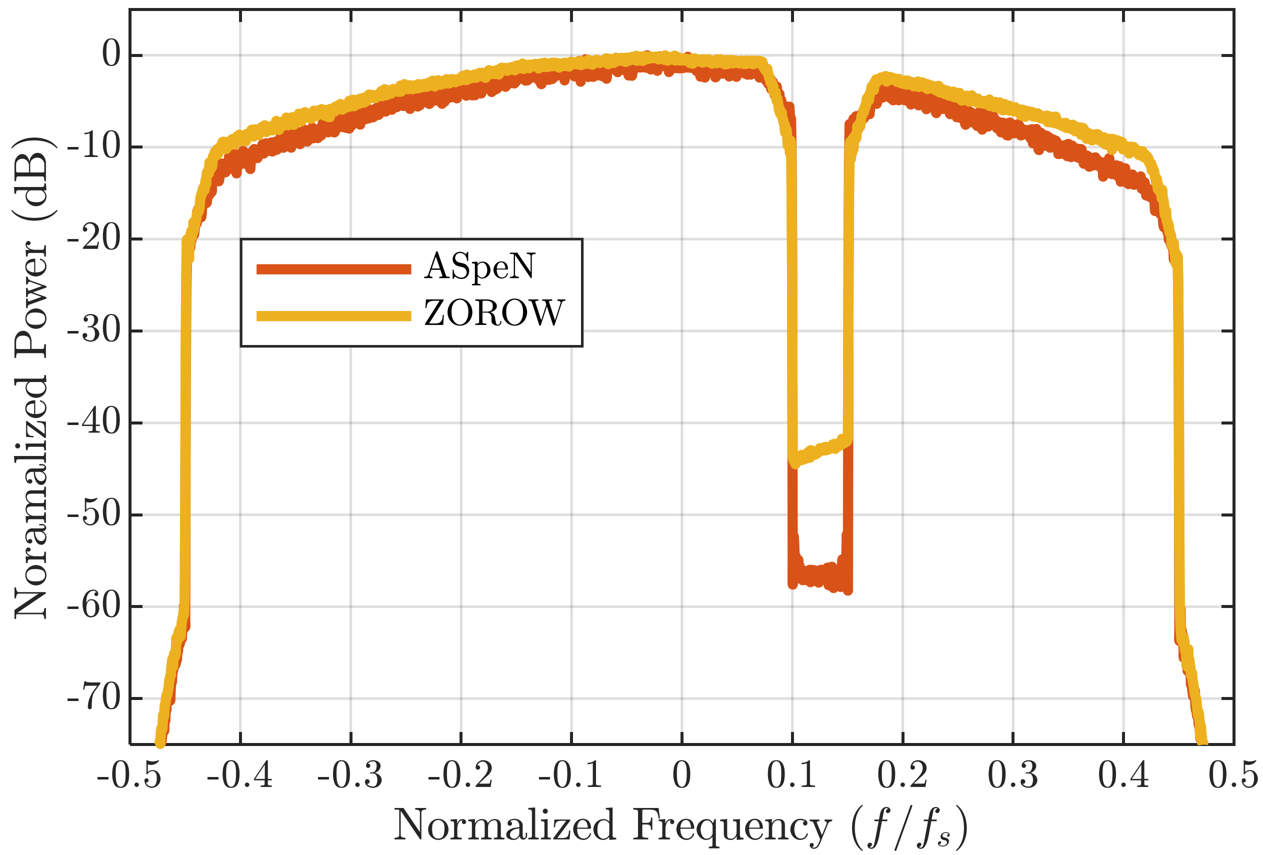

[3] J. Jakabosky, S. D. Blunt, B. Himed, "Spectral-shape optimized FM noise radar for pulse agility," *2016 IEEE Radar Conf.*, May 2016.

# Loopback Experiments: High-Performance AWG

On a high performance AWG (10 GHz DAC rate compared to 200 MHz 3-dB bandwidth):

- ASpeN reaches –57 dB notch depth
- ZOROW performs poorly and loses ~33 dB of notch depth (compared to sim)

- But ZOROW was not designed for such a high DAC rate. **<u>This is model mismatch</u>**
- High performance is <u>only</u> achieved with the right implementation.




**Let's see how ZOROW performs with the implementation it was designed for …**

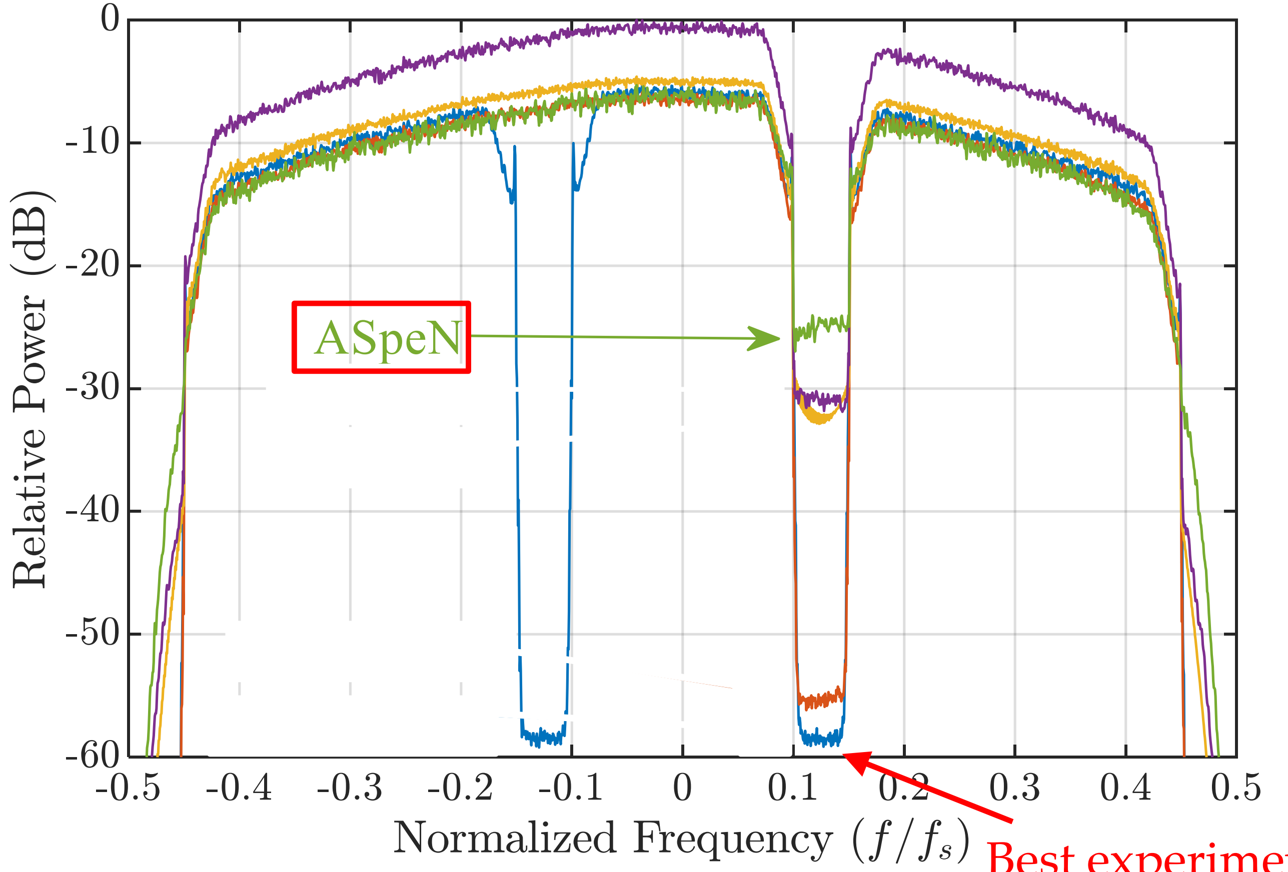
# Loopback Experiments: X310 SDR

The goal of these tests is to isolate the impact of various non-ideal hardware effects as a function of physically realizable notch depth in loopback.

**ASPeN** - with Tukey taper, subscale output

- Tukey taper reduces distortion from fast pulse rise/fall times
- Subscale output reduces distortion from amplitude clipping (lower overall power)
- ASPeN is designed to realize extremely deep notches on high DAC rate equipment. (experimentally shown down to –57 dB).

Here, model mismatch produces **~30 dB loss in notch depth**.



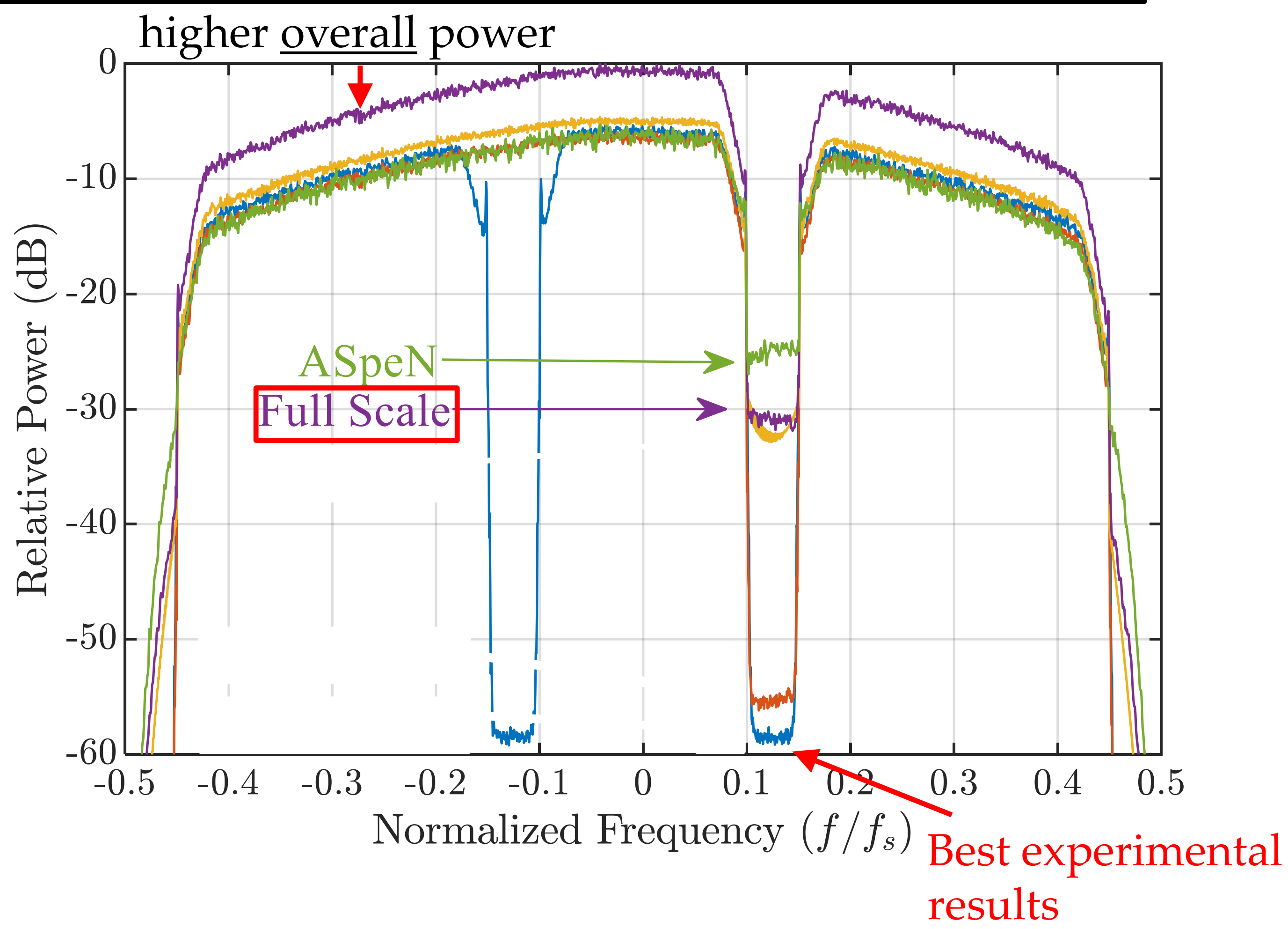

Best experimental results

# Loopback Experiments: X310 SDR

The goal of these tests is to isolate the impact of various non-ideal hardware effects as a function of physically realizable notch depth in loopback.

**ZOROW** - with Tukey taper, **full-scale** output

- Tukey taper reduces distortion from fast pulse rise/fall times
- Full-scale output and digital interpolation filter result in amplitude clipping. (but higher overall power)



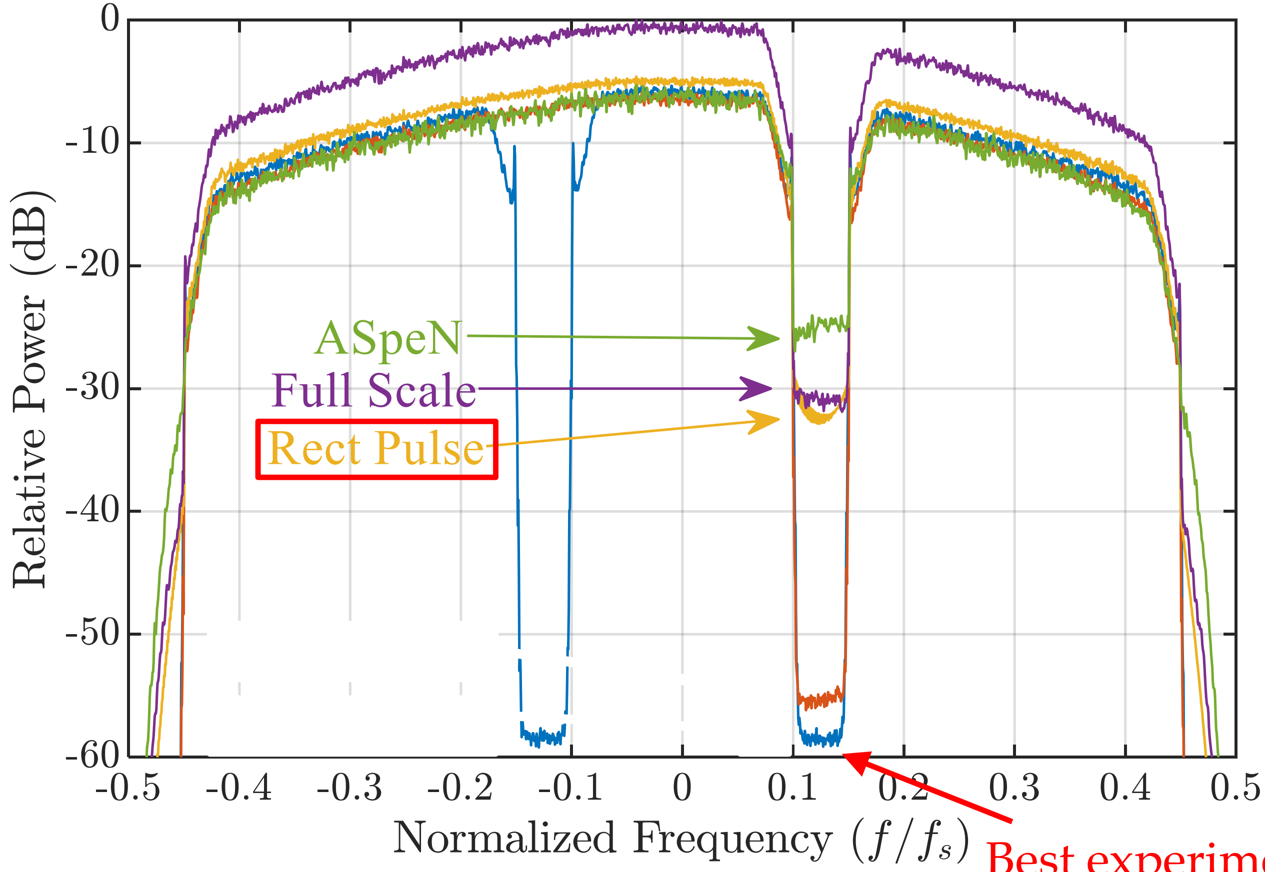

Amplitude clipping results in ~28 dB notch depth loss vs. best case.

# Loopback Experiments: X310 SDR

The goal of these tests is to isolate the impact of various non-ideal hardware effects as a function of physically realizable notch depth in loopback.

**ZOROW** - no Tukey taper, subscale output

- Tukey taper reduces distortion from fast pulse rise/fall times
- Full-scale output and digital interpolation filter result in amplitude clipping. (but higher overall power)




Best experimental results

Even at subscale output (minimal amplitude clipping), the fast rise/fall times result in ~27 dB loss in notch depth.

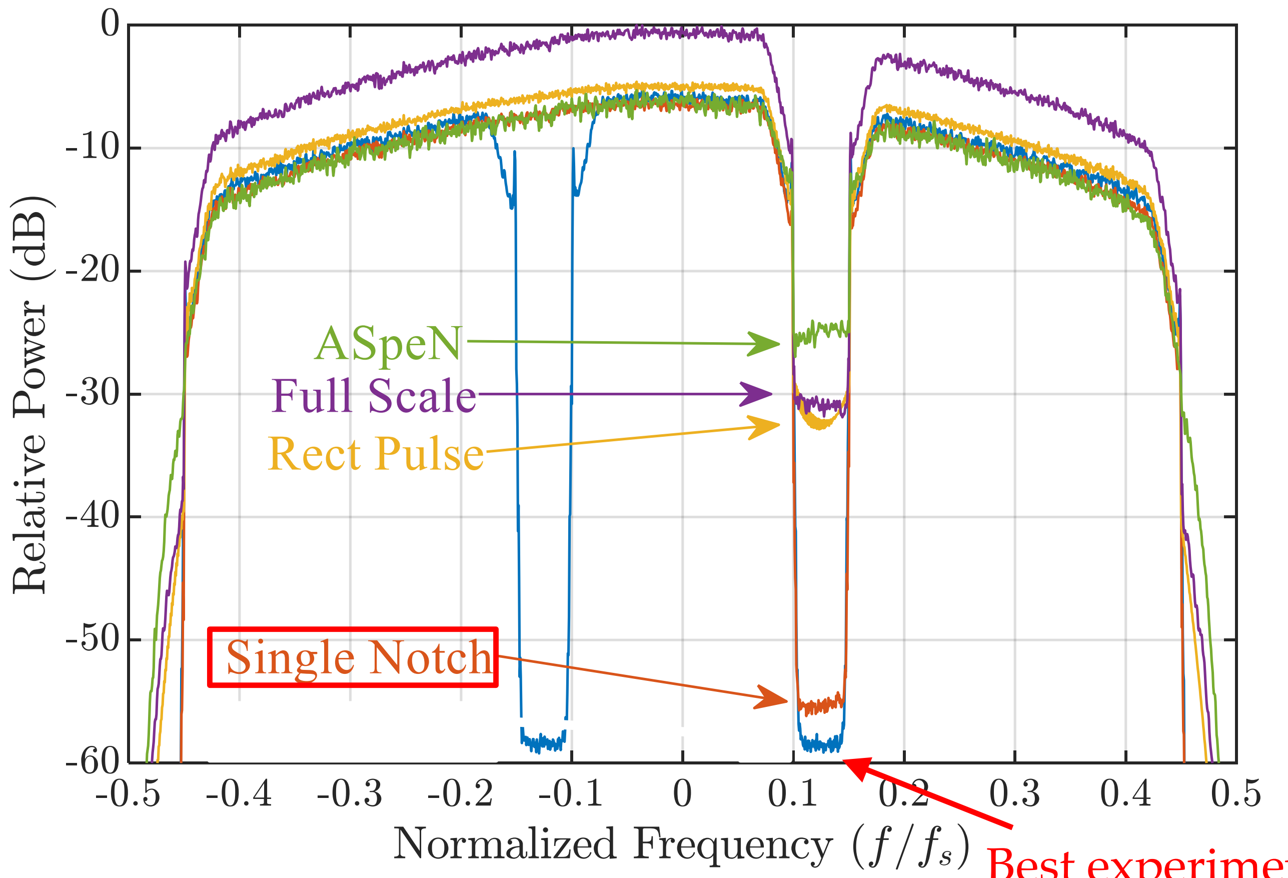
# Loopback Experiments: X310 SDR

The goal of these tests is to isolate the impact of various non-ideal hardware effects as a function of physically realizable notch depth in loopback.

**<u>ZOROW</u>** <u>– with Tukey Taper, subscale output</u>

- Tukey taper reduces distortion from fast pulse rise/fall times
- Subscale output reduces distortion from amplitude clipping (lower overall power)

With several distortion effects accounted for, ZOROW realizes ~50 dB notch depth




Best experimental results

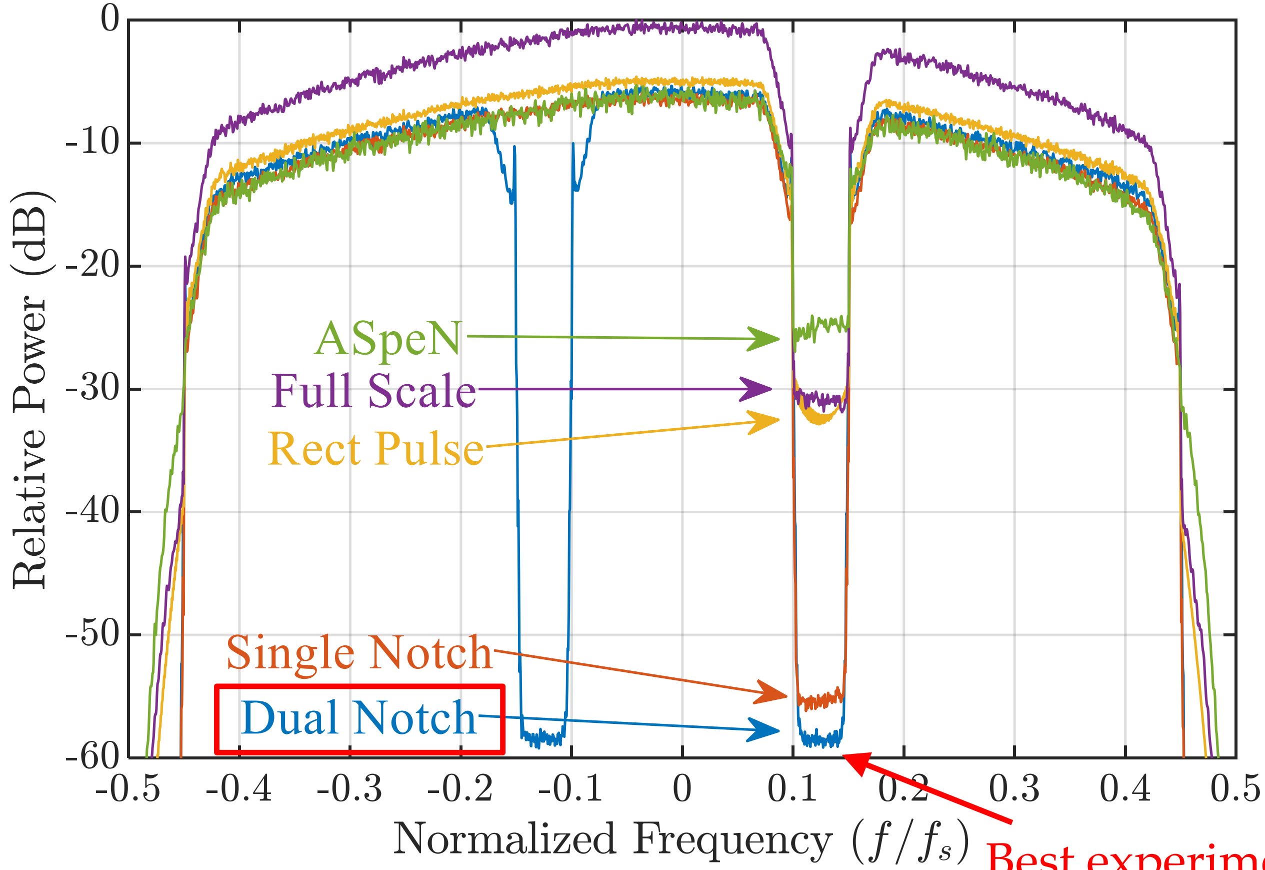
# Loopback Experiments: X310 SDR

The goal of these tests is to isolate the impact of various non-ideal hardware effects as a function of physically realizable notch depth in loopback.

**<u>ZOROW</u>** <u>– with Tukey Taper, subscale output</u>

- Tukey taper reduces distortion from fast pulse rise/fall times
- Subscale output reduces distortion from amplitude clipping (lower overall power)
- Two symmetric notches to demonstrate IQ imbalance effect

Symmetric notches illustrates what is otherwise a loss in fidelity due to IQ imbalance. Notch depth improves by ~ 3dB.



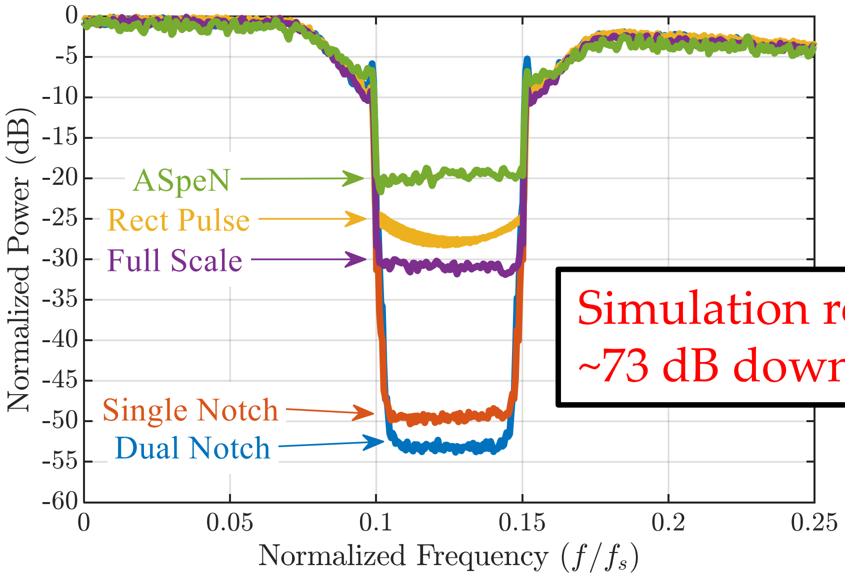

Best experimental results

# Loopback Experiments: X310 SDR

Based on these loopback measurements, the sources of distortion (from most to least significant) are:

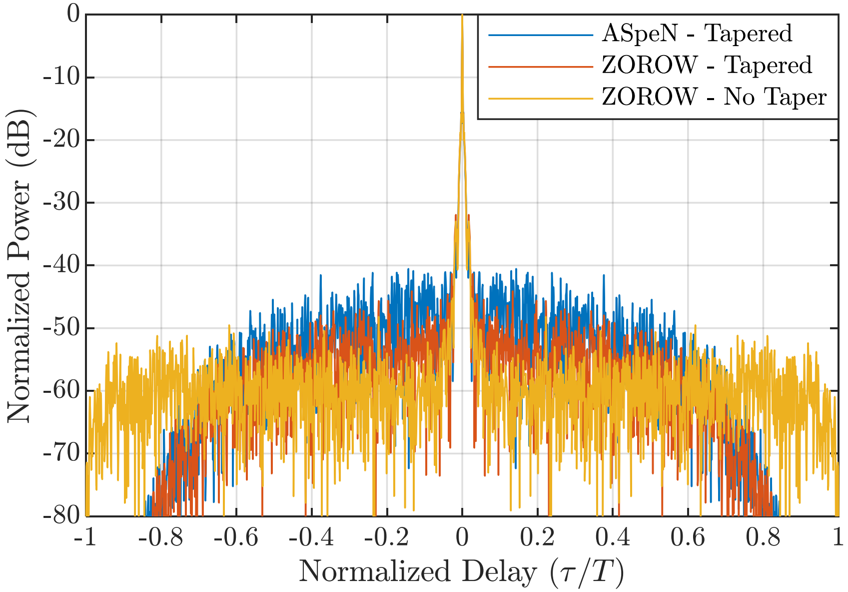
1. Implementation model mismatch
2. Pulse rise/fall time
3. Amplitude clipping
4. IQ imbalance (distant fourth)

Loopback mean power spectra




Simulation reached ~73 dB down

Loopback autocorrelation single notch



# Conclusions

- When implementing highly-fidelity radar waveforms (spectral notches, very low autocorrelation sidelobes), it is crucial to design those waveforms with a model that explicitly captures the hardware implementation.

- Beyond the waveform implementation model, other sources of distortion (amplitude clipping, sharp rise/fall times, IQ imbalance) can cause further distortion on modest-fidelity systems (COTS SDRs).

- The ZOROW waveform model accounts for the hardware implementation and is amenable to other compensation to realize extremely deep spectral notches on modest DAC rate systems. See **[2]** for real-time implementation results.

[2] J.W. Owen, C.A. Mohr, B.H. Kirk, S.D. Blunt, A.F. Martone, K.D. Sherbondy, "Demonstration of real-time cognitive radar using spectrally-notched random FM waveforms," *2020 IEEE International Radar Conf.*, Washington, DC, Apr. 2020.