Hybrid Signal Processing Techniques for

Shared Spectrum Multistatic Radars

by

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Abstract

Multiple radars transmitting a waveform at the same time on the same band will cause interference that most pulse compression algorithms cannot suppress effectively. The Multistatic Adaptive Pulse Compression (MAPC) algorithm and other adaptive algorithms have demonstrated the ability to suppress interference from other radars transmitting on the same band and the sidelobes that form due to the waveform that the radar of interest transmits. Another algorithm that has been used to mitigate the effects of sidelobes using pulse compression is the CLEAN algorithm which has been used by radio astronomers since the 1970's as a way to deconvolve a received signal.

To improve the performance of the MAPC algorithm, two variants of the CLEAN algorithm were developed to eliminate scatterers with a large SNR that are causing interference within the received radar signal so that the MAPC algorithm is able to further suppress interference from other radars. Also two different methods for integrating the newly developed CLEAN algorithms with the MAPC algorithm have been developed and tested in this thesis to create a hybrid algorithm. Compared to the MAPC algorithm the one of the hybrid algorithms is able to detect a scatterer that has 10 dB less signal to noise ratio (SNR) at a probability of detection of 0.9. By combining the MAPC and CLEAN algorithms the probability of detecting scatterers with a small signal to noise ratio improves along with the mean squared error of the range profile.

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List of Acronyms

ADT	Average Detection Threshold
APC	Adaptive Pulse Compression
CA-CFAR	Cell Averaging Constant False Alarm Rate
CFAR	Constant False Alarm Rate
DSP	Digital Signal Processing
FIR	Finite Impluse Response
GO-CFAR	Greatest Of Constant False Alarm Rate
IC-MAPC	Integrated CLEAN Multistatic Adaptive Pulse Compression
LO-CFAR	Least Of Constant False Alarm Rate
MAPC	Multistatic Pulse Compression
MIMO	Multiple-Input Multiple-Output (radar)
MMSE	Minimum Mean Squared Error
MPCR	Multistatic Adaptive Pulse Compression Repair
MRP-CLEAN	Multiple Repetitions Projected CLEAN
MSE	Mean Squared Error
OS-CFAR	Ordered Statistics Constant False Alarm Rate
OTH	Over The Horizon (radar)
PRI	Pulse Repition Interval
RF	Radio Frequency
RMMSE	Reiterative Minimum Mean Squared Error
SNR	Signal-to-Noise Ratio

Chapter 1 Introduction

One of the most common radar systems is a pulsed radar system. A pulsed radar system operates on a specified band where it transmits a waveform. After a specific waveform has been transmitted the radar will turn off the transmitter and listen for the echoes from the scatterers within the environment. The echoes received when the transmitter is turned off are downconverted into a baseband signal and sampled with an analog-to-digital converter.

All radar systems operate on a specific band of frequencies and that frequency band is subject to interference from other radars. If the other radars are cooperative and each radar system knows the transmitted waveform of the other radars then this information could potentially be used to each radar's advantage to share the same frequency band [1]. If the radar creating the interference was uncooperative but the transmitted waveform could be estimated through some means these processing techniques would still be applicable.

1.1 Shared Spectrum Multistatic Radars

Shared spectrum multistatic radars make use of the same spectrum by transmitting distinct radar pulses into the environment. Returns from each of the transmitted waveforms are received by all of the radars. Each scatterer has a different radar cross section that is dependent upon the angle of incidence of the RF energy, which gives each radar a different perspective of the scatterers within the environment. Each radar may receive a different signal from all of the scatterers due to the angle of incidence of the waveform upon the scatterers and the position of the receiver relative to each scatterer [1].

Another situation in which a similar received signal is experienced is for a monostatic radar system that has encountered RF fratricide. In this situation the radar is expecting to receive returns from just its transmitted waveform but instead it receives returns from itself and other radars' transmitted waveforms. This situation occurs more frequently in arid environments where signals experiences a ducting effect through the ionosphere [2]. Most of the time ducting is unintentional and occurs sporadically as the as many different factors change the environment. Most of these factors are dependent upon the weather and the operating frequency. If the radar experiencing the RF fratricide is able to obtain the transmitted waveform of the interfering radar through cooperative or uncooperative means then the received signal could be processed in a similar manner.

Having the ability to remove interference caused by other radars may make it feasible to create radar sensor networks and also reduce spectrum demand. Currently, multistatic radar networks operate by using separate frequency bands and suppress the interference using bandpass filters but some of the RF energy from the radar does bleed over into the other bands. Multiple radars operating on the same frequency can also use spatial beamforming to reduce the interference from other radars or allow only one radar to transmit each

Pulse Repetition Interval (PRI). While all of these methods have been implemented or can be implemented they still have some deficiencies. Using adaptive signal processing techniques may enable multiple radars to operate in close proximity and in the same band allowing multiple radars the ability to transmit a waveform without blinding other cooperative radars. Each radar is still able to take advantage of the suppression that occurs from spatial beamforming.

1.2 Prior Work

The received radar signal for shared spectrum multistatic radars contains enough information to generate a monostatic range profile for the waveform that was transmitted from that particular radar and bistatic range profiles for each of the other radars that transmitted a waveform during the PRI. One way to retrieve this information is using the Multistatic Adaptive Pulse Compression (MAPC) algorithm [3] which is a Reiterative Minimum Mean Squared Error (RMMSE) algorithm [4]. This algorithm has a finite number of adaptive degrees of freedom to suppress the scatterers from other range profiles and suppress the sidelobes for the desired range profile. The MAPC algorithm is the multistatic implementation of the Adaptive Pulse Compression (APC) algorithm [5] which is able to suppress range sidelobes.

An alternative algorithm is the CLEAN algorithm. The CLEAN algorithm's subtractive approach is able to suppress many scatterers detected within a received radar signal as long as the information about the scatterers is fairly accurate in terms of the location of the scatterer and the estimated complex amplitude [6]. Other approaches to suppressing interference in multistatic radar signals have also been explored by Abramovich in the late 1970's [7]. These approaches are similar to the CLEAN algorithm. By coupling the strengths of the CLEAN and MAPC algorithms a higher precision in the estimation of the range profile can be achieved while increasing the probability of detection for scatterers in other range profiles.

It should also be noted that a paper was published recently by Zhang and Amin [8] where two Over The Horizon (OTH) radars were used in a multistatic radar situation. The authors describe a cross-radar cancellation procedure that is similar to the MRP-CLEAN algorithm that will be discussed and formulated in section 4.2.3 of this thesis. One of the main differences between the two methods is that the MRP-CLEAN algorithm performs multiple repetitions of the same cancellation procedure discussed in the paper. The analysis in section 4.2.3 shows why multiple repetitions of this procedure are necessary.

1.3 Motivation of the Thesis

The MAPC algorithm is able to suppress interference from other radars in the same frequency band while being able to obtain an accurate estimate of the range profile. When there are large amounts of interference present from large scatterers or if a significant number of different radars are present the accuracy of the range profile estimate degrades and the probability of detecting small scatterers decreases. Integrating the MAPC algorithm with other algorithms allows for more accurate range profile estimates and a greater probability of detection for small scatterers.

The improvement in multistatic radar signal processing will allow radars to gain more information about a certain environment. Using multiple radars at the same time allows radar systems the ability to place more RF energy on the scatterers that are within the environment which would give radar operators the ability to detect scatterers that they might not be detectable with a monostatic radar system or the previous multistatic radar systems discussed. This type of receive signal processing can help improve Multiple-Input Multiple-Output (MIMO) radar operations but it is not specifically intended to address the problems that MIMO radars encounter.

1.4 Organization of the Thesis

This thesis is organized into six chapters. The first chapter is the introduction followed by the radar signal model and background in the second chapter. The radar signal models are used for the algorithms discussed in the second chapter which includes the algorithms used to develop the hybrid processing techniques discussed in this paper. The third chapter discusses the implementation of the algorithms developed in this thesis. The fourth chapter discusses the results found from evaluating these algorithms while the final chapter contains the concluding remarks for the thesis and possible future work.

Chapter 2 Background

In all radar systems, scatterers within an environment are illuminated by a source. The source that illuminates the scatterers transmits a distinct waveform. This waveform then returns to the receiver by reflecting off scatterers within the environment and in some radar systems the signal is converted to a baseband signal where the information about the scatterers can be determined.

Once the waveform is received, pulse compression of a radar signal is used to determine the location and amplitude of targets and other scatterers within an environment. Some pulse compression schemes operate in the frequency domain. In these pulse compression schemes a linear frequency modulated signal is transmitted and then the received signal is modulated with itself. In this setup the modulated signal is viewed in the frequency domain to determine the position and amplitude of the scatterers in the environment [9].

2.1 Pulsed Radar

A radar system consists of a source transmitting a pulse and then receiving the echoes of the same pulse from the scatterers that are within the environment. In this radar system the pulse is transmitted from the mainbeam of the antenna of the radar and the pulse is then reflected off of the scatterers in the environment and returns through the same mainbeam of the antenna [1]. There are cases when the pulse can hit a scatterer within the environment and the reflected pulse then hits another scatterer within the environment and then the twice reflected pulse returns through the sidelobe or mainbeam of the antenna's beam pattern [1]. Due to the losses that occur in a multipath case like this one it has not been removed from the monostatic model but it is not explicitly addressed either.

2.2 Monostatic Radar Received Signal

This signal model consists of a single radar operating in a certain frequency band. The received signal consists of one PRI where the transmitted waveform is convolved with the scatterers that are in the environment. Then the echoes from the scatterers in the environment are received and sampled using and analog-to-digital converter. The sampled version of the received signal, $\mathbf{y} = [y(l) \quad y(l+1) \quad \ldots \quad y(l+L-1)]^T$ is L samples long and is represented by the equation

$$y(l) = \tilde{\mathbf{x}}^T(l)\mathbf{s} + v(l) \tag{2.1}$$

the $(\bullet)^T$ and the $(\bullet)^H$ denote the transpose and the complex conjugate of the transpose known as the Hermitian, respectively. The term $l = 1, \quad 2, \quad \ldots \quad L$ determines the size of the processing window and $\mathbf{s} = [s_0 \quad s_1 \quad \ldots \quad s_{N-1}]^T$ is the transmitted waveform that is N samples long. The term $v(l)$ is the random noise for that sample of the received signal which is the same size as the vector $\mathbf{y}$ which is length L. For this model $v(l)$ has been modeled as additive noise. The vector $\tilde{\mathbf{x}}(l)$ consists of N samples of the complex amplitude of the scatterers in the range profile indexed by l. The vector

$$\tilde{\mathbf{x}}(l) = [x(l) \quad x(l+1) \quad \dots \quad x(l+N-1)]^T \tag{2.2}$$

is the composite range profile from all of the reflected illumination that has been illuminated by the radar system.

2.3 The Matched Filter

There are many methods in the time domain to determine the position and amplitude of a scatterer. The most commonly used time domain method for pulse compression is matched filtering. The matched filter is a time domain version of the transmitted waveform where the indices of each sample have been reversed and then the complex conjugate of the signal is taken. Then the received signal is convolved with the matched filter to give a pulse compressed version of the range profile.

The transmitted waveform is represented with $\mathbf{s}$ where $\mathbf{s} = [s_0 \quad s_1 \quad \dots \quad s_{N-1}]^T$ which is N samples long. Then the matched filter $\tilde{\mathbf{s}}$ is represented by $\tilde{\mathbf{s}} = [s_{N-1}{}^* \quad s_{N-2}{}^* \quad \dots \quad s_0{}^*]^T$ where the $(\bullet)^*$ denotes the complex conjugate of the scalar. Then this signal is convolved with the received signal and normalized to form the pulse compressed signal $\hat{\mathbf{x}}$ [10].

$$\hat{\mathbf{x}} = \tilde{\mathbf{s}} * \mathbf{y} \tag{2.3}$$

The other commonly used method for representing the matched filter is in terms of vectors. This notation uses the term l to indicate the position in the range profile where the vector $\mathbf{y}(l) = [y(l) \quad y(l+1) \quad \dots \quad y(l+N-1)]^T$ and the term $\hat{x}(l)$ is a scalar. This is represented by an inner product of the transmitted waveform and the received signal

$$\hat{x}(l) = \mathbf{s}^H \mathbf{y}(l) \tag{2.4}$$

for each sample of the received signal unless there are not $N-1$ samples following. It is common for the matched filtered response to be normalized by the inner product of the transmitted waveform $\mathbf{s}^H \mathbf{s}$.

It has been shown that the matched filter optimizes the signal-to-noise ratio (SNR) of a single point scatterer in white Gaussian noise [9]. This improved SNR increases the delectability of the scatterer. When the matched filter is used for other targets such as extended targets (i.e. a target that occupies more than one range cell) the matched filter is unable to accurately determine the complex amplitude of the extended target.

Matched filtering is analogous to finding the cross correlation between the transmitted signal and the received signal [10]. One of the effects from matched filtering is that cross correlation terms cause sidelobes in the matched filtered signal which causes ambiguities in the complex amplitude of other scatterers. These sidelobes can also mask small scatterers. The autocorrelation of the waveform will determine the characteristics of the sidelobes when the matched filter is used. Sidelobes for a single point scatterer which is located within one range cell will be present in the range cells that are $N-1$ before and after the scatterer [9].

2.4 Least Squares

One method used to mitigate sidelobes from matched filtering is to use the least squares method to determine the least squares estimate of the range profile. The least squares method solves the equation $\mathbf{Sx} = \mathbf{y}$ when $\mathbf{S}$ is a matrix that can have more linearly independent rows than columns [11]. Since $\mathbf{S}^H\mathbf{S}$ is a non-singular matrix, it is invertible and a solution can be found that allows for an approximation of $\mathbf{x}$ which is $\hat{\mathbf{x}}$.

$$\hat{\mathbf{x}} = \left(\mathbf{S}^H\mathbf{S}\right)^{-1}\mathbf{S}^H\mathbf{y} \tag{2.5}$$

Using the monostatic radar signal model a least squares solution can be derived that finds a solution. The matrix

$$S = \begin{bmatrix} s_0 & 0 & \cdots & \cdots & 0 \\ \vdots & s_0 & & & \vdots \\ s_{N-1} & \vdots & \ddots & & \\ 0 & s_{N-1} & & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ & & & \ddots & s_0 \\ \vdots & & & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 & s_{N-1} \end{bmatrix} \tag{2.6}$$

consists is a $L+N-1$ by L matrix. Then the monostatic signal model can be rewritten in matrix form,

$$\mathbf{y} = \mathbf{Sx} + \mathbf{v} \tag{2.7}$$

where $\mathbf{y}$ and $\mathbf{v}$ are column vectors that are $L+N-1$ samples long and the vector $\mathbf{x}$ is a vector that is L samples long [12].

2.5 Beamforming

When an M element linear array is used to transmit and receive for a monostatic radar, the radar signal model changes allowing the signal to be steered using beamforming. This is done by using the properties of constructive and destructive interference to create a gain at one angle or direction and nulls in other directions. Beamforming allows the signal to be spatially filtered so that the gain in a certain direction will increase while smaller gains will be seen in other directions. Beamforming also allows the radar to suppress unwanted signals like jamming or interference [13]. It also allows the user to quickly steer the mainbeam of the antenna in other directions if no mechanical parts are needed to move the mainbeam in that direction. When the signal model is changed to allow for beamforming the received signal becomes $\mathbf{y}_{\mathbf{m}}$ which is the signal that is received at the m^{th} element of the linear array. Each of the received signal vectors are placed in a matrix $\mathbf{y} = [\mathbf{y}_1 \quad \mathbf{y}_2 \quad \cdots \quad \mathbf{y}_M]$. This matrix can be multiplied by the vector $\mathbf{r}_k = \left[1 \quad e^{j\theta_k} \quad \ldots \quad e^{j(M-1)\theta_k}\right]^T$ to create a single column vector of the beamformed signal [13]. The received signal from a single antenna element will be modeled using

$$y_m(l) = \tilde{\mathbf{x}}^T(l)\mathbf{s}\ e^{j(m-1)\theta_k} + v(l) \tag{2.8}$$

where θ_k is the electrical angle of the M element linear array.

Beamforming on the multistatic signal allows the interference outside the receiver mainbeam to be reduced. Assuming that the receiving radar has M linear array elements the received radar signal can be written as

$$y_m(l) = \sum_{k=1}^{K} \mathbf{x}_{k,m}{}^T (l) \mathbf{s}_k e^{j(m-1)\theta_k} + v(l) . \tag{2.9}$$

As in the monostatic case $y_m(l)$ can be placed into a matrix and then multiplied by the vector $\mathbf{r}_k$ to create the beamformed signal which consists of a single column vector.

2.6 Adaptive Pulse Compression

Due to the effects of sidelobes as mentioned in the matched filter and least squares approach to pulse compression, considerable efforts have been made to mitigate these effects. As mentioned these processing techniques cause the masking of small targets in the presence of large targets and can cause ambiguities in the estimation of the complex amplitude if more than one significant target exists in the range profile.

One approach to reducing the effects of sidelobes is to reduce how well a signal correlates with itself for time shifts not equal to zero. Many different waveforms have been developed that can accomplish this task. Research in waveform design has provided some improvement in decreasing the sidelobes though there will always be sidelobes for any practical waveform when the signal is filtered using a deterministic Finite Impulse Response (FIR) filter [14].

Another approach to reducing the interference caused by sidelobes is to adaptively change the filter to null interference from scatterers and other unwanted interference. An algorithm that has these processing characteristics using a minimum mean squared error (MMSE) approach known as APC [5]. The APC algorithm uses a matched filtering approach to pulse compression by adaptively changing the filter

$$\hat{x}(l) = \mathbf{w}(l)^H \tilde{\mathbf{y}}(l) \tag{2.10}$$

to determine the processed range profile, $\hat{x}(l)$, the estimate of the range profile and $\tilde{\mathbf{y}}(l) = [y(l) \;\; y(l+1) \;\; \ldots \;\; y(l+N-1)]^T$ is the received signal. The MMSE filter for the range cell of interest is $\mathbf{w}(l)$. This means that $\mathbf{w}(l)$ adaptively changes for each individual range cell. Then using the cost function for the MMSE where $E[\bullet]$ represents the expectation.

$$J(l) = E\left[\left|x(l) - \mathbf{w}^H(l)\tilde{\mathbf{y}}(l)\right|^2\right] \tag{2.11}$$

This cost function is then minimized for each sample of l. One assumption made is that the neighboring impulse response terms are uncorrelated. Then after differentiating with respect to $\mathbf{w}^*(l)$ the filter

$$\mathbf{w}(l) = \rho(l)(\mathbf{C}(l) + \mathbf{R})^{-1}\mathbf{s}\,. \tag{2.12}$$

The term $\rho(l) = |x(l)|^2$ and $\mathbf{R}$ is the covariance matrix of the noise $v(l)$. The matrix

$$\mathbf{C}(l) = \sum_{n=-N+1}^{N-1} \rho(l+n)\mathbf{s}_n\mathbf{s}_n{}^H \tag{2.13}$$

where $\mathbf{s}_n$ contains the transmitted waveform shifted by n samples. An example of this is $\mathbf{s}_{-3} = [s_3 \quad ... \quad s_{N-1} \quad 0 \quad 0 \quad 0]^T$. If the subscript of $\mathbf{s}$ were 3 then the first three elements would be zero followed by s_0 and the last element would be s_{N-4}. It is important to note that the matrix $\mathbf{C}(l)$ is positive semi-definite and $\mathbf{R}$ is positive definite. When both matrices are summed together their sum is invertible since the sum is positive definite.

An important aspect in the implementation of the APC algorithm is the stability of the algorithm. Extensive testing of the algorithm has shown that the algorithm has some stability issues when the exponent in the equation $\rho(l) = |x(l)|^2$ is 2. A more suitable value for this exponent usually ranges from 1.7 to 1.4 and decreases for each stage of the algorithm [5].

Since the first iteration of APC has no prior information and considering that the noise power is negligible, the filter for all range cells in the first iteration can be written as the following.

$$\hat{\mathbf{w}} = \left(\sum_{n=-N+1}^{N-1} \mathbf{s}_n\mathbf{s}_n{}^H \right)^{-1} \mathbf{s} \tag{2.14}$$

This filter can then be used like an FIR filter applied to the received signal. In the testing of this algorithm it was found that this filter had cross correlation sidelobes that were approximately the same power level as the matched filter [5]. To save on computing the inverse for this stage the matched filter can be used to determine the initial knowledge of the range profile but this filter can also be precomputed easily.

APC has been shown to significantly reduce the effects of sidelobes for pulse compression of a monostatic received signal. Compared to the matched filter and the Least Squares algorithm APC is able to uncover small scatterers that have been masked by large scatterers.

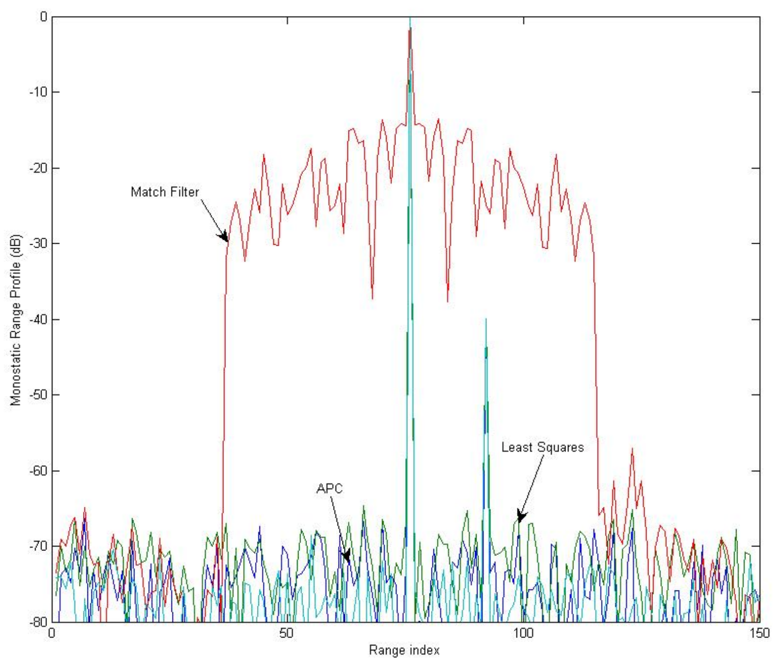

Figure 2.1 Various pulse compression techniques applied to a monostatic signal with a small scatterer that is 40 dB lower than the large scatterer.

Figure 2.1 shows a larger scatterer that is masking a small scatterer that is 40 dB lower when the signal is pulse compressed using the matched filter. Both APC and Least Squares are able to unmask the scatterer. The sidelobes for the least squares algorithm are slightly larger than the sidelobes for APC. This leads to the mean squared error for the least squares algorithm to be higher than APC.

One of the drawbacks to the APC algorithm is that it requires a large number of computations. APC requires a matrix inverse for each range cell for the formulation shown above. A fast matrix update has been developed for the APC algorithm that uses the matrix inversion lemma. This fast matrix update coupled with the use of the matched filter on the first stage has allowed the APC algorithm a greater computational efficiency compared to using the previous mentioned formulation. The original formulation requires $O(N^3L)$ calculations while using the matrix inversion lemma and the matched filter on the first stage require $O\left((M-1)N^2L\right)$ where N is the length of the waveform, M is the number of stages and L is the number of cells in the range profile. There are also other research efforts that have increased the computational efficiency of the algorithm while accepting some degradation in performance [15].

2.7 Adaptive Pulse Compression Repair

A variant method of APC was developed using a similar formulation of the APC algorithm for in-service radars. Adaptive Pulse Compression Repair (PCR) [16] was developed so that radars that are currently in use could use a form of APC to improve the sensitivity in their range profiles. Some radars that are in use today have the pulse compression stage integrated into their systems so that it is unfeasible to remove it to apply the APC algorithm. The PCR algorithm operates on the matched filtered range profile of the signal.

The signal model for the PCR algorithm changes since the signal that is being operated on is the matched filtered response.

$$x_{mf}(l) = \mathbf{s}^H \mathbf{y}(l) \tag{2.15}$$

From this point in the discussion of the PCR algorithm the term

$$\tilde{y}(l) = \mathbf{s}^H \mathbf{A}^T(l)\mathbf{s} + \mathbf{s}^H \mathbf{v}(l) \tag{2.16}$$

will be referred to as the matched filtered response of the signal. Where the matrix

$$\mathbf{A}(l) = \begin{bmatrix} x(l) & x(l+1) & \dots & x(l+N-1) \\ x(l-1) & x(l) & \ddots & \vdots \\ \vdots & \ddots & \ddots & x(l+1) \\ x(l-N+1) & \cdots & x(l-1) & x(l) \end{bmatrix} \tag{2.17}$$

is an N by N matrix that contains samples of the actual range profile.

Since the received signal has been correlated with the matched filter as previously mentioned the matched filter will produce sidelobes when there is a correlation with the transmitted waveform. When viewing the targets from the perspective of the output from the matched filter it can be viewed as superposition of sums of the autocorrelation of the waveform summed with the correlation of the noise. The term

$$\tilde{y}(l) = \tilde{\mathbf{x}}^T(l)\mathbf{r} + u(l) \tag{2.18}$$

where $u(l)$ is the noise correlated with the matched filter or $\mathbf{s}^H \mathbf{v}(l)$ and $\mathbf{r}$ is an $2N-1$ vector of the autocorrelation of the transmitted waveform. It is important to note that the new received signal model for PCR has a similar form to the monostatic signal model presented in this chapter. From this point we are able to use the same cost function for APC where $\tilde{\mathbf{y}}(l)$ is the matched filtered response of the received waveform.

$$J(l) = E\left[\left|x(l) - \tilde{\mathbf{w}}^H(l)\tilde{\mathbf{y}}(l)\right|^2\right] \tag{2.19}$$

One of the main differences in this cost function compared to APC is the filter $\tilde{\mathbf{w}}(l)$ is length $2N-1$. Again this cost function is minimized by taking the derivative with respect to $\tilde{\mathbf{w}}^*(l)$ and a filter similar to the APC algorithm's filter is found.

$$\tilde{\mathbf{w}}(l) = \hat{\rho}(l)(\mathbf{C}(l) + \mathbf{R})^{-1}\mathbf{r} \tag{2.20}$$

Where $\hat{\rho}(l)$ is the power estimate for the range profile estimate $\hat{x}(l)$ which is found by using a similar range of exponents as mentioned in the APC algorithm. The matrix $R = E\left[\mathbf{u}(l)\mathbf{u}(l)^H\right]$ is the covariance matrix of the noise correlated with the matched filter. Then the matrix $\mathbf{C}(l)$ is represented by

$$\mathbf{C}(l) = \sum_{n=-2N+2}^{2N-2} \hat{\rho}(l-n)\mathbf{r}_n \mathbf{r}_n{}^H \tag{2.21}$$

where the term $\mathbf{r}_n$ contains the shifted samples of the autocorrelation of the transmitted waveform. These samples are shifted the same way that $\mathbf{s}_n$ is shifted in the formulation of the APC algorithm.

This algorithm was developed for in-service radars and shows similar performance characteristics as the APC algorithm. This algorithm allows more radars that are currently in use to gain the added benefits of APC without suffering substantial costs by trying to integrate the algorithm into their systems. The one drawback to the implementation of this algorithm is that it is more computationally expensive than the APC algorithm. When the fast matrix update is used in the APC algorithm and the matched filter is used as the first stage, APC uses $O((M-1)N^2L)$ computations while PCR uses $O((M-1)(2N-1)^2L)$ since the PCR algorithm operates on the matched filtered response instead of the received signal [16].

2.8 Multistatic Radar Received Signal

A multistatic radar as discussed in this thesis refers to multiple radars that are concurrently transmitting in the same frequency band. Each radar has its own transmitter and receiver and each radar transmits a distinct waveform[1]. In general, interference from other radars is undesirable[1]. If the proper pulse compression techniques to suppress the interference are used, the received radar signal for a multistatic radar contains enough information to generate a monostatic profile for the waveform that was transmitted from the radar of interest and bistatic profiles for each of the other radars that transmitted a waveform during the PRI.

Modeling a shared spectrum multistatic radar signal involves multiple radars transmitting a waveform such that the returns from the scatterers in the received signal overlap but do not necessarily align like MIMO radars. Each of these overlapping signals are received by each radar as a sum of their complex amplitudes. Each received waveform can be represented as the transmitted waveform of a single radar convolved with the complex amplitude of the scatterers in the each of the other range profiles and its own range profile. Then each of these signals containing the transmitted waveform convolved with the other actual range profiles are summed together to create the total received signal $\mathbf{y}$.

In the model used for this thesis, there are K radars transmitting and K receive. An important aspect of this multistatic radar scenario is that each individual radar will receive a different radar signal from the environment due to their physical position in the environment. The received signal model for this scenario can be represented as

$$y(l) = \sum_{k=1}^{K} \mathbf{x}_{i,n}{}^{T}(l)\mathbf{s}_i + v(l) \tag{2.22}$$

where as before in the monostatic case $v(l)$ represented the random noise in each sample. The term $\mathbf{s}_i = [s_{0,i} \quad s_{1,i} \quad \dots \quad s_{N,i}]^T$ is the transmitted waveform from the i^{th} radar and the first term in the subscript is the index of the sampled waveform.

The actual complex amplitude of the scatterers from each range profile are indicated by the term $\mathbf{x}_{i,n}(l) = [x_{i,n}(l) \quad x_{i,n}(l-1) \quad \ldots \quad x_{i,n}(l+N-1)]^T$ which is N samples long.

The term n in the subscript of $\mathbf{x}_{i,n}(l)$ indicates the n^{th} beamformed direction. Each radar is receiving a received signal from itself, which is the case when $i = k$, and from one of the other K radars (when $i \neq k$). All of the terms that are within $x_{i,n}(l)$ is the composite range profile from all of the reflected illumination that has been illuminated by the i^{th} radar. In most situations the desired portions of the radar received signal are the portions that are transmitted and received on the main beams of the radars' antenna.

2.9 Multistatic Least Squares

The least squares solution can also be generalized for the multistatic radar signal model. The multistatic radar model can be rewritten in a similar matrix form as the monostatic model. There are only two modifications to the formulation that need to be made. The first is that the vector $\hat{\mathbf{x}}$ is a concatenation of the range profiles for the multistatic signal. The vector $\hat{\mathbf{x}}$ thus has the form

$$\hat{\mathbf{x}}_m = \begin{bmatrix} \mathbf{x}_{1,m} \\ \mathbf{x}_{2,m} \\ \vdots \\ \mathbf{x}_{m,m} \end{bmatrix} \tag{2.23}$$

where the first index in the subscript indicates the transmitter and the second index indicates the receiver in the multistatic arrangement. The index m indicates the m^{th} radar in the multistatic arrangement. For instance if there were a multistatic configuration that had two radars operating and the least squares solution is applied to the received signal of one radar, the least squares solution would contain the range profile of the monostatic range profile for the radar and a bistatic range profile between the two radars. This example can be extended to K radars where $\hat{\mathbf{x}}_m$ will contain one monostatic range profile and $K-1$ bistatic range profiles [12].

The matrix $\tilde{\mathbf{S}}$ is a concatenation of $\mathbf{S}$ matrices that contain the transmitted waveform for each of the radars as

$$\tilde{\mathbf{S}} = \begin{bmatrix} \mathbf{S}_1 & \mathbf{S}_2 & \cdots & \mathbf{S}_K \end{bmatrix}. \tag{2.24}$$

The subscript for each of the $\mathbf{S}$ matrices indicates the radar's shifted transmitted waveform within each $\mathbf{S}$ matrix. The solution to the multistatic least squares problem is

$$\hat{\mathbf{x}}_{LS,m} = \left(\tilde{\mathbf{S}}^H \tilde{\mathbf{S}}\right)^{-1} \tilde{\mathbf{S}}^H \mathbf{y}\,. \tag{2.25}$$

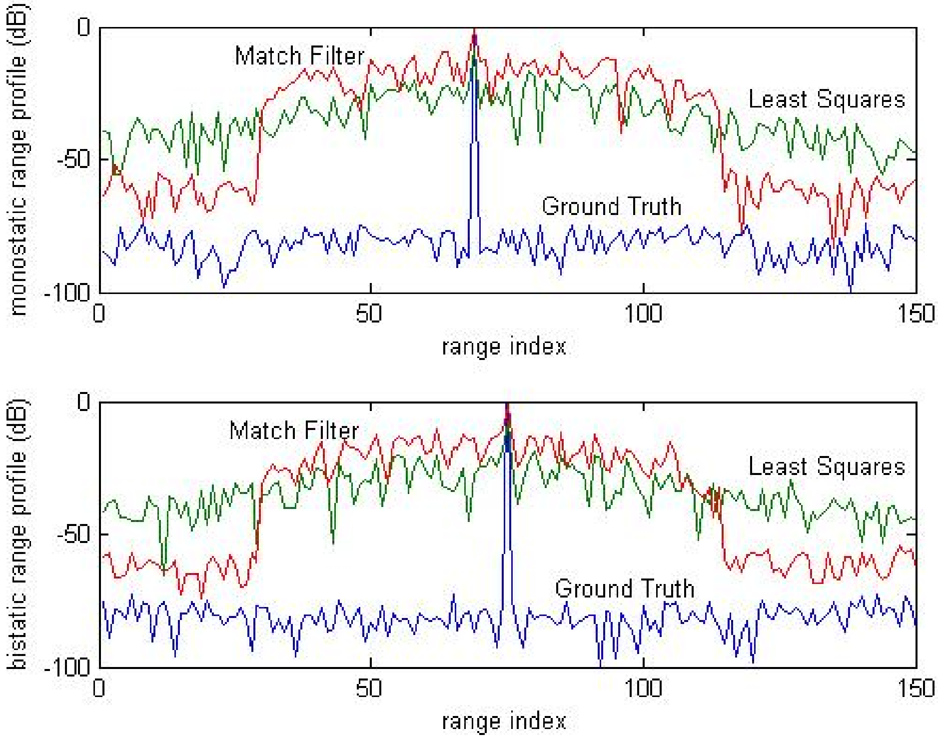

Figure 2.2 Range Profile from Multistatic Least Squares Processing

This solution does have lower sidelobes than the matched filter in both the multistatic and monostatic cases but it does have its own shortcomings. As seen in figure 2.2 the sidelobes extend further than the $N-1$ sidelobes of the matched filter. Although these sidelobes are lower they extend further because the error in the estimation of the range profile is spread somewhat equally throughout the estimated range profile. The least squares solution does not perform well when there are scatterers in the first $N-1$ samples of the received signal $\mathbf{y}$ [12]. The least squares solution also requires a matrix inverse which is computationally expensive but can be precomputed if all of the waveforms are known.

2.10 Multistatic Adaptive Pulse Compression

Another variation to the APC algorithm is used to suppress interference from other radars in a multistatic setting. This scenario occurs when multiple radars transmit a signal on the same band. The interference from the other radars in this setup make using other deterministic methods, like the matched filter or least squares algorithm, inadequate to detect scatterers unless they are in an environment that is very sparse or the radars transmitted waveforms have cross-correlations that are low to minimize the interference. These scatterers will still need to have a high SNR to be detectable using these deterministic methods. The MAPC algorithm [3] is able to detect small scatterers near the noise floor by suppressing the interference of scatterers from other range profiles and suppressing the sidelobes of scatterers within its own range profile so that smaller scatterers that are within the sidelobes of larger scatterers are detectable.

MAPC is a MMSE based algorithm like APC and PCR and uses the same adaptive approach where the filter changes for every range cell. MAPC also uses a matched filter based approach to filtering the signal. The MAPC algorithm was developed by minimizing the cost function

$$J_{i,n}(l) = E\left[\left|\tilde{x}_{i,n}(l) - \mathbf{w}_{i,n}^{H}(l)\mathbf{y}_{n}(l)\right|^{2}\right] \qquad (2.26)$$

with respect to $\mathbf{w}_{i,n}^{*}(l)$. In this cost function the i term indicates the i^{th} radar and the term n indicates the n^{th} beamformed direction.

Once this function is minimized it has a similar equation for the filter as APC where

$$\mathbf{w}_{i,n}(l) = \rho_{i,n}(l)\left(\sum_{k=1}^{K}\mathbf{C}_{k,n}(l) + \mathbf{R}_n\right)^{-1}\mathbf{s}_i \tag{2.27}$$

and

$$\rho_{i,n}(l) = E\left[\ \left|\tilde{\mathbf{x}}_{i,n}(l)\right|^2\right]\ . \tag{2.28}$$

The matrix

$$\mathbf{C}_{k,n}(l) = \sum_{\tau=-N+1}^{N-1}\rho_{k,n}(l+\tau)\mathbf{s}_{k,\tau}\mathbf{s}_{k,\tau}^H \tag{2.29}$$

where $\mathbf{s}_{k,\tau}$ contains the transmitted waveform for the k^{th} radar shifted by τ samples similarly to the APC algorithm. As in APC this algorithm can be adapted for the first iteration of the MAPC algorithm since no information exists about the range profile. This filter

$$\overline{\mathbf{w}}_i = \left(\sum_{k=1}^{K}\overline{\mathbf{C}}_k(l)\right)^{-1}\mathbf{s}_i \tag{2.30}$$

where the term i refers to the i^{th} radar and it’s transmitted waveform. The term $\overline{\mathbf{C}}_k(l)$ is found similarly to the formulation in (2.14) as

$$\overline{\mathbf{C}}_k = \sum_{\tau=-N+1}^{N-1}\mathbf{s}_{k,\tau}\mathbf{s}_{k,\tau}^H\ . \tag{2.31}$$

Since there is no initial range profile information the term $\rho_{k,n}(l+\tau)$ in (2.29) becomes all ones which makes the first stage of the MAPC filter deterministic. Unlike APC this filter cannot be replaced by the matched filter to achieve the same performance but it can be precomputed. Just like APC and PCR the MAPC algorithm can use the fast matrix update that was developed for the APC algorithm this significantly reduces the computational complexity of the algorithm so that there is only one matrix inversion for each stage of the MAPC algorithm.

2.11 Constant False Alarm Rate Processors

Once the radar signal is received and processed, most radar systems are intended to determine the location of scatterers within the range profile. Most received signals have not only the returns from desired scatterers but they are also contaminated by noise. This noise can be thermal noise or internal noise from the system.

Some simple methods for detection involve setting a constant threshold and if the amplitude of the range cell exceeds the threshold then a scatterer is detected. For detecting a large scatterer that exists in a single range cell with a constant noise this type of detection process is adequate. If the noise power begins to change then this type of detection process would begin to fail. If the noise power increased then there could be a significant number of false detections and if they started to decrease the scatterer might not be detectable because the threshold would not be able to adapt.

This desire for adaptively in the detection process leads to a detector that is more adaptive to the changing conditions within the processing window. The most common detector that is able to adapt to these changes is the Constant False Alarm Rate (CFAR) processor [17]. There are many types of CFAR processors that are used for a variety of different situations. The most common type of CFAR processor is the Cell Averaging CFAR (CA-CFAR). A CA-CFAR finds the mean of the magnitude of the cells that are before and after the range cell of interest and then multiplies the average by T, a value greater than one, and then compares this product with the magnitude of the range cell of interest. If the product of the averaged cells and T is greater than the magnitude of the range cell of interest nothing is detected but if the product is less than the magnitude of the range cell of interest then it is considered a detection [17].

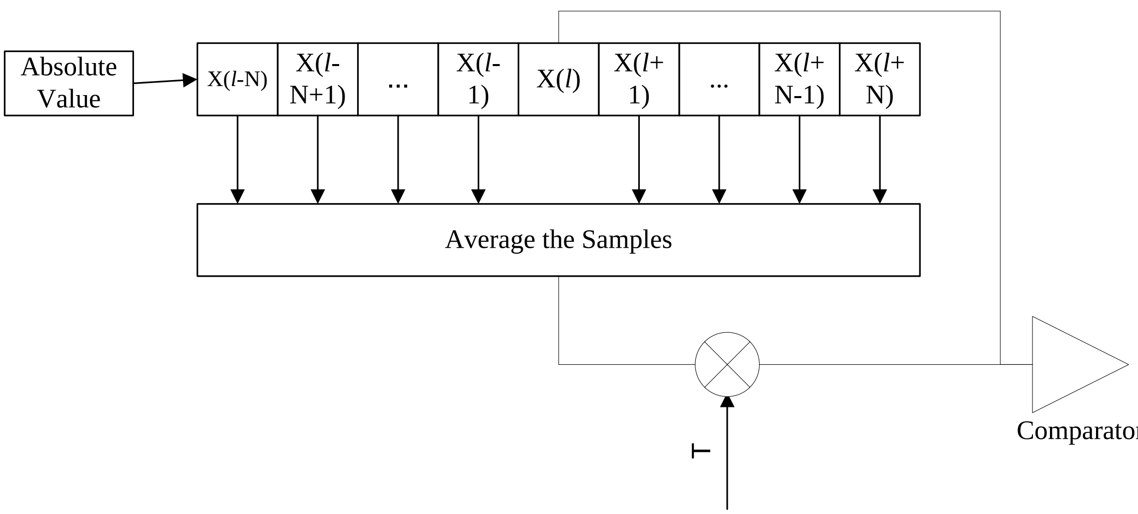

Figure 2.3 Block Diagram representation of a CA-CFAR

For practical purposes a CA-CFAR processor has averaging windows that are usually 8, 16, or 32 cells on each side of the range cell of interest. A certain number of guard cells can be used around the range cell of interest [18]. This allows for a greater probability of detection of extended targets that raise overall threshold of the CFAR processor so that the extended target is undetectable. Other CFAR processors that can be used to overcome this issue are the Least-Of CFAR (LO-CFAR), Greatest-Of CFAR (GO-CFAR) and the Ordered-Statistics CFAR (OS-CFAR). Each of these CFAR processors are discussed in [17-18] in greater detail.

Another important aspect to CFAR processors is the ability to determine the probability of false alarm. CFAR processors will have a constant probability of false alarm that is dependent on the threshold and the size of the window on both sides of the range cell of interest. The probability of false alarm is the probability that the CFAR will detect a target that is not present in the range cell of interest. Using the analysis from [17] the probability of false alarm is

$$P_{fa} = \frac{1}{\left(\frac{T^2}{N}+1\right)^N} \tag{2.32}$$

where P_{fa}, the probability of false alarm and N is is the length of the CA-CFAR window. This equation can also be rewritten so that the threshold can be determined by the window length and a desired probability of false alarm where

$$T = \sqrt{N(P_{fa}{}^{-\frac{1}{N}} - 1)}\,. \tag{2.33}$$

An important difference between [17] and the way that the equations have been presented in this section is that the term ADT (average detection threshold) in [17] is referenced to as T in this section and throughout the rest of this paper.

2.12 The CLEAN Algorithm

An algorithm known as CLEAN is a method to mitigate the effects of sidelobes that mask small targets. CLEAN is a deconvolution technique that was developed in the radio astronomy community during the 1970's and has also been used in seismic exploration [19]. Abramovich independently added his contribution to the algorithm in the late 1970's [7].

The first step in the CLEAN algorithm which is also known as Coherent CLEAN in [6] is to find the largest scatterer in the range profile. The range profile has been pulse compressed and a detection algorithm is used to find the largest scatterer. Next the algorithm measures the complex amplitude of the scatterer and determines the detected scatterer's location. This information is then saved into memory. Then a new received signal is created by using information about the detected scatterer. The transmitted waveform is multiplied by the complex amplitude of the detected scatterer. Then the location of the scatterer is used to subtract the return of the scatterer from the received signal using the product of the transmitted waveform and the estimated complex amplitude of the brightest target.

Due to the additive nature of the received signal model this newly formed received signal should not contain the detected scatterer. The only problem with this assumption is that the method for pulse compression will most likely have some ambiguities in the estimation of the complex amplitude so most likely there will be some residual part of the signal that is left behind.

Then this newly formed received signal is pulse compressed and a new range profile is formed from this signal. The previously mentioned steps are followed until the newly formed range profile is at the same level as the noise floor of the signal or no targets are detectable.

As previously mentioned there are some drawbacks to this algorithm. The CLEAN algorithm relies on a precise estimate of the detected scatterer. If the pulse compression method used creates a false target or if this target’s location is incorrect then the newly created signal will be contaminated with a false return which could lead to the detection of scatterer that does not exist.

There have been methods created that would help mitigate these effects. One of these algorithms is called Sequence CLEAN [6]. This algorithm uses a tree search that measures the total signal power of the range profile in each iteration of the algorithm. If the total power of the range profile is greater than the previous range profile that branch of the tree search is terminated. This method insures a better way to accurately locate and detect signals but it is very costly in terms of computations.

Chapter 3 Novel Methods for Processing Multistatic Signals

To obtain a better level of precision in the estimate of the range profile varying methods of pulse compressing the received multistatic signal have been proposed. These methods combine different approaches to process the multistatic signal so that a better estimate of the range profile is obtained and the small scatters are easily detectable. Different methods for the CLEAN algorithm have been proposed that more effectively uses the estimates from the range profiles of adaptive algorithms compared to the pervious versions of the CLEAN algorithm.

3.1 Multistatic Pulse Compression Repair

As mentioned previously, the PCR algorithm was developed for in-service radars where it is difficult to remove the pulse compression component of the radar system [16]. In this thesis, a similar algorithm called Multistatic Pulse Compression Repair (MPCR) for in-service radars to suppress multistatic interference from radars for the multistatic radar configuration described in section 2.8 or to mitigate the effects of RF fratricide.

3.1.1 Formulation of the MPCR Algorithm

Like the PCR algorithm the received signal is matched filtered using the waveform for the desired returns. For instance if the desired radar of interest was the range profile for the monostatic returns it would matched filter using the waveform that it transmitted but if it wanted the range profile for one of the bistatic returns then it would matched filter using the waveform from the radar that created the returns for the bistatic profile. Matched filtering the received signal creates cross-correlations for the other interfering signals that will need to be suppressed but these cross-correlations have less power compared to the desired returns after they are filtered by the matched filter.

Another way to view the matched filtering process is similar to beamforming. When beamforming, a gain is placed in a certain direction while signals from other directions receive significantly less gain compared to the mainbeam of the beamformed antenna array. When matched filtering a signal that has a similar phase response as the matched filter the power of that signal is estimated to be proportional to the power of the actual return. Other signals that are not completely phase matched to the matched filter will receive a partial gain as in the cross-correlation terms similar to the sidelobes of a beamformed signal [10]. The MAPC algorithm was designed for signals that have been beamformed and can also be adapted for signals that have been correlated with a matched filter. This matched filtered adaptation is the MPCR algorithm.

The MPCR algorithm uses the multistatic signal model,

$$y(l) = \sum_{n=1}^{K} \mathbf{A}_n^T(l)\mathbf{s}_n + v(l) \tag{3.1}$$

to create a received signal model for this algorithm. The term $\mathbf{y}(l) = [y(l) \quad y(l+1) \quad \ldots \quad y(l+N-1)]^T$ is the received signal and

$\mathbf{v}(l) = [v(l) \quad v(l+1) \quad \ldots \quad v(l+N-1)]^T$ is the noise within the system. Both vectors are N samples long. The term $\mathbf{s}_n$ is the transmitted waveform for the n^{th} radar. The matrix,

$$\mathbf{A}_n(l) = \begin{bmatrix} x_n(l) & x_n(l+1) & \ldots & x_n(l+N-1) \\ x_n(l-1) & x_n(l) & \ddots & \vdots \\ \vdots & \ddots & \ddots & x_n(l+1) \\ x_n(l-N+1) & \cdots & x_n(l-1) & x_n(l) \end{bmatrix} \tag{3.2}$$

and the term n refers to the terms that are created by the n^{th} radar. This is similar to the matrix $\mathbf{A}(l)$ found in the derivation of the PCR algorithm (2.17).

Then the multistatic signal is matched filtered with the transmitted waveform from one of the radars. The matched filtered estimate can be represented as

$$\hat{x}_{mf}(l) = \mathbf{s}_m^H \mathbf{y}(l) = \sum_{n=1}^{K} \mathbf{s}_m^H \mathbf{A}_n^T(l)\mathbf{s}_n + \mathbf{s}_m^H \tilde{\mathbf{v}}(l) \tag{3.3}$$

and m indicates the transmitted waveform that is used as the matched filter. Then the matched filtered response can be written as

$$\hat{x}_{mf}(l) = \sum_{n=1}^{K} \tilde{\mathbf{x}}_n^T(l)\mathbf{p}_{m,n} + u(l) \tag{3.4}$$

and $u(l) = \mathbf{s}_m^H \tilde{\mathbf{v}}(l)$ which is the noise correlated with the matched filter. The term

$$\mathbf{p}_{m,n} = \mathbf{s}_m * \mathbf{s}_n \tag{3.5}$$

which is a column vector of the correlation between two transmitted waveforms which is $2N-1$ samples long.

At this point we are able to represent the matched filtered output in a similar manner as in (3.1) where

$$\tilde{y}(l) = \sum_{n=1}^{K} \mathbf{B}_n^T(l)\mathbf{p}_{m,n} + \tilde{u}(l) \tag{3.6}$$

and the vectors $\tilde{\mathbf{y}}(l) = [\hat{x}_{mf}(l) \quad \hat{x}_{mf}(l+1) \quad \ldots \quad \hat{x}_{mf}(l+2N-2)]$ and the vector $\tilde{\mathbf{u}}(l) = [u(l) \quad u(l+1) \quad \ldots \quad u(l+2N-2)]$ are both $2N-1$ samples long. The matrix

$$\mathbf{B}_n(l) = \begin{bmatrix} x_n(l) & x_n(l+1) & \ldots & x_n(l-N+1) \\ x_n(l-1) & x_n(l) & \ddots & \vdots \\ \vdots & \ddots & \ddots & x_n(l+1) \\ x_n(l-2N+2) & \cdots & x_n(l-1) & x_n(l) \end{bmatrix}. \tag{3.7}$$

Now the signal model is in a similar form as the received signal model for the multistatic model (3.1). Where the noise is now correlated with the matched filter instead of the noise in (3.1) and the transmitted waveform is now replaced with the correlations of transmitted waveforms and the matched filter.

Just as the three previous variations of the APC algorithm the MMSE cost function is

$$J(l) = E\left[\left|x(l) - \mathbf{w}_m^H(l)\tilde{\mathbf{y}}(l)\right|^2\right]. \tag{3.8}$$

The filter $\mathbf{w}_m(l)$ is $2N-1$ samples long and then by taking the derivative with respect to $\mathbf{w}_m{}^*(l)$, the adaptive filter is

$$\mathbf{w}_m(l) = \left(E\left[\tilde{\mathbf{y}}(l)\tilde{\mathbf{y}}^H(l)\right]\right)^{-1} E\left[\tilde{\mathbf{y}}(l)x^*(l)\right]. \tag{3.9}$$

Then substituting $\sum_{n=1}^{K}\mathbf{B}_n^T(l)\mathbf{p}_{m,n}+\tilde{u}(l)$ for the signal model $\tilde{y}(l)$ the adaptive filter is

$$\mathbf{w}_m(l)=\left(E\left[\left[\sum_{n=1}^{K}\mathbf{B}_n^T(l)\mathbf{p}_{m,n}+\tilde{u}(l)\right]\left[\sum_{n=1}^{K}\mathbf{B}_n^T(l)\mathbf{p}_{m,n}+\tilde{u}(l)\right]^H\right]\right)^{-1}E\left[\left(\sum_{n=1}^{K}\mathbf{B}_n^T(l)\mathbf{p}_{m,n}+\tilde{u}(l)\right)\mathbf{x}^*(l)\right]$$

(3.10)

and $u(l)$ and the range profiles are all uncorrelated this equation can be simplified into

$$\mathbf{w}_m(l)=\left(E\left[\sum_{n=1}^{K}\mathbf{B}_n^T(l)\mathbf{p}_{m,n}\mathbf{p}_{m,n}^H\mathbf{B}_n^*(l)\right]+\mathbf{R}\right)^{-1}\mathbf{p}_{m,n}\left|x_m(l)\right|^2 \tag{3.11}$$

where $\mathbf{R}$ is the correlation matrix for the correlated noise $\tilde{u}(l)$ which is $\mathbf{R}=E\left[\ \left|\tilde{u}(l)\right|^2\right]$.

Then this equation for $\mathbf{w}_m(l)$ can be written in a similar form as the MAPC algorithm. The adaptive filter

$$\mathbf{w}_m(l)=\rho_m(l)\left(\sum_{n=1}^{K}\mathbf{C}_n(l)+\mathbf{R}\right)^{-1}\mathbf{p}_{m,n} \tag{3.12}$$

and $\rho_m(l)=\left|x_m(l)\right|^2$ is the power estimate of the range cell l. The matrix

$$\mathbf{C}_n(l)=\sum_{\tau=-2N+2}^{2N-2}\rho(l-\tau)\mathbf{p}_{m,n,\tau}\mathbf{p}_{m,n,\tau}{}^H \tag{3.13}$$

is a $2N-1$ by $2N-1$ matrix and the vector $\mathbf{p}_{m,n,\tau}$ is the correlation vector $\mathbf{p}_{m,n}$ which is shifted by samples of τ similar to the $\mathbf{s}$ vector in the APC derivation. Just like the previously mentioned variations to the APC algorithm this algorithm uses the fast matrix update to compute the matrix inverse for each step. Another important aspect of the MPCR algorithm is that it requires a $2N-1$ matrix inverse for each stage of the algorithm compared to the MAPC algorithm which requires a N matrix inverse.

3.2 Hybrid Processing

When the MAPC is used as a pulse compression algorithm the range profile that it produces contains the targets with sidelobes that have been significantly suppressed. The MAPC algorithm is also able to accurately determine the complex amplitude of the scatterers within the range profile. Since the sidelobes of the scatterers have been suppressed and smaller scatterers are detectable within the range profile this leads to changes in the way that the CLEAN algorithm can be implemented since the smaller scatterers are not masked by sidelobes from other returns.

3.2.1 The Hybrid Processing Model

The MAPC algorithm is able to produce very accurate information about the scatterers within each range profile causing interference. The CLEAN algorithm is able to subtract the detectable large scatterers from the received radar signal so that the large detectable scatters do not interfere with the pulse compression in the second iteration of an adaptive algorithm to suppress the smaller undetectable scatterers from the range profile. This will reduce the error in the estimate of the range profile and increase the probability of detection for smaller scatterers that could be masked by larger returns from other range profiles.

Since the MAPC algorithm is able to accurately determine the scatterers within the received signal the CLEAN algorithm has be modified for the data flow model. These variations of the CLEAN algorithm will be explained in more detail in the following sections.

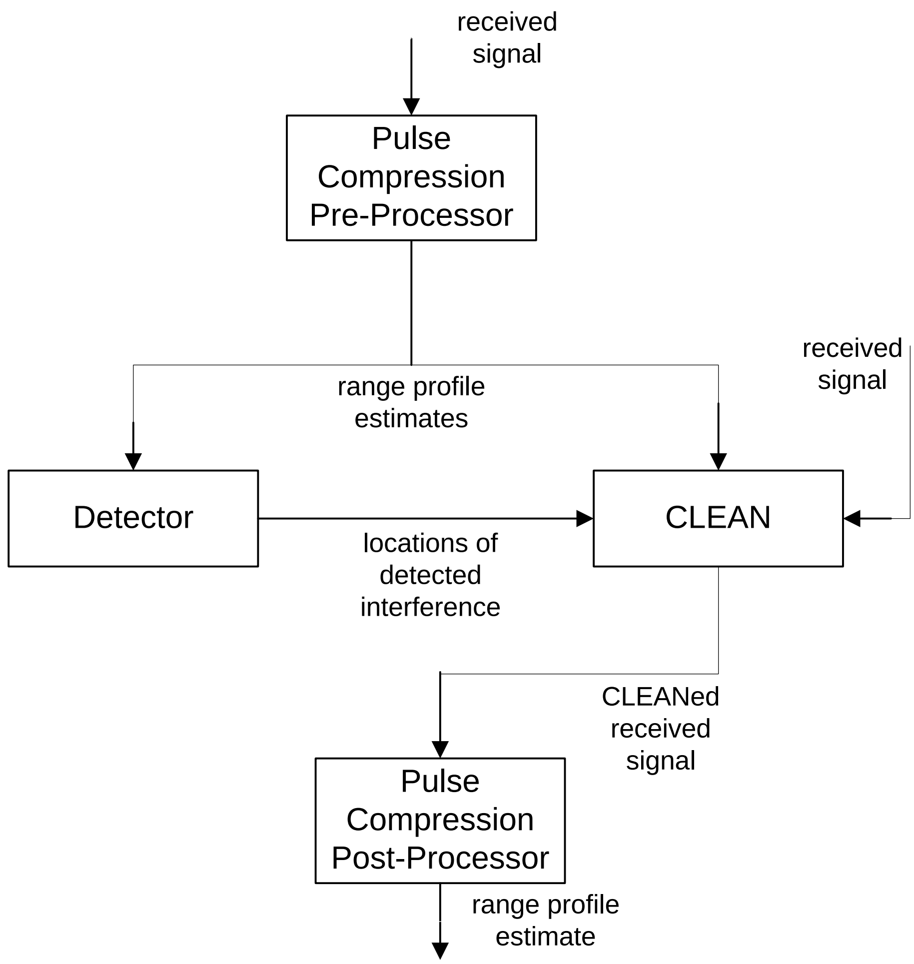

Figure 3.1 Data Flow model of the Hybrid processing scheme

In the data flow model in figure 3.1 the pulse compression algorithm can be any of the previously mentioned pulse compression algorithms and in general reference to this model this stage of the algorithm will be referred to as the pre-processor. The most obvious choice would be the MAPC algorithm due to its ability to suppress noise and other interference making small scatters more detectable. To increase the computational efficiency of the Hybrid algorithm the first stage could be the matched filter or least squares algorithm but as previously mentioned those algorithms do have some deficiencies.

The detector used in this setup is the CA-CFAR as previously mentioned. In most cases this would not be the best detector to use to detect scatterers and other detection methods should be considered depending on the characteristics of the problem but for the purposes of this thesis it is the easiest to understand conceptually and remains as a “constant” throughout the testing of this algorithm. Other CFAR detectors to consider have been mentioned in section 2.10.

Then the received signal along with the processed signal and the information about the location and the complex amplitude of the detected scatterers are sent to the CLEAN algorithm. Then the CLEANed received radar signal is sent to a pulse compression algorithm. This will be referred to as the post-processor.

It is important to note that the pre-processor and post-processor can be replaced with deterministic pulse compression algorithms but these techniques are unable to suppress the interference as effectively as the adaptive techniques mentioned but they have the potential relieve some of the computational burden of the hybrid algorithm.

Each iteration of the CLEAN algorithm was intended to perform a pulse compression stage like the matched filter. The matched filter is not difficult to perform in terms of computations but it creates a range profile that has large sidelobes. These sidelobes coupled with the computational costs are the main reason that the CLEAN algorithm will perform multiple iterations of the pulse compression stage because once a scatterer is removed so are its sidelobes and smaller scatterers that are below the sidelobes are unmasked and could be detected.

When an adaptive algorithm, like MAPC, is used as the pulse compression algorithm the range profile that MAPC produces contains the targets with the sidelobes and noise that have been significantly suppressed [3]. The adaptive algorithm is also able to accurately determine the complex amplitude of the scatterers within the range profile [3]. Since the sidelobes of the scatterers have been suppressed and smaller scatterers are detectable within the range profile this leads to changes in the way that the CLEAN algorithm can be implemented since the smaller scatterers are not masked by sidelobes from other returns. An example of the accuracy of the MAPC algorithm is shown in figure 3.2.

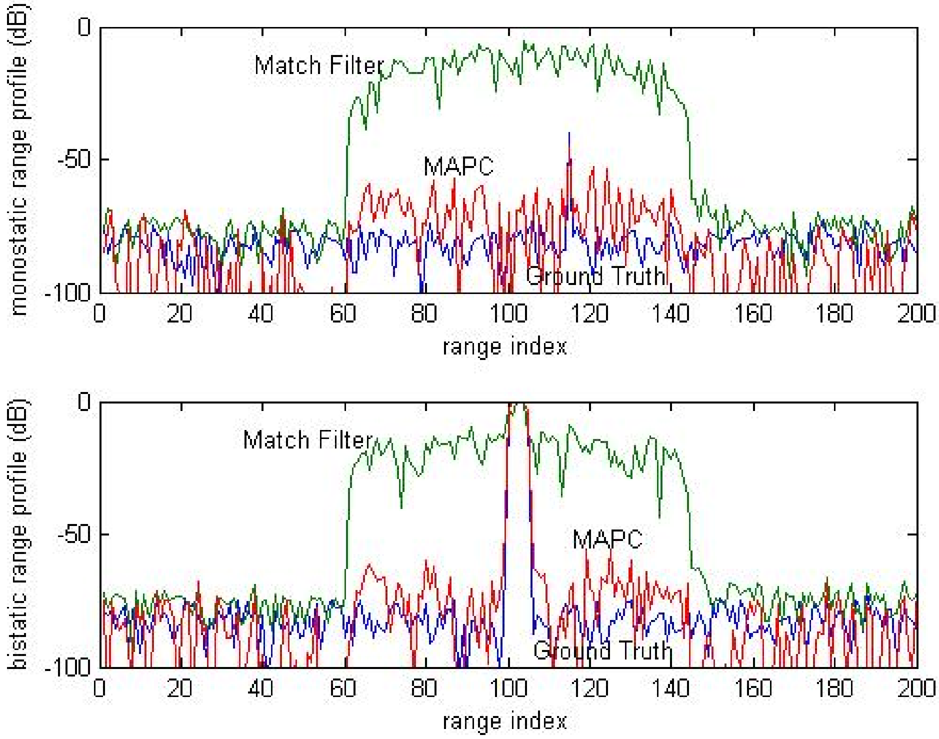

Figure 3.2 Two Multistatic Radars where the extended target in the bistatic profile masks the smaller scatterer in the monostatic profile.

In this figure the MAPC algorithm estimates the power both the extended target and the small scatter and the matched filter masks the small scatter and has an incorrect estimate of the power of the extended target.

For the multistatic approach to CLEAN using an adaptive algorithm, CLEAN will only need to perform one iteration of subtractions for each of the scatterers that are detectable within the range profile and the range profile that is created is used to determine the complex amplitude of all of the detectable scatterers causing interference. The scatterers that cause interference within the range profile of interest are found and subtracted so that a new received signal is created. This new received signal can now be processed by the MAPC algorithm which will further suppress the interference from smaller undetectable scatterers that were left behind.

Both the MAPC, MPCR and the CLEAN algorithm have the same objective which is trying to suppress the interference from unwanted scatterers but each algorithm takes a different approach in suppressing these scatterers. The adaptive algorithms are MMSE based approaches which try to estimate the interference by nulling scatterers. These nulls are adaptively placed and there are a limited number of degrees of freedom that the algorithm has to cancel interference. When there are dense scattering environments, this algorithm has a hard time suppressing the interference from all of the scatterers and the adaptive algorithms try to balance the suppression of all of the scatterers within the range profile to minimize the mean squared error.

The CLEAN algorithm is not limited by adaptive degrees of freedom to suppress interference from scatterers. The CLEAN algorithm is limited by the CA-CFAR detection of an interfering scatterer and by the accuracy of the estimate of the complex amplitude of the scatterer. If either of these two aspects is compromised the CLEAN algorithm can create a false target or a significant amount of the interfering scatterers can be left behind in the received signal by the CLEAN algorithm.

The characteristics of both the CLEAN and the adaptive algorithms make up for the deficiencies of the other algorithm. Using an adaptive algorithm as the first pulse compression stage creates an estimate of the range profile that has accurate information about the complex amplitude and location of other scatterers that are within each range profile. Then the CLEAN algorithm is able to use this information to detect these scatterers which tend to be scatterers with a large SNR and place a fixed null by subtracting the interfering signal. Then the new received signal is given to the adaptive algorithm for the second iteration of the pulse compression algorithm. The adaptive algorithm is able to suppress the undetectable interference more accurately compared to the previous iteration since it does not have to suppress the larger interfering targets that were subtracted by the CLEAN algorithm.

3.2.2 Hybrid Clean

Due to the accuracy of the range profile estimates of the adaptive algorithms a variant of the CLEAN algorithm was developed to remove bistatic interference. This CLEAN algorithm is called Hybrid Clean. Hybrid Clean receives the information about the location of the detected scatterers and obtains the complex amplitude of those scatterers from the pulse compressed signal. At this point the Hybrid CLEAN algorithm uses this information and subtracts the detected interference signals from the received radar signal. The signal left behind after the subtraction after all of the detected inferring scatterers have been removed is the new radar received signal $\tilde{\mathbf{y}}(l)_n$. Initially $\tilde{\mathbf{y}}(l)_n$ equals $\mathbf{y}(l)_n$ before any of the subtractions of the received signal occur. Then for each scatterer detected in the interfering range profiles the following equation is used to create the CLEANed version of the received radar signal.

$$\tilde{\mathbf{y}}(l)_n = \mathbf{y}(l)_n - \mathbf{s}_i \hat{x}_{i,n}(l) \tag{3.14}$$

Where $\tilde{\mathbf{y}}(l)_n = [\tilde{y}(l)_n \quad \tilde{y}(l+1)_n \quad \ldots \quad \tilde{y}(l+N-1)_n]$ and $\mathbf{y}(l)_n = [y(l)_n \quad y(l+1)_n \quad \ldots \quad y(l+N-1)_n]$ which are both N samples long. The term $\hat{x}_{i,n}(l)$ is a complex scalar that represents the estimated complex amplitude of the detected scatterer and $\mathbf{s}_i$ is the transmitted waveform of the detected interference.

Once all of the detected interference has been removed from $\tilde{\mathbf{y}}(l)_n$ this new received signal is used as the received signal for the post-processor. The post-processor uses the newly formed received signal to determine the range profile whose scatterers were left in this signal. The scatterers that are in the range profile of interest are not removed from this signal.

3.2.3 Multiple Repetition Projected CLEAN

Another method using the CLEAN approach to remove interfering targets is by projecting part of the received signal that contains the interference into another subspace so that when a pulse compression technique like the matched filter is used, there is a very low correlation with the interference. The projection matrix should project the scatterer to a subspace that is perpendicular to subspace that the interfering transmitted waveform occupies with in the received signal,

$$\tilde{\mathbf{y}}(l)_n = \mathbf{P}_i \mathbf{y}(l)_n . \tag{3.15}$$

This projection matrix,

$$\mathbf{P}_i = \mathbf{I} - \frac{\mathbf{s}_i \mathbf{s}_i^H}{\mathbf{s}_i^H \mathbf{s}_i} \tag{3.16}$$

projects the samples within the vector $\mathbf{y}(l)_n$ on to a subspace that is perpendicular to the subspace that was occupied by the interfering signal [11]. One important aspect of this projection is that it is a particular implementation of the Hybrid CLEAN algorithm that uses the matched filter estimate to determine the complex amplitude of the subtracted interference. The only difference is that the amplitude estimate is determined after each subtraction compared to using the amplitude estimates from the preprocessor's output if the preprocessor was the matched filter.

Using the projection method for the CLEAN algorithm has some interesting properties that are not seen by the Hybrid CLEAN implementation of the CLEAN algorithm. First if we look at a monostatic signal,

$$\mathbf{y}(l) = \alpha \mathbf{s}_{1,0} + \beta \mathbf{s}_{1,1} \tag{3.17}$$

that has two returns from two scatters that occupy two different range cells. The terms α and β are the amplitudes of both of the returns. The term $\mathbf{y}(l) = [y(l) \quad y(l+1) \quad \ldots \quad y(l+N-1)]$ and is N samples of the received waveform and

$\mathbf{s}_{k,\tau}$ is the transmitted waveform transmitted by the k^{th} radar and shifted by τ samples this vector is N samples long. For example $\mathbf{s}_{1,1} = [0 \quad s_0 \quad s_1 \quad \cdots \quad s_{N-2}]^T$. When the projection is applied to $\mathbf{y}(l)$ and then applied to $\mathbf{y}(l+1)$ the following received signal is found.

$$\mathbf{y}(l) = \alpha\mathbf{s}_{1,0} + \beta\mathbf{s}_{1,1} \tag{3.18}$$

$$\mathbf{P}_1\mathbf{y}(l) = \left(I - \frac{\mathbf{s}_{1,0}\mathbf{s}_{1,0}^H}{\mathbf{s}_{1,0}^H\mathbf{s}_{1,0}} \right)\alpha\mathbf{s}_{1,0} + \left(I - \frac{\mathbf{s}_{1,0}\mathbf{s}_{1,0}^H}{\mathbf{s}_{1,0}^H\mathbf{s}_{1,0}} \right)\beta\mathbf{s}_{1,1} \tag{3.19}$$

$$\mathbf{P}_1 y(l) = \alpha\mathbf{s}_{1,0} - \alpha\mathbf{s}_{1,0} + \beta\mathbf{s}_{1,1} - \beta\frac{\mathbf{s}_{1,0}\mathbf{s}_{1,0}^H}{\mathbf{s}_{1,0}^H\mathbf{s}_{1,0}}\mathbf{s}_{1,1} \tag{3.20}$$

$$\tilde{\mathbf{y}}(l) = \mathbf{P}_1\mathbf{y}(l) = 0 + \beta\mathbf{s}_{1,1} - r\beta\mathbf{s}_{1,0} \tag{3.21}$$

Where the term

$$r = \frac{\mathbf{s}_{k,\tau}^H\mathbf{s}_{k,\lambda}}{\mathbf{s}_{k,\tau}^H\mathbf{s}_{k,\tau}} \tag{3.22}$$

represents the maximum value found in the autocorrelation of the transmitted waveform when $\lambda \neq \tau$.

Then the projection is applied to the shifted term.

$$\tilde{\mathbf{y}}(l+1) = \beta\mathbf{s}_{1,0} - r\beta\mathbf{s}_{1,-1} \tag{3.23}$$

$$\mathbf{P}_1\tilde{\mathbf{y}}(l+1) = \mathbf{P}_1(\beta\mathbf{s}_{1,0} - r\beta\mathbf{s}_{1,-1}) \tag{3.24}$$

$$\mathbf{P}_1\tilde{\mathbf{y}}(l+1) = \left(I - \frac{\mathbf{s}_{1,0}\mathbf{s}_{1,0}^H}{\mathbf{s}_{1,0}^H\mathbf{s}_{1,0}} \right)\beta\mathbf{s}_{1,0} + \left(\mathbf{I} - \frac{\mathbf{s}_{1,0}\mathbf{s}_{1,0}^H}{\mathbf{s}_{1,0}^H\mathbf{s}_{1,0}} \right)(-r\beta\mathbf{s}_{1,-1}) \tag{3.25}$$

$$\mathbf{P}_1\tilde{\mathbf{y}}(l+1) = \beta\mathbf{s}_{1,0} - \beta\mathbf{s}_{1,0} - r\beta\mathbf{s}_{1,-1} + r^2\beta\mathbf{s}_{1,0} \tag{3.26}$$

$$\tilde{\mathbf{y}}(l) = 0 - r\beta\mathbf{s}_{1,0} + r^2\beta\mathbf{s}_{1,1} \tag{3.27}$$

After both projections have been applied it leaves a signal that is similar to the unaltered received signal. The only difference is that the α term has been removed and the $-r\beta$ term has replaced it and the β term is now scaled by the r^2 term for the second scatterer. It is important to note that the term r will be much less than one so the newly formed receive signal will have returns that are in the same place as the previous two returns. Both of the returns will be significantly smaller and neither return will retain it's phase but the intention is to suppress both of the scatters so the information about their phase is unimportant. If these two projections were applied M times in this order the newly formed received signal will have the following form.

$$\tilde{\mathbf{y}}(l) = -r^{2M-1}\beta\mathbf{s}_{1,0} + r^{2M}\beta\mathbf{s}_{1,1} \tag{3.28}$$

Each time both of these projections are applied the targets in the received signal would be significantly diminished. The other interesting aspect of this situation is that the α has been removed and if the first projection had occurred at $\mathbf{y}(l+1)$ instead of $\mathbf{y}(l)$ the β term would have been removed. This is one of the reasons why the scatterer with the largest SNR is removed in the implementation of the CLEAN algorithm. The other two obvious reasons are that it might be the only detectable scatterer to remove and by removing it there is a greater possibility that its sidelobes might be covering other smaller targets due to its size.

When viewing the projection matrix from the multistatic point of view a similar outcome occurs. One thing that we will assume about this implementation is that α is greater than β to show that when it is projected first the largest amount of interference is removed. The term $\mathbf{s}_{2,1}$ is the transmitted waveform of the desired returns and the term Δ is the amplitude of the desired scatterer.

$$\mathbf{y}(l) = \alpha\mathbf{s}_{1,0} + \beta\mathbf{s}_{1,1} + \Delta\mathbf{s}_{2,1} \tag{3.29}$$

$$\tilde{\mathbf{y}}(l) = \mathbf{P}_1\mathbf{y}(l) = \beta\mathbf{s}_{1,1} - r\beta\mathbf{s}_{1,0} + \Delta\mathbf{s}_{2,1} - c\Delta\mathbf{s}_{1,0} \tag{3.30}$$

Where the term

$$c = \frac{\mathbf{s}_{k,\tau}^H\mathbf{s}_{j,\lambda}}{\mathbf{s}_{k,\tau}^H\mathbf{s}_{k,\tau}} \tag{3.31}$$

represents the maximum value of one of the terms of the cross correlations of the two transmitted waveforms which means that $k \neq j$.

Then applying the projection to the shifted signal the following is found.

$$\tilde{\mathbf{y}}(l+1) = \mathbf{P}_1\mathbf{y}(l+1) = -(r\beta + c\Delta)\mathbf{s}_{1,-1} + (r\beta + c\Delta)r\mathbf{s}_{1,0} + \Delta\mathbf{s}_{2,0} - c\Delta\mathbf{s}_{1,0} \tag{3.32}$$

$$\tilde{\mathbf{y}}(l) = -(r\beta + c\Delta)\mathbf{s}_{1,0} + (r\beta + c\Delta)r\mathbf{s}_{1,1} + \Delta\mathbf{s}_{2,1} - c\Delta\mathbf{s}_{1,1} \tag{3.33}$$

It is important to note the role of both terms c and r. These two terms are significantly less than one which cause both interfering scatterers to be reduced but the desired term $\Delta\mathbf{s}_{2,1}$ remains unaffected even though both projections took place. The other interesting thing that occurs is that a false target appears in place of the target that is being projected if another target is overlapping, which means that target exists within its sidelobes. A false target can occur in any variation of the CLEAN algorithm if there is a false alarm in the detector. An example of this occurrence is easily demonstrated with a monostatic received signal

$$\mathbf{y}(l) = \alpha \mathbf{s}_{1,0} \tag{3.34}$$

that contains one target. If the received signal is shifted by one sample and then projected a false target will appear.

$$\mathbf{y}(l+1) = \alpha \mathbf{s}_{1,-1} \tag{3.35}$$

$$\tilde{\mathbf{y}}(l+1) = \mathbf{P}_1 \mathbf{y}(l+1) = \left(\mathbf{I} - \frac{\mathbf{s}_{1,0}\mathbf{s}_{1,0}^H}{\mathbf{s}_{1,0}^H \mathbf{s}_{1,0}} \right) \alpha \mathbf{s}_{1,-1} \tag{3.36}$$

$$\tilde{\mathbf{y}}(l+1) = \alpha \mathbf{s}_{1,-1} - r\alpha \mathbf{s}_{1,0} \tag{3.37}$$

$$\tilde{\mathbf{y}}(l) = \alpha \mathbf{s}_{1,0} - r\alpha \mathbf{s}_{1,1} \tag{3.38}$$

This demonstrates the need for accuracy when detecting targets and how important a low probability of false alarm is. Another interesting conclusion that can be made from this false target analysis is that it shows why there needs to be some form of detection instead of projecting every range cell to possibly remove a target.

This analysis leads to the derivation of the Multiple Repetition Projected CLEAN (MRP-CLEAN) algorithm. This method of the CLEAN algorithm would be placed in the block labeled CLEAN that is shown in the block diagram, figure 3.1. It would be used when a deterministic filter is used as the preprocessing stage instead of an adaptive filter. Due to the lack of accuracy from the estimates of a deterministic filter, the detected targets can be projected multiple times to suppress the interference

The MRP-CLEAN algorithm would use the detector to determine the location of the detected interfering scatterers. Then the brightest detected scatterer is found and projected. Then the next brightest detected scatter is found and projected from the range profile. This process is repeated until all of the detected scatters have been projected once.

Then multiple rounds of projections are applied to the all of the detected scatters in same order that they were projected in the first round. The number of rounds of projections would be determined by the radar operator's computational requirements and by how far the radar operator would like the signal to be suppressed. When testing the MRP-CLEAN algorithm, it performed five rounds of projections on the detected scatterers.

Using MRP-CLEAN will never completely eliminate an interfering target which can be seen in the monostatic case (3.28). The only way to completely remove a target is to have an exact estimate of the target's complex amplitude and even with pulse compression algorithms like MAPC this is still unachievable.

3.2.4 Hybrid Pre-processors

As previously discussed the complex amplitude estimate of the range profile is one of the factors in the performance of the CLEAN algorithm. If there is a good amplitude estimate the detected interference can be removed from the range profile but if the estimate is incorrect it could generate a larger sidelobe or create more multistatic interference.

This can be illustrated using a simple monostatic model as the received signal. In this received signal,

$$\mathbf{y}(l) = \alpha \mathbf{s}_{1,0} \tag{3.39}$$

there is one scatterer that has a complex amplitude α. Then using some form of pulse compression to determine the estimate, $\hat{\alpha}$. Error from the pulse compression algorithm is introduced as δ so that the estimate is

$$\hat{\alpha} = \alpha - \delta . \tag{3.40}$$

The Hybrid CLEAN algorithm is then applied to the received signal using the amplitude estimate found from the pulse compression stage and the following received signal is formed.

$$\tilde{\mathbf{y}}(l) = \mathbf{y}(l) - \hat{\alpha}\mathbf{s}_{1,0} \tag{3.41}$$

$$\tilde{\mathbf{y}}(l) = \alpha\mathbf{s}_{1,0} - \alpha\mathbf{s}_{1,0} + \delta\mathbf{s}_{1,0} \tag{3.42}$$

$$\tilde{\mathbf{y}}(l) = \delta\mathbf{s}_{1,0} \tag{3.43}$$

This analysis shows the importance in using an approach that reduces the error in the estimate of the range profile. When multiple targets are removed using the Hybrid CLEAN algorithm multiple residual δ terms are left behind and if δ has a large magnitude, then the residual term could be considered a target. It is important to use a pre-processor that will have very little error when determining the estimate of the target.

There are many options for the preprocessing stage to generate the estimate of the range profile. For instance if there were only two radars operating and the bistatic and monostatic range profile contained only one scatterer within each profile and each scatterer did not overlap. The most computationally efficient way to get a good estimate for the complex amplitudes of each scatterer would be the matched filter. In this type of situation applying the MAPC algorithm would provide the best range profile estimate compared to using a hybrid algorithm because the MAPC algorithm would not need the subtractive nulls from the CLEAN algorithm to suppress the interference. In other cases when the range profiles are more dense, the matched filter and the least squares algorithm are both unable to generate range profiles that have significantly low sidelobes to detect small scatters but the larger targets maybe detectable and the complex amplitude estimate may have some slight ambiguities. If there were not a high level of computational constraints the MAPC algorithm would generate the best range profile for pre-processor block of the Hybrid Model.

The following shows the range profile of a scatterer in one range profile that is masked by the extended target within the other range profile.

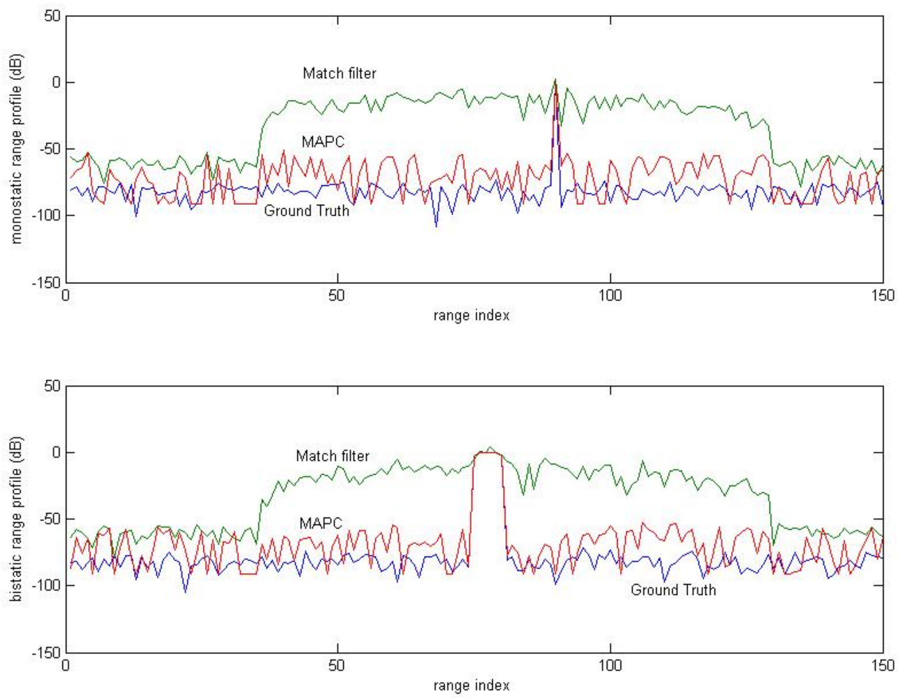

Figure 3.3 Range Profiles from the MAPC algorithm

From figure 3.3, using this range profile the mean squared error was determined for both the matched filter and the MAPC algorithm. This error would be represented by the δ term in the analysis.

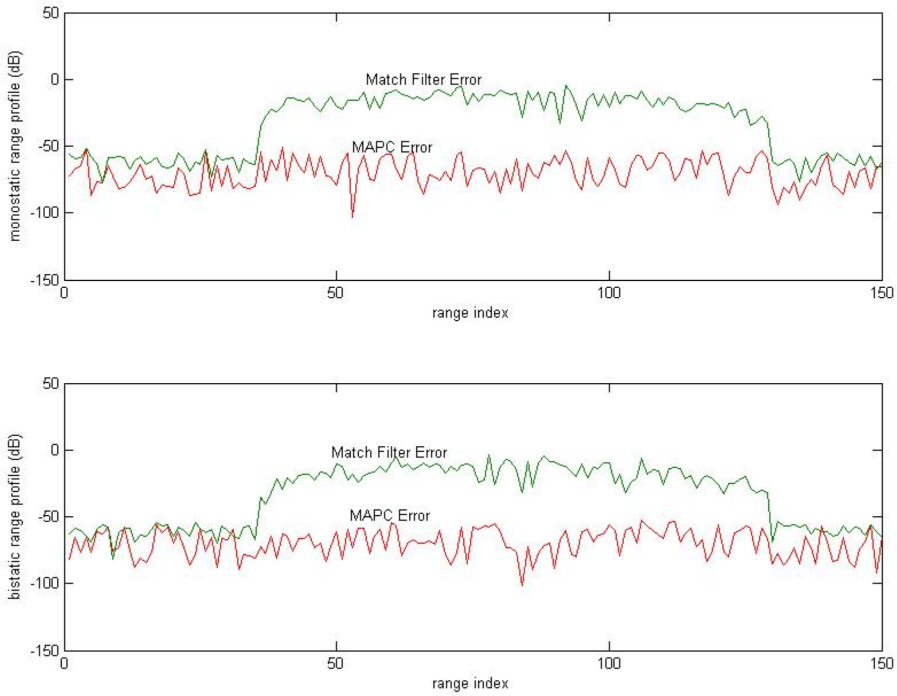

Figure 3.4 Error from the Matched filter and the MAPC algorithm

By using the MAPC algorithm the amount of error introduced when subtracting the range profile estimate is reduced significantly compared to the matched filter and should be a consideration when choosing a pre-processor for the Hybrid algorithm. Figure 3.4 shows the error for each processed range profile and the error for the MAPC algorithm is significantly less when the extended target is causing interference.

The choice for the pre-processor algorithm depends on the environment and the computational demands of the DSP processor but like all engineering decisions there are tradeoffs. The two tradeoffs when determining the pre-processor are between the computational efficiency of the pre-processor and the precision in the final range profile. The MAPC algorithm can generate a very precise range profile but the computational demands are high compared to the matched filter.

3.2.5 Hybrid Post-processors

When deciding on a post-processor there are fewer options to choose from. If a deterministic post-processor were used this algorithm would be similar to a few iterations of the Coherent CLEAN algorithm. The performance gains from the Coherent CLEAN algorithm are noticed after all of the detectable targets have been removed. In most cases this would require significantly more iterations of the algorithm because large targets have been removed from the received signal and smaller targets have been uncovered. The other problem with the matched filter and the multistatic least squares algorithm as a post-processor is that both algorithms have large sidelobes while the adaptive approaches can suppress the sidelobes thus reducing the error in the range profile estimate.

The sidelobe levels in the pre-processor in certain situations will not have a significant effect on the range profile in the final iteration since only the complex amplitude and location are necessary to remove the target with the Hybrid CLEAN algorithm discussed in section 3.2.2 and only the location of the interference is needed in the MRP-CLEAN algorithm as it was shown in section 3.2.3. Once the location and complex amplitude are affected by the sidelobes of the targets, in cases where a target is undetectable or there are significant ambiguities in the estimation of the amplitude, then an adaptive approach is needed for the preprocessor.

The post-processor in the hybrid model needs the suppression of the sidelobes and the smaller undetectable interference to reduce the error in the range profile thus making small scatterers detectable when other interfering scatterers are present. APC, PCR, MAPC or MPCR can be used as the postprocessor for the hybrid processing model. If the post-processor is a deterministic processing method the hybrid algorithm will experience significant since an adaptive approach is not used as the post-processor and in most cases it would have been better to have just used an adaptive algorithm or to just use the Coherent CLEAN algorithm.

3.2.6 Detector

In the block diagram the range profile from the pre-processor is sent to the detector which is a CA-CFAR. This is the same CA-CFAR that has been mentioned in Chapter 2. The threshold T, for the deterministic pre-processors was set to 8 dB due to the sidelobe effects while the threshold for the adaptive pre-processors was set to 12 dB. The guard cells on each side of the CA-CFAR was 10 range cells long and the processing windows on each side was 30 range cells long on each side of the guard cells.

This CA-CFAR can be replaced by another detector depending on the statistics of the signal received. It is important to note that the only information that the Hybrid CLEAN algorithm needs about the detected interfering scatters is only the complex amplitude and location. The MRP-CLEAN algorithm needs the location of the detected interference since it estimates the complex amplitude using the matched filter in the formulation of the projection. The detection process for this algorithm was simplified to demonstrate the power of CLEAN and the adaptive algorithms combined.

3.3 Integrated CLEAN

Due to the computational complexity of running the MAPC algorithm as both the pre-processor and the post-processor other methods were explored to reduce the burden on the DSP processor. One method previously discussed is to use a deterministic preprocessor to reduce the amount of computations. Another way is to place the CLEAN algorithm within each iterative stage of the MAPC algorithm. As discussed previously the MAPC algorithm performs multiple stages of pulse compression to obtain better estimates of the range profile. The Coherent CLEAN algorithm uses multiple iterations of a pulse compression algorithm to find and subtract targets from the received signal. Using the implementation of the CLEAN algorithm described in section 3.2.2 the CLEAN algorithm can be integrated into the MAPC algorithm.

3.3.1 Integrated CLEAN MAPC

The Integrated CLEAN Multistatic Adaptive Pulse Compression (IC-MAPC) algorithm implements the first stage of the MAPC algorithm. After the estimate of that stage is computed it is used in the CA-CFAR which detects the targets within that estimate. This information is then sent to the Hybrid CLEAN algorithm which removes the detected scatters from the range profile. The new received signal is then used for the next iteration of the MAPC algorithm.

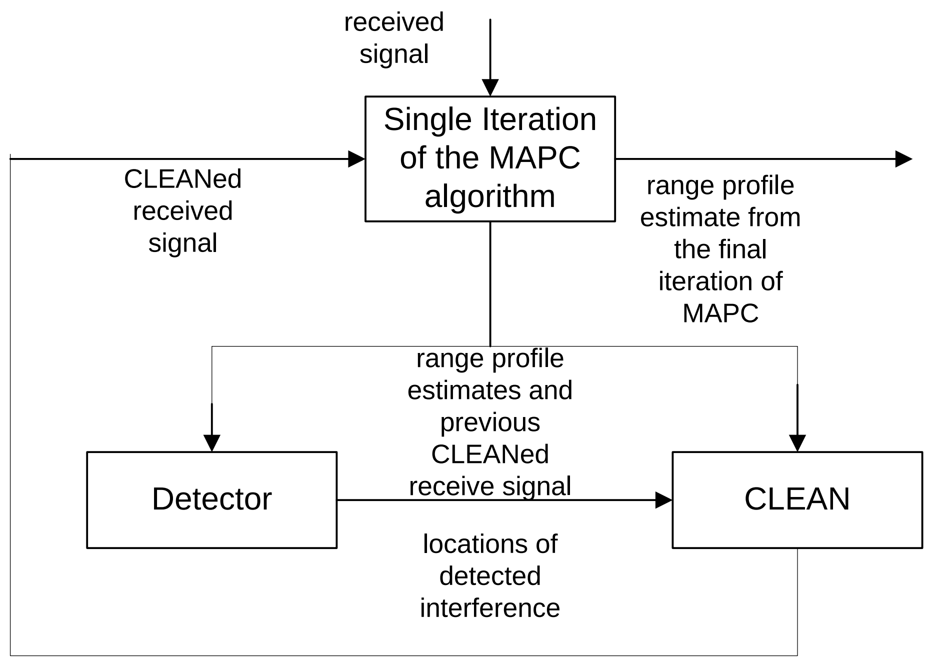

Figure 3.5 Data flow model of the IC-MAPC algorithm

Figure 3.5 has a similar structure as the Hybrid Processing model in figure 3.1 the difference is that this model is placed within each iteration of the MAPC algorithm compared to allowing the MAPC algorithm to process each stage and then apply the CLEAN algorithm to the received signal. This method requires fewer computations than the Hybrid model when an adaptive algorithm is used as the preprocessor.

Another consideration that is made in the development of this algorithm was removing the detected scatter from the estimate of the range profile so that the MAPC algorithm on the following stage would not consider the removed scatter when creating the filters for the range profile. This was done by subtracting the estimated amplitude of the scatter from the estimate used to calculate the filters for the next reiterative stage. This lead to instability in the algorithm so the amplitude estimates were unaltered. The MAPC algorithm decreased the amplitude of the range cell where the detected scatter was removed adaptively on the next stage. Changing the amplitude estimates at each stage was not necessary.

3.3.2 IC-MAPC Detector and CLEAN algorithm

Both of the CLEAN algorithms that were discussed in section 3.2.2 and section 3.2.3 can be used in the IC-MAPC algorithm. The Hybrid CLEAN method is a better choice for the IC-MAPC algorithm. After the first stage of the MAPC algorithm the estimates of the range profile have less error than the matched filter. The matched filter is the basis of the MRP-CLEAN algorithm which was shown in section 3.2.3.

The detector used in the algorithm can also be the same as in the Hybrid processing model. The detector used in the implementation of this algorithm was the CA-CFAR. Other detectors might be able to detect where the targets are more accurately and could possibly improve the performance of the IC-MAPC algorithm. Varying the threshold level T at each iteration of the IC-MAPC algorithm can help obtain better estimates. Future research into other detection algorithms is needed to improve the probability of detection while reducing the computational complexity of the algorithm.

Chapter 4 Simulation Results

Simulations in this chapter used waveforms that were constant modulus and had a random phase. The phase of each of the samples of each transmitted waveform were determined by a uniform random variable that ranged from 0 to 2π. The noise power in range profiles is a Gaussian distribution that has constant power. Targets in the model are considered to be point targets. The extended targets modeled in this simulation are a series of point targets that have been placed side by side. The outermost range cells of each extended target are 3 dB lower than the range cells on the inside of the extended targets. All of the probability of detection simulations in this section where run using a CA-CFAR that had a probability of false alarm equal to 10^{-6}. All of the simulations that are presented in this chapter are Monte Carlo simulations that had at least 1000 iterations or more. The CA-CFAR that determined the probability of detection did not use guard cells and the windows on each side of the CA-CFAR were 32 range cells long.

4.1 The MPCR Algorithm

When comparing the MPCR algorithm to the MAPC algorithm the most important metric is the mean squared error (MSE). Since both algorithms have an objective function that tries to minimize the MSE of the range profile it is important to compare the algorithms in terms of MSE. The MPCR algorithm has shown comparable results to the MAPC algorithm in terms of MSE. One of the expectations for this algorithm was that it would be able to show significant improvements in MSE over the MAPC algorithm due to the increase in adaptive degrees of freedom in the matched filtered domain but this was not the case. For example when there is a single target in the presence of the sidelobes of large extended target in the bistatic range profile and the MSE of the estimated range profile of the single target is 51 dB.

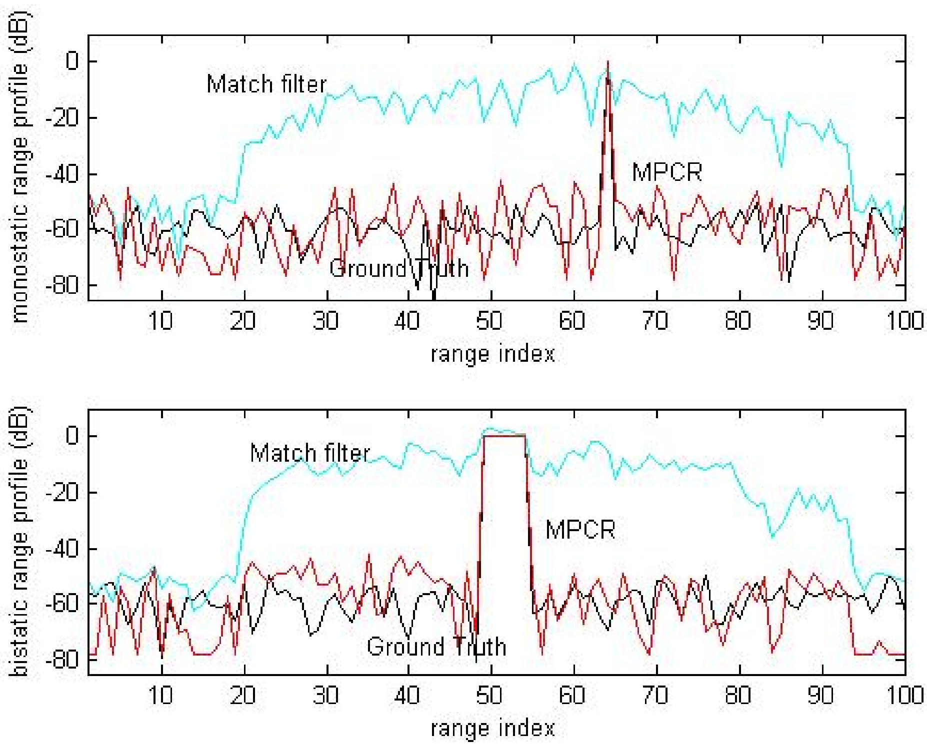

Figure 4.1 MPCR applied to a multistatic received signal with a single target in the presence of the sidelobes an extended target 6 range cells long

While using a similar received signal for the MAPC algorithm, MAPC is able to generate an estimate that has an MSE of 49 dB.

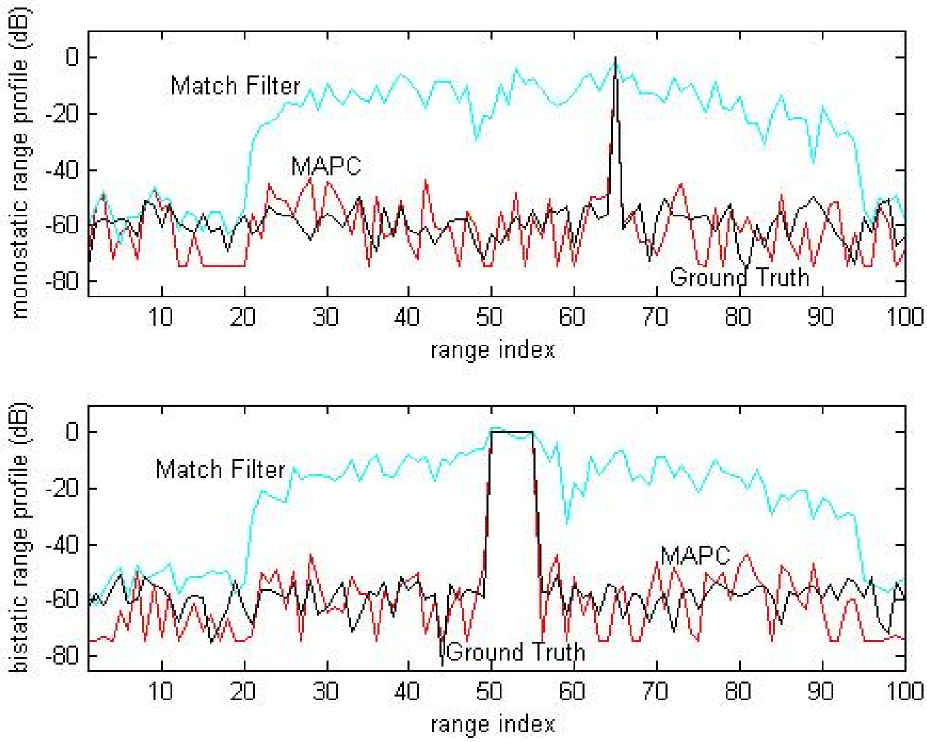

Figure 4.2 MAPC applied to a multistatic received signal with a single target in the presence of the sidelobes an extended target 6 range cells long

The MPCR algorithm does have a better range profile estimate than the MAPC algorithm for these two specific cases there is one detail about the two algorithms that is not immediately apparent from the derivation of either algorithm. MAPC requires the samples from the received signal to be the twice the number of reiterative stages multiplied by $N-1$ plus the number of cells in the range profile of interest while the MPCR requires the samples from the received signal to be the twice the number of reiterative stages multiplied by $2N-1$ plus the number of cells in the range profile of interest. So the MPCR algorithm needs more data to generate a range profile that is the same length as the MAPC algorithm.

Taking this difference into consideration both algorithms were tested on a received signal containing the same number of samples. Each of the algorithms used the maximum number of adaptive stages allowed to generate a range profile the same size and place within the range profile using the identical received signals. The received signals contained returns from range profiles that were randomly generated to simulate a range profile with a dense number of scatterers.

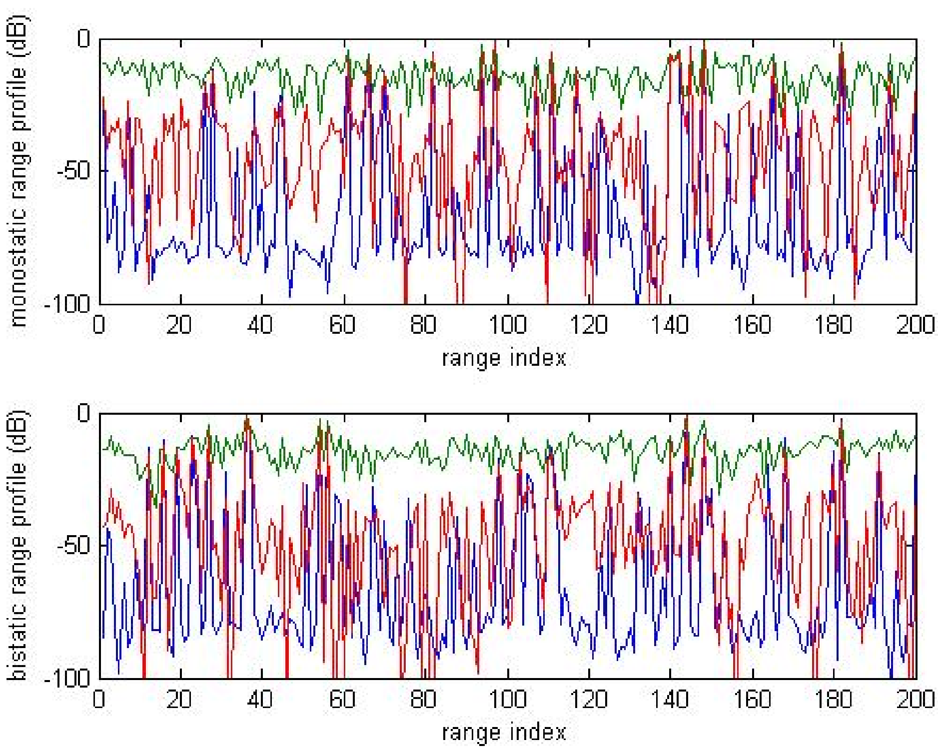

Figure 4.3 MAPC using a dense channel

This figure is a dense range profile that is a example of the range profiles that the MAPC and the MPCR algorithm were tested on. The red line is the MAPC algorithm and the blue line is the ground truth. The green line is the matched filtered response. Range profiles like this one were used to produce the following plot to show the processing capabilities of the MAPC and MPCR algorithms.

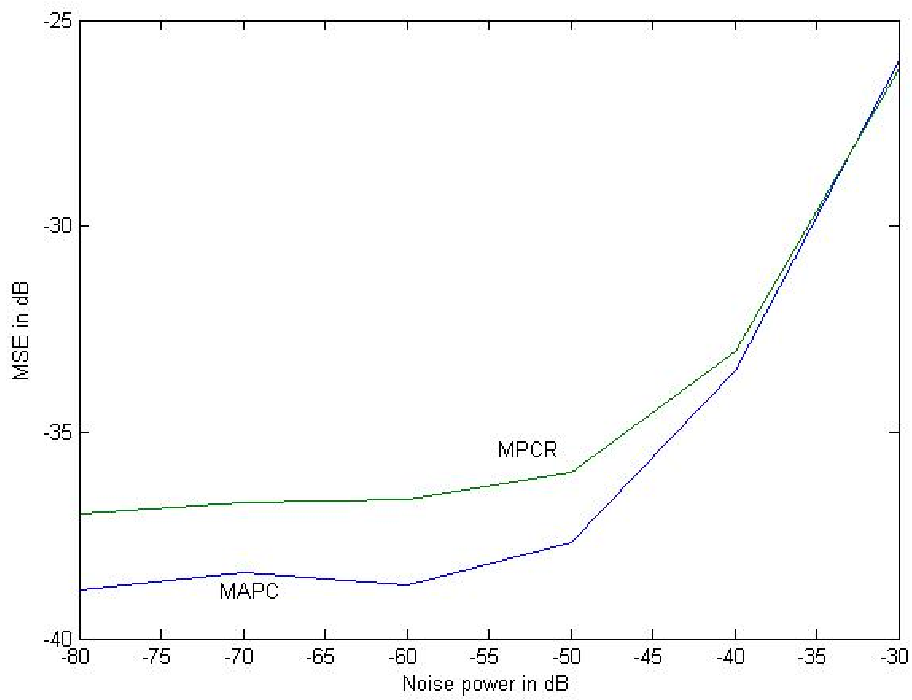

Figure 4.4 MSE of MAPC and MPCR using a dense range profile

In this plot the MAPC algorithm had seven reiterative stages while the MPCR had four reiterative stages. In this plot it is clearly evident that the MAPC algorithm outperforms the MPCR algorithm in terms of MSE. This performance comparison lead to further research in improving the MAPC algorithm instead of the MPCR algorithm but it is important to note that many of the hybrid techniques that are mention in this paper can be easily applied to the MPCR algorithm since it is similar to the MAPC algorithm.

There are some capabilities that the MPCR algorithm has over the MAPC algorithm. The MPCR algorithm is able to process returns for in-service radars and mitigate RF fratricide for in-service radars. Since the algorithm is able to operate in the matched filtered domain of the signal it is able operate on the matched filtered response of the signal from legacy radars.

Another aspect of this algorithm that was explored, is the MPCR algorithm is also able to operate on signals that are filtered with other types of deterministic filters instead of the matched filter. These filters could be designed to reduce the amount of cross-correlation that the transmitted waveforms have with each other or increase the gain for a certain Doppler shift. It is important to note that some of the filters described in [14] were used and the algorithm was able to converge. Some filters were $N+1$ samples long or longer which created cross correlations with the transmitted signal that were greater than $2N-1$ and these cross correlations were used as the column vector $\mathbf{p}_{m,n}$. Where the subscript m represents the deterministic filter used to filter the received signal and n represents the transmitted waveform. Further research would need to be done to develop these filters and waveforms to achieve desired levels of performance [14].

4.2 Deterministic Pre-processors for the Hybrid Model

As discussed in section 3.2.4 deterministic pre-processors can be used in the hybrid model with varying levels of performance. When discussing the different configurations of the pre-processor, the CLEAN algorithm and the post-processor that were used in the hybrid model they will be referred to in the order that was just mentioned. For instance if the Pre-Processor is the multistatic least squares algorithm and the CLEAN algorithm is MRP-CLEAN and the post-processor was the MAPC algorithm this specific hybrid processing algorithm would be referred to as Least Squares MRP-CLEAN MAPC. This description will be found throughout the rest of this paper and on the plots. The detector within the hybrid model will always be the CA-CFAR unless indicated differently.

For the deterministic pre-processors there are different configurations between the Hybrid CLEAN algorithm and the MRP-CLEAN algorithm that have varying results. The situation that was simulated in this scenario was a single point scatterer in the monostatic profile whose SNR was varied between 0 and 60 dB. While an extended target that was six range cells long was in the middle of the bistatic profile. The inner range cells of the extended target had a SNR of 60 dB. An example of what this range profile would look like can be found in figure 3.3.

When comparing the different algorithms one of the baselines for comparison is the MAPC algorithm. Since most of the algorithms presented in this paper will use the MAPC algorithm.

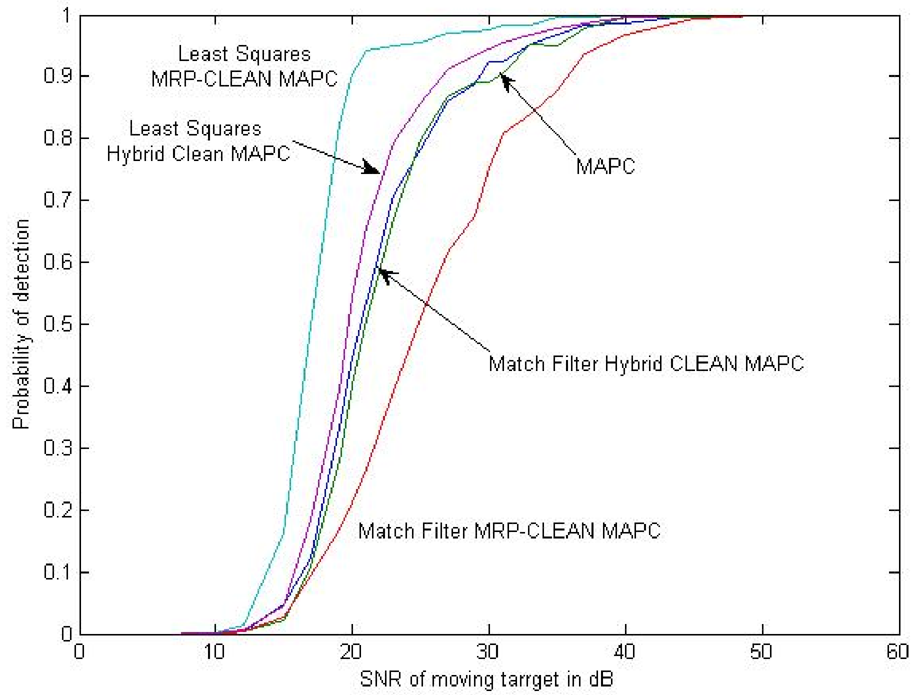

Figure 4.5 Probability of Detection of different Hybrid algorithms with deterministic pre-processors

In figure 4.5 the probability of detection of the small moving target decreases for the Matched Filter MRP-CLEAN MAPC algorithm. This is due to the inability of the CA-FAR to correctly identify the extended target within the bistatic profile. Using the Matched filter the CA-CFAR is able to identify the some of the innermost range cells of the extended target but there are still some large residual parts left over on the outermost range cells of the extended target. The MAPC algorithm can achieve a smaller MSE when there is one extended target compared to an extended target where the power level varies dramatically across the range cells that it occupies. This type of received signal is generated when only part of the extended target is detected.

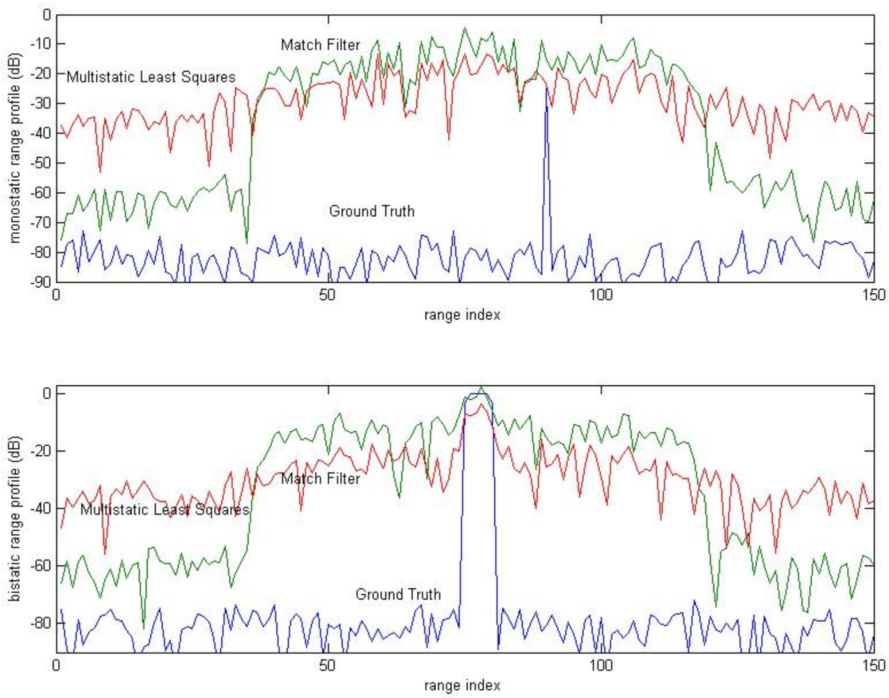

Figure 4.6 Least Squares and the Matched Filter applied to the extended target

The range profile in figure 4.6 there is part of the extended target which is more detectable compared to the rest of the extended target. The Matched Filter Hybrid CLEAN MAPC implementation seems to perform the same as the MAPC algorithm or has some slight degradation. This is occurs because the complex amplitude of the extended target is not the same as the complex amplitude estimated by the matched filter. When this subtraction is made it can only increase the power of the extended target by 3dB which will diminish the probability of detection for the small scatter but not by a significant amount. The two hybrid algorithms that use the multistatic least squares algorithm as the pre-processor seem to have a higher probability of detection compared to the MAPC algorithm.

Using the least squares algorithm the sidelobes are lower but there is loss in the estimated power of the extended target. Since there is a loss in power of the estimated power of the extended target the Hybrid Clean algorithm will be unable to remove a large portion of the extended target since it uses the amplitude estimate from the least squares algorithm. The MRP-CLEAN algorithm, as mentioned earlier, is derived from the matched filter in its formulation. This allows it to remove more of the extended target by having a better estimate and then it is able to perform multiple projections to further suppress the interference from the extended target.

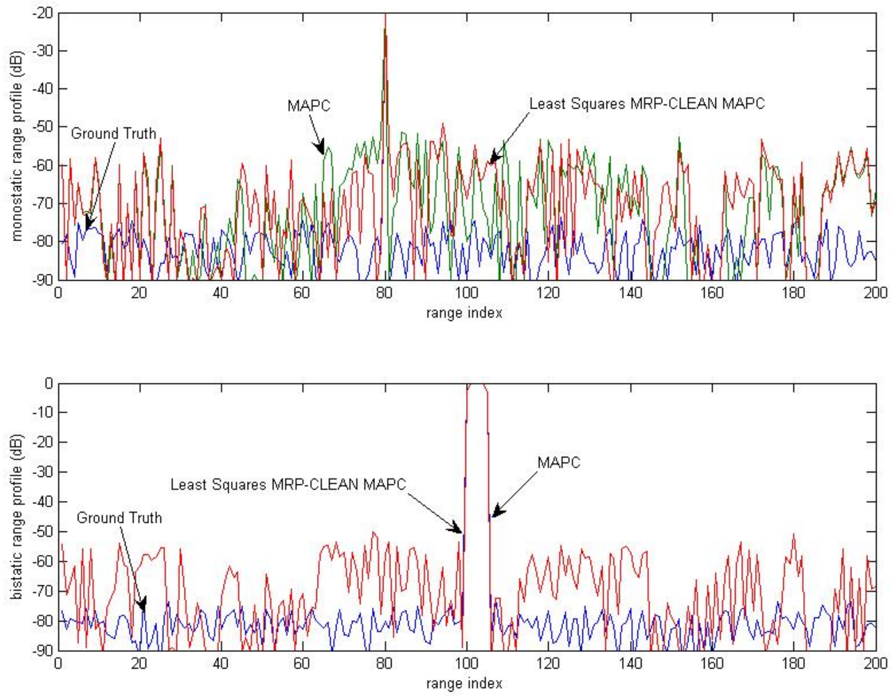

Figure 4.7 MAPC and Least Squares MRP-CLEAN MAPC applied to the extended target

Figure 4.7 shows some of the suppression seen from the Least Squares MRP-CLEAN MAPC algorithm compared to the MAPC algorithm and the ground truth. The hybrid algorithm is able to suppress some of the interference around the small scatter. The bistatic profile for both algorithms is identical since the small scatterer is initially undetectable to the least squares algorithm.

The MSE of the algorithms, shown in figure 4.8, also show why the deterministic filters have the probability of detection shown in that figure.

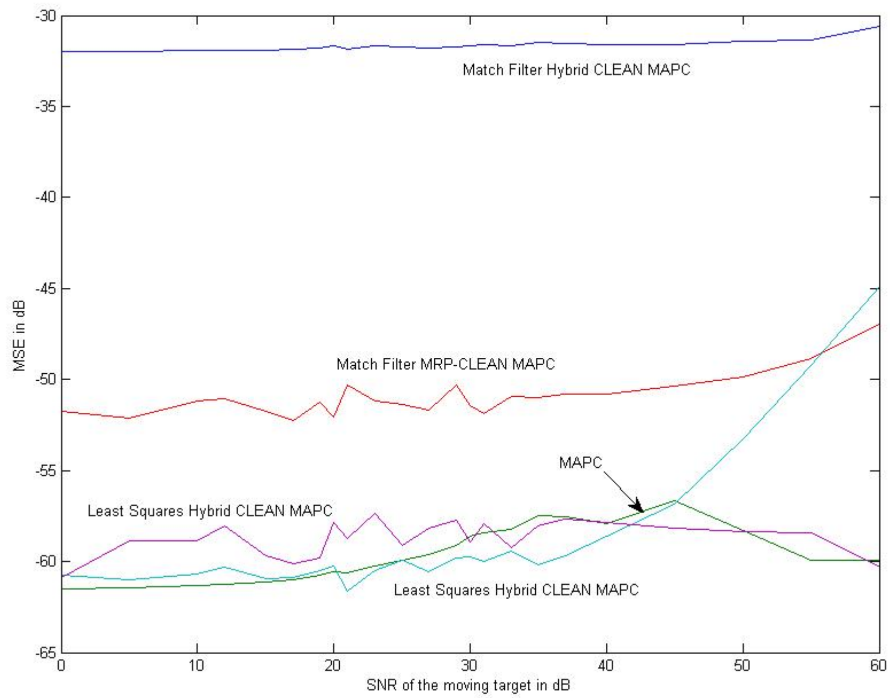

Figure 4.8 MSE of the different hybrid algorithms with deterministic pre-processors

From figure 4.8 both of the hybrid algorithms with the matched filter as the preprocessor have a worse MSE than the MAPC algorithm. This is due to the false detections discussed earlier or lack of detection. What is interesting about both algorithms is that even though the Matched Filter MRP-CLEAN MAPC algorithm has a better estimate of the range profile compared to the Matched Filter Hybrid CLEAN MAPC algorithm it performs worse in terms of probability of detection.

The Least Squares MRP-CLEAN MAPC algorithm has a large amount of MSE when the scatter is has a large SNR compared to when the SNR decreases. This is due to the scatter in the monostatic profile interfering with the estimate in the biastatic profile. Since the MRP-CLEAN algorithm is based off the matched filter estimate the large SNR of the scatter in the monostatic profile causes ambiguities in the estimation of the amplitude of the extended target for the projection. Once the SNR of the target in the monostatic profile starts to decrease better estimates are made and the MSE improves. This effect is not seen in the Least Squares Hybrid CLEAN MAPC algorithm because the Multistatic Least Squares algorithm tries to spread the MSE throughout the range profile so a better estimate is made and subtracted from the range profile.

4.3 Adaptive Pre-processors for the Hybrid Model

The other class of pre-processors that can be used to determine the location of interfering targets are the adaptive pulse compression algorithms. Only the multistatic algorithms like MAPC and MPCR are able to suppress interference from the scatterers in the other range profiles and provide accurate estimates of the range profiles. Due to the performance of the MPCR algorithm compared to the MAPC algorithm and the computational efficiency of the MAPC algorithm, compared to the MPCR, the MAPC algorithm was simulated for these results.

Although the APC algorithm is not well suited as a pre-processor it could be used as a post-processor. Assuming that the large scatters are removed or significantly suppressed in the CLEAN section of the hybrid model the resulting received signal would be similar to a monostatic signal with small amounts of interference from the other radars.

The other aspect of using an adaptive pre-processor like the MAPC algorithm is that it will have very accurate information about the amplitude of the scattterers in the bistatic range profile as mentioned in section 3.2.4. The Hybrid CLEAN algorithm will be able to suppress the interference better than the MRP-CLEAN algorithm due to the accuracy of the estimates. Just as it was mentioned in section 3.2.3, the MRP-CLEAN algorithm is unable to completely remove interference. Whereas the Hybrid Clean algorithm is able to remove a target with an accurate estimate of the range profile and only leave a small residual term behind. So when using an adaptive preprocessor like the MAPC algorithm the best CLEAN algorithm to use is the Hybrid CLEAN due to its ability to suppress the targets further and its computational efficiency.

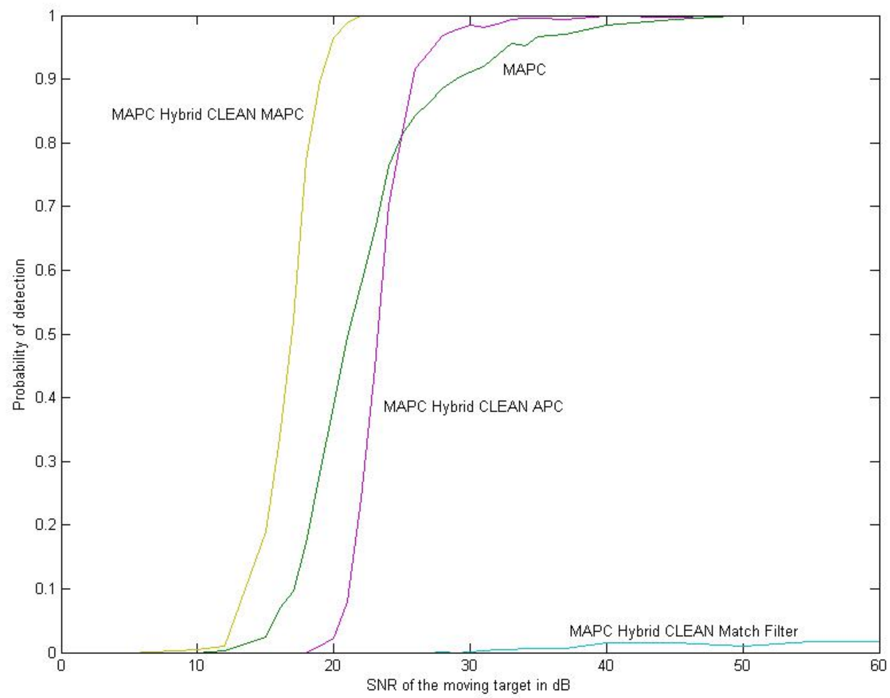

Figure 4.9 Probability of detection of Hybrid Algorithms with MAPC pre-processors

Figure 4.9 shows the probability of detection for three different implementations of the hybrid algorithm using the MAPC pre-processor. The MAPC Hybrid CLEAN Matched Filter algorithm does not perform well compared to the other algorithms but is shown here to demonstrate why it is necessary to have an adaptive algorithm as the post-processor. The inability to suppress the monostatic sidelobes and interference left over from the Hybrid CLEAN algorithm is what makes this algorithm perform poorly. This indicates that even after having accurate information about where the targets are located and their estimated complex amplitudes there will still be some interference left behind that needs to be adaptively suppressed.

The MAPC Hybrid CLEAN APC algorithm shows significant improvement over the MAPC algorithm. At a 0.9 probability of detection the MAPC Hybrid CLEAN APC algorithm is able to detect a target that has 3 dB less SNR than the MAPC algorithm at the same probability of detection. What is interesting about this algorithm compared to the MAPC Hybrid CLEAN Matched Filter is that is shows how important it is to suppress the monostatic sidelobes to obtain a range profile where the small target is easily detected.

The other interesting observation that can be made about the APC algorithm is its ability to deal with lower levels of interference and still be able to process the received signal. This shows how robust the APC algorithm is. This simulation shows that even when the signal model changes slightly the APC algorithm is still able to produce a usable range profile.

The most impressive probability of detection results from the MAPC Hybrid CLEAN MAPC algorithm. This algorithm has both the MAPC algorithm as a pre-processor and a post-processor and by having the MAPC algorithm as a post processor it is able to suppress the interference caused by not just the sidelobes but also the residual interference that is left over from the Hybrid CLEAN algorithm. The suppression of the residual interference from the MAPC algorithm in the post-processor is apparent from the increased probability of detection over MAPC Hybrid CLEAN APC algorithm. At a 0.9 probability of detection the MAPC Hybrid CLEAN MAPC algorithm is able to detect a target that has 10 dB less SNR than the MAPC algorithm at the same probability of detection and is able to detect a target that has 7 dB less SNR for the MAPC Hybrid CLEAN APC algorithm at the same probability of detection.

The MAPC Hybrid CLEAN MAPC approach has the best probability of detection for small scatters within the monostatic profile but also provides a better MSE compared to the other approaches.

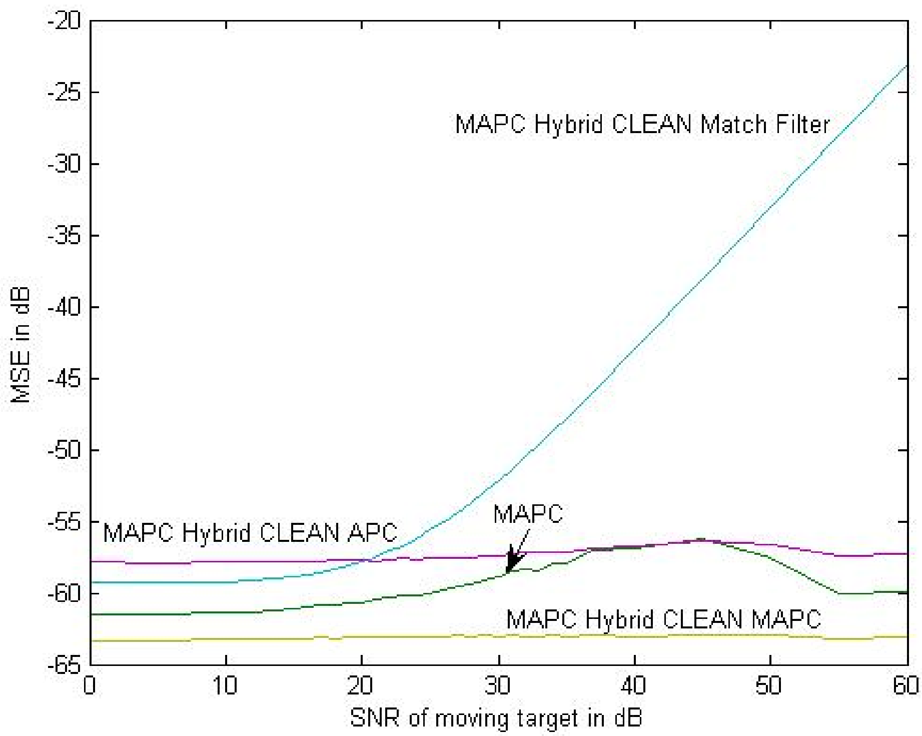

Figure 4.10 MSE of hybrid algorithms with MAPC pre-processors

From figure 4.10 the MAPC Hybrid CLEAN MAPC algorithm has a lower MSE than any of the other approaches which makes it easier to detect targets with a low SNR. The MAPC Hybrid CLEAN Matched Filter's MSE appears to improve as the SNR of the small target decreases. This is due to the sidelobes produced by the Matched filtering process. When the SNR of the moving target decreases the sidelobes will decrease proportionally and the range profile starts to resemble the noise floor.

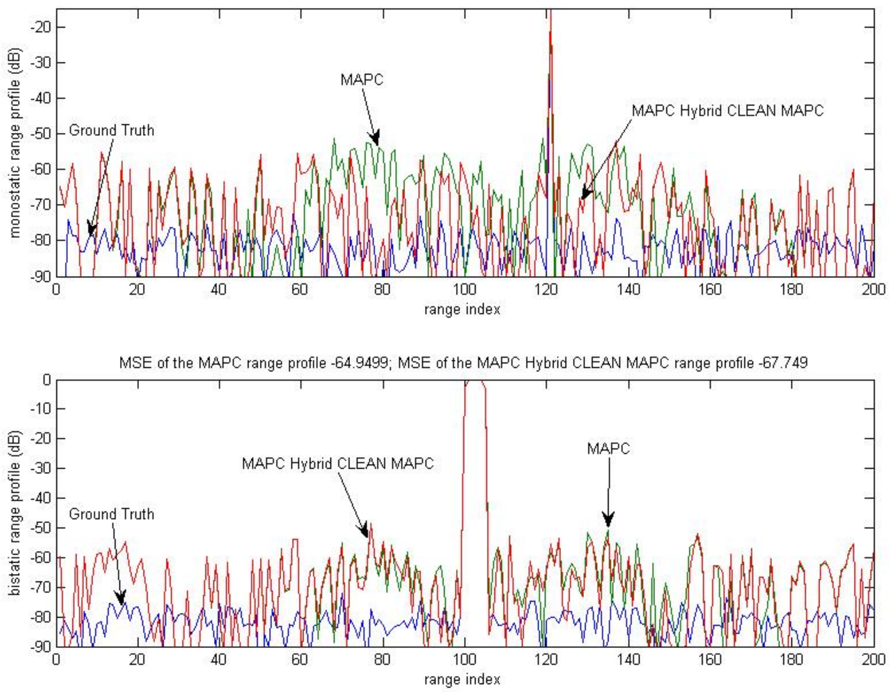

Figure 4.11 Pulse compressed range profile using MAPC Hybrid CLEAN MAPC and the MAPC algorithm

Figure 4.11 is a range profile estimate of the MAPC Hybrid CLEAN MAPC approach compared to the MAPC algorithm. The sidelobes in the monostatic profile have been significantly reduced compared to the MAPC algorithm which leads to a smaller MSE and a greater probability of detection.

To test the limits of the MAPC Hybrid CLEAN MAPC algorithm the size of the extended target was varied and the probability of detection was found for the larger extended targets and compared to the extended target used in the previous probability of detection simulation. Just as before the extended targets in each of these range profiles have a SNR of 60 dB for each of the range cells that it occupies except for the outer most range cells which have an SNR of 57 dB. The detector used in this simulation was not the CA-CFAR. The detector was a simple threshold detector which means that anything that had a certain power level in the pulse compression domain was considered detected and then removed with the CLEAN algorithm using the estimates from the range profile. The threshold in this simulation was 17 dB below the initial power level of the inner range cells of the extended target. In this simulation it is also important to consider the length of the transmitted waveform. The transmitted waveform was 40 samples long.

It is important to note that the detector in the Monte Carlo simulation did not change. This algorithm is still a CA-CFAR that has a probability of false alarm equal to 10^{-6}. The only detector that changed is the detector that is within the Hybrid Model.

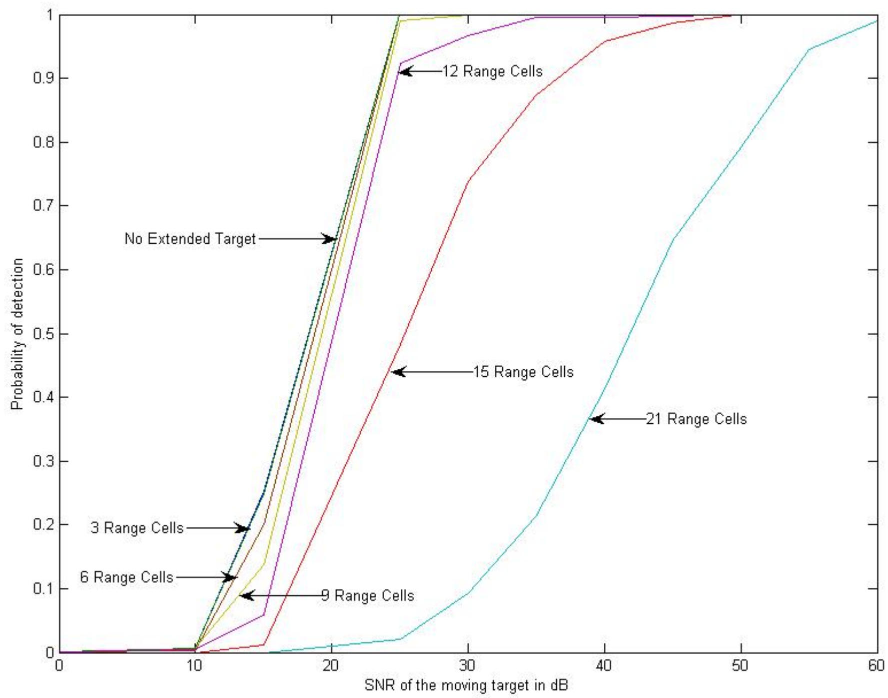

Figure 4.12 Probability of detection of the MAPC Hybrid CLEAN MAPC algorithm varying the size of the extended target

One of the things that should be noted about figure 4.12 is that the SNR is varied in increments of 5dB compared to figure 4.9 where the SNR varied in increments of 1 dB between an SNR of 15 to 35 dB so the probability of detection curve for 6 range cells will appear differently compared to figure 4.9. This plot does give some insight into how the algorithm will perform when larger extended targets are present. One of the interesting effects is the slight degradation in performance when the extended target is increased from the situation when an extended target is not present to an extended target of 9 range cells. When the extended target is about 9 range cells long it is almost a quarter the length of the transmitted waveform. Once the size of the target starts to increase to 12 ranges cells or more the probability of detection for the moving scatter in the monostatic range profile starts to decrease significantly.

4.4 IC-MAPC Results

The IC-MAPC algorithm has shown some significant improvement over the MAPC algorithm. The probability of detection improved 5 dB at probability of detection of 0.9 from the MAPC algorithm.

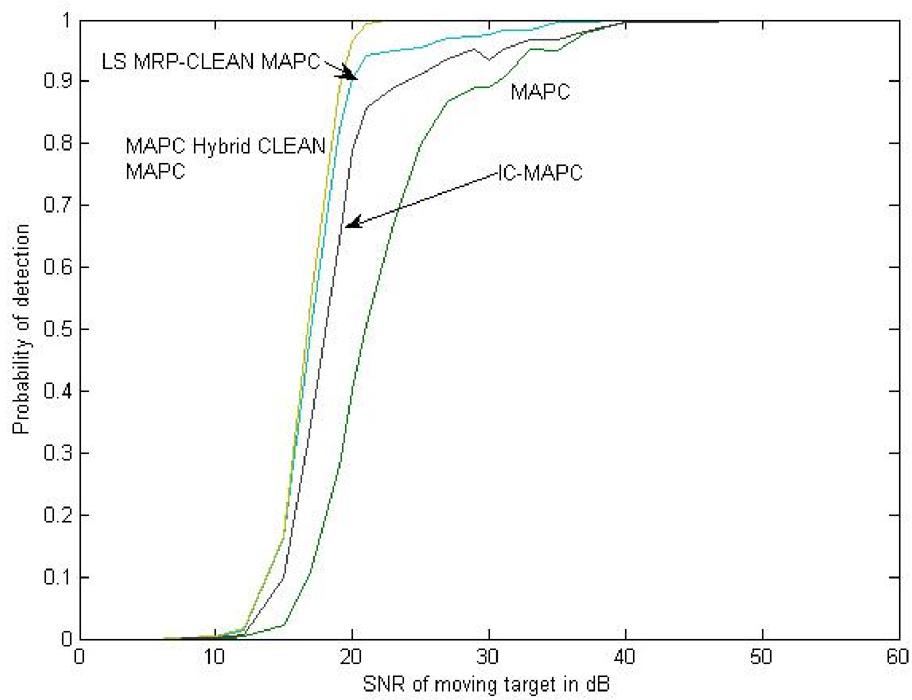

Figure 4.13 Probability of Detection of the IC-MAPC algorithm compared to other approaches.

Although the IC-MAPC algorithm did not have an improvement in the probability detection over the Least Squares MRP-CLEAN MAPC algorithm or the MAPC Hybrid CLEAN MAPC algorithm in figure 4.13, it does require fewer computations than either two algorithms. The MAPC Hybrid CLEAN MAPC algorithm uses two cycles of the MAPC algorithm while the Least Squares Hybrid CLEAN MAPC algorithm uses one cycle of MAPC and one implementation of the Least Squares algorithm which does require a large matrix inverse.

In terms of MSE the IC-MAPC algorithm mirrors the MAPC algorithm except that it has about 2 dB less error than MAPC which is seen in figure 4.14.

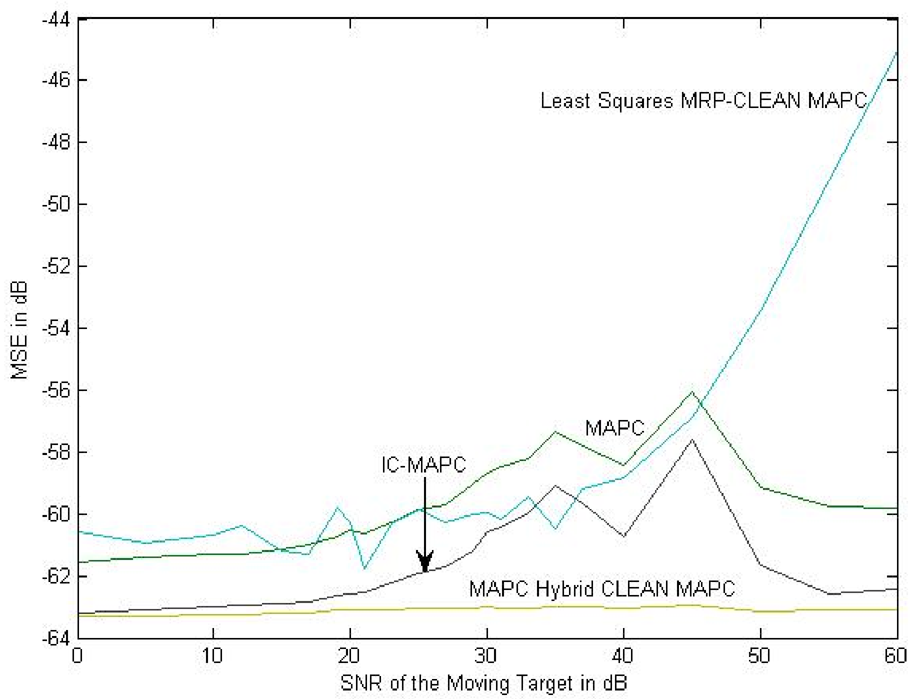

Figure 4.14 MSE of the IC-MAPC algorithm compared to other approaches.

The IC-MAPC algorithm does see improvement over the both the Least Squares MRP-CLEAN MAPC algorithm and the MAPC algorithm but does not improve over the MAPC Hybrid CLEAN MAPC approach. Since the MAPC Hybrid CLEAN MAPC approach has much more precise estimates it is the most effective in removing the interference from the other range profiles.

Chapter 5 Conclusion

5.1 Conclusion

The underlying theme throughout this thesis has been finding techniques to improve the sensitivity of the adaptive multistatic pulse compression algorithms while keeping the computational cost low. These two tradeoffs will influence the use of these hybrid algorithms in future radar systems. Throughout the testing of the algorithms the MAPC Hybrid CLEAN MAPC algorithm has shown the most impressive results in terms of probability of detection and mean squared error. The only drawback to this algorithm is the computational costs of using the MAPC algorithm twice. The other promising methods such as using the multistatic least squares algorithm as the preprocessor in the Hybrid Model and the IC-MAPC algorithm can reduce the computational complexity but for certain situations does not offer the performance that the MAPC Hybrid CLEAN MAPC approach does.

To further increase the performance of these hybrid algorithms the detection algorithm needs to be improved. In the simulation a simple CA-CFAR was used except when the extended target's size was varied then a constant threshold detector was employed as the detector in the Hybrid Model. The detector might provide some increase in the performance and robustness in the algorithms but the key factor in the performance of the hybrid algorithms is the accuracy of estimator in the pre-processor and post-processor. Both the CLEAN algorithms and the detector rely on the accuracy of the pre-processor. In the IC-MAPC algorithm the estimate in the stage before each detection and CLEAN stage is important in the final processed range profile.

These hybrid processing approaches should be used to improve the performance of the MAPC algorithm on multistatic received signals. The CLEAN algorithms coupled with the MAPC algorithm should help further the development of multistatic radars so that the hybrid algorithms are implemented in future radar systems.

5.2 Future Work

From the hybrid processing techniques that were researched and discussed in this thesis other avenues of research are available to further the processing capabilities of multistatic radars. These areas of research include but are not limited to the following:

- Development or use of mismatched filters for the MPCR algorithm that would decrease the interference caused by the cross-correlation effects.
- Development of efficient detection algorithms to increase the accuracy of the detection processes to find smaller interfering scatters for the Hybrid Model.
- Determine a method to alter the amplitude estimates within the IC-MAPC algorithm so that it is stable and can make better estimates.

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