Online Scheduling of Moldable Task Graphs under Common Speedup Models

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Scheduling Problems

Taxonomy of scheduling problems:

- **Offline Scheduling vs. Online Scheduling**
  - Offline: All tasks are known in advance (NP-hard problems)
  - Online: Tasks are released on the fly (over time or one-by-one)

- **Scheduling Independent Tasks vs. Task Graphs**
  - Independent tasks: There are no dependencies among tasks
  - Task graphs: Tasks have dependencies in the form of a directed acyclic graph (DAG)

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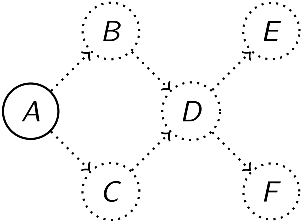
  - Task graphs: Tasks have dependencies in the form of a directed acyclic graph (DAG)

In this work, we focus on **online scheduling of task graphs**

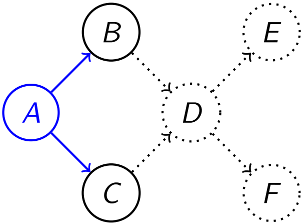
- A task is not known until all predecessors are completed
- Has applications in dynamic workflow scheduling

Example



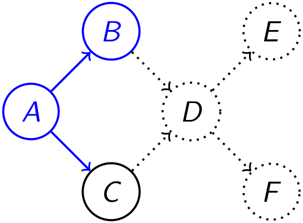
- At first, only task A is known, and others are unknown yet

Example



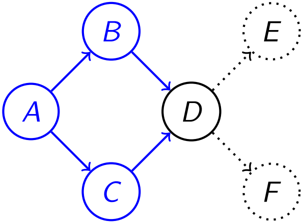
- When task A is done, the scheduler discovers tasks B and C

Example



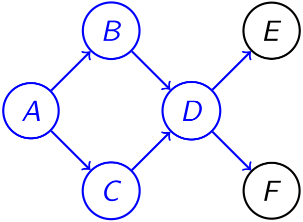
- When task B is done, task D is still not known yet

Example



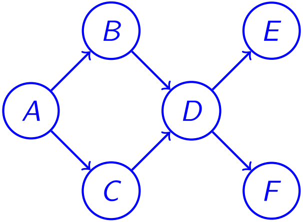
- Only when task C is also done, task D becomes known

Example



- Finally, when task D is done, tasks E and F are discovered

Example



- Tasks E and F are then processed to complete whole graph

Parallel Tasks

Taxonomy of parallel tasks:

- **Rigid tasks**: Processor allocation is fixed
- **Moldable tasks**: Processor allocation is decided by the system but cannot be changed once task starts running
- **Malleable tasks**: Processor allocation can be dynamically changed during runtime

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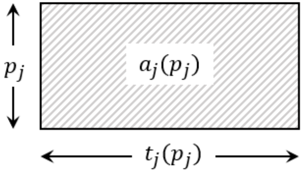
In this work, we focus on **moldable tasks**

- Easily adapt to amount of available resources (contrarily to rigid tasks)
- Easy to design and implement (contrarily to malleable tasks)

Scheduling Model

- A graph of n moldable tasks. Each task only becomes known when all of its predecessors are completed (i.e., online)
- P identical processors to process the tasks
- For each task j:
  - Execution time $t_j(p_j)$ depends on number of processors p_j allocated to it, and this function also becomes known when the task is discovered
  - Area is $a_j(p_j) = p_j \times t_j(p_j)$



Speedup Models

We mainly focus on a general speedup model:

$$t_j(p_j) = \frac{w_j}{\min(p_j, \bar{p}_j)} + d_j + (p_j - 1)c_j$$

which contains several common models as special cases

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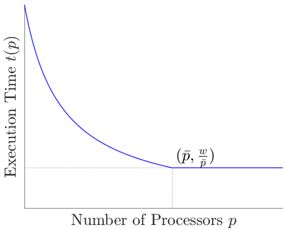
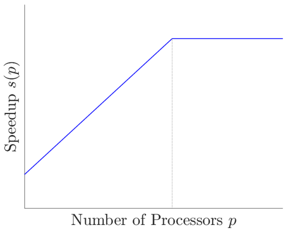
- Roofline model: $t_j(p_j) = \frac{w_j}{\min(p_j, \bar{p}_j)}$
  where $\bar{p}_j$ is maximum degree of parallelism





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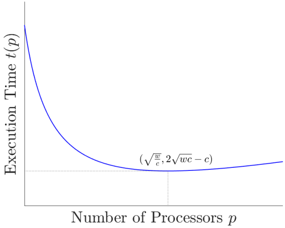
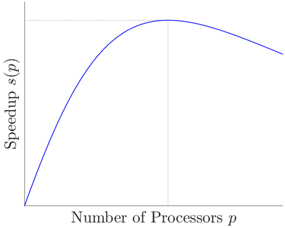
- Communication model: $t_j(p_j) = \frac{w_j}{p_j} + (p_j - 1)c_j$
  where c_j is communication overhead





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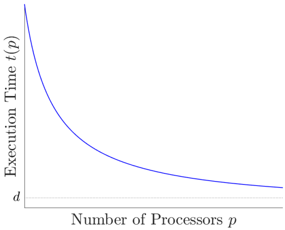
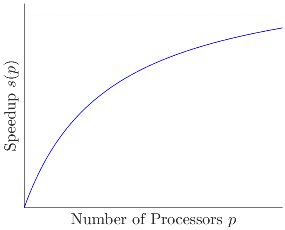
- Amdahl's model: $t_j(p_j) = \frac{w_j}{p_j} + d_j$
  where d_j is inherently sequential work





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- Additionally, we consider the arbitrary model, where $t_j(p_j)$ can be an arbitrary function of p_j

Scheduling Objective

Find an online moldable schedule (i.e., processor allocation p_j and starting time s_j for each task j):

- minimizes makespan: $T = \max_j(s_j + t_j(p_j))$
- subject to processor constraint: $\sum_{j \text{ active at time } t} p_j \leq P, \forall t$
- subject to precedence constraint: $j_1 \rightarrow j_2 \;\Rightarrow\; s_{j_2} \geq s_{j_1} + t_{j_1}$

Competitive Ratio:

An online algorithm is said to be *r-competitive* if its makespan T for any task graph satisfies:

$$\frac{T}{T_{\mathrm{OPT}}} \leq r$$

where T_{OPT} is the optimal offline makespan for the same graph

Our Main Results

- New online algorithm with almost tight competitive ratios for several common speedup models

Model	Roofline	Comm.	Amdahl	General
Upper bound	2.62	3.61	4.74	5.72
Lower bound	2.61	3.51	4.73	5.25

- Negative result for the arbitrary speedup model:
  Any deterministic online algorithm is $\Omega(\ln(D))$-competitive, where D is the length of the longest path in the graph

(Closely) Related Work

- **Feldmann, Kao, Sgall, Teng (1998)**:
  - Online scheduling of moldable task graphs in "*non-clairvoyant*" setting (i.e., work of a task is unknown until completion)
  - A 2.62-competitive algorithm for roofline model

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- **Ye, Chen, Zhang (2018)**:
  - Online scheduling of independent moldable tasks in "*one-by-one*" setting (i.e., tasks are released sequentially and each task must be scheduled immediately upon release)
  - A 16.74-competitive algorithm for arbitrary model

Outline

Lower Bound on Makespan

For each task j:

- Minimum area: $a_j^{\min} = \min_p a_j(p)$
- Minimum execution time: $t_j^{\min} = \min_p t_j(p)$

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For task graph:

- Minimum total area: $A_{\min} = \sum_{j=1}^{n} a_j^{\min}$
- Minimum critical-path length: $C_{\min} = \max_f \sum_{j \in f} t_j^{\min}$

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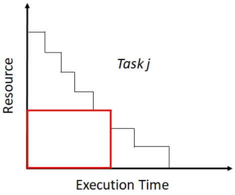
Proposition

The ***optimal makespan*** *satisfies:*

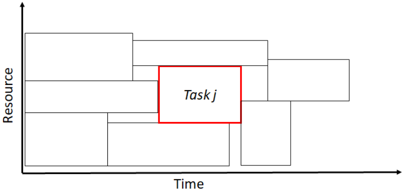
$$T_{\text{OPT}} \geq \max\left(\frac{A_{\min}}{P}, C_{\min}\right)$$

Two-Phase Approach [Turek et al. '92]

- **Phase 1**: Determine a resource allocation for each task once it becomes available



- **Phase 2**: Construct a schedule based on resource allocations of the available tasks



Phase 1: (Local) Resource Allocation

- Step (1): Initial allocation [Benoit et al. 20]
  Find an allocation $p_j \in [1, P]$ from the following problem:

$$\min_{p} \ \alpha(p) \triangleq \frac{a_j(p)}{a_j^{\min}}$$

$$\text{s.t. } \beta(p) \triangleq \frac{t_j(p)}{t_j^{\min}} \leq \frac{1-2\mu}{\mu(1-\mu)}$$

$\Rightarrow$ Allocate resource locally for each task: minimize area subject to a time constraint

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- Step (2): Adjusted allocation [Lepère et al. 01]
  If $p_j > \lceil \mu P \rceil$ **then** $p'_j \leftarrow \lceil \mu P \rceil$ **else** $p'_j \leftarrow p_j$
  $\Rightarrow$ Reduce high allocation to increase overall resource utilization: choice of $\mu \in (0, 0.5)$ depends on speedup model

Phase 2: (Online) List Scheduling

- Insert a task in a list (i.e., waiting queue) as it becomes available
- Whenever an existing task completes, which releases resources, scan the list and schedule each task that fits

Note: when a task becomes available, it is not required to be immediately scheduled (one-by-one model)

Outline

(1) Local Analysis

Can we say something about each individual task?

Proposition

For a given speedup model M, there exists an (α, β) pair and an initial resource allocation p_j for any task j such that:

$$a_j(p_j) \leq \alpha \cdot a_j^{\min}$$
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These local bounds will carry over to the global analysis!

$$\sum_{j \in J} a_j(p_j) \leq \alpha \cdot \sum_{j \in J} a_j^{\min}$$
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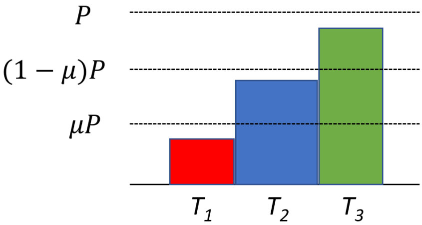
$$\sum_{j \in J} a_j(p_j) \leq \alpha \cdot \sum_{j \in J} a_j^{\min} \leq \alpha \cdot A_{\min}$$
$$\sum_{j \in f} t_j(p_j) \leq \beta \cdot \sum_{j \in f} t_j^{\min} \leq \beta \cdot C_{\min}$$

(2) Global Analysis

Total makespan interval $[0, T]$ divided in three sets [Lepère et al. 01]:

- T_1: Less than μP processors are used.
- T_2: Between μP and $(1-\mu)P$ processors are used
- T_3: More than $(1-\mu)P$ processor are used



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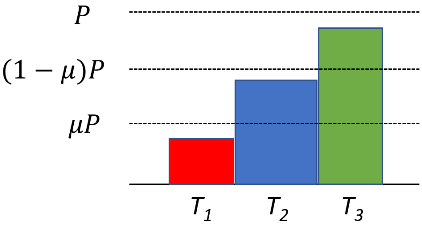
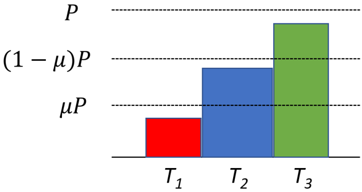


T_1 and T_2 can be charged to the critical-path length
T_2 and T_3 can be charged to the total area

(3) Combining Two Analyses



Critical-path bound: $\frac{T_1}{\beta} + \mu T_2 \leq C_{\min}$

Total area bound: $\mu T_2 + (1 - \mu) T_3 \leq \frac{\alpha \cdot A_{\min}}{P}$

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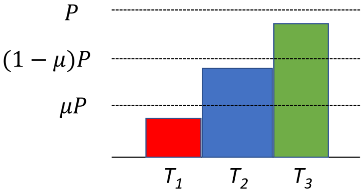

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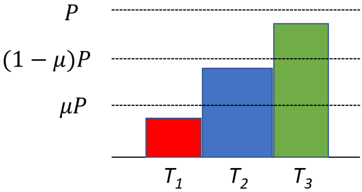

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Proposition

Combining the two bounds with $T = T_1 + T_2 + T_3$, we get:

$$\frac{T}{T_{\text{OPT}}} \leq \frac{\mu\alpha + 1 - 2\mu}{\mu(1-\mu)} \quad \textit{subject to} \quad \beta \leq \frac{1-2\mu}{\mu(1-\mu)}$$

Final Results

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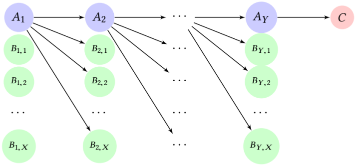
Optimization procedure for a given speedup model:

1. Find an upper bound for α as a function of μ
2. Find μ minimizing the ratio subject to β constraint

Model	Roofline	Comm.	Amdahl	General
Choice of μ	0.382	0.324	0.271	0.211
Upper bound	2.62	3.61	4.74	5.72

Outline

Instance for Common Speedup Models



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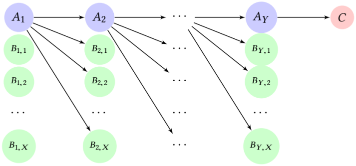
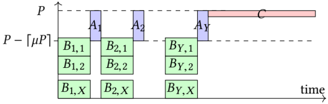


Task parameters are chosen so that:

- **For online algorithm** (a): Barely impossible to process a full layer in parallel
- **For optimal algorithm** (b): First process all A's and then B's and C's in parallel



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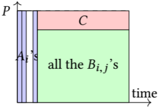
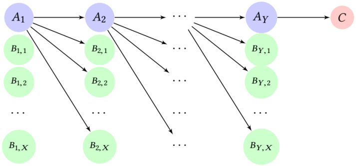


(b)

Instance for Common Speedup Models



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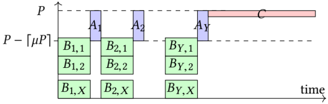

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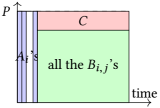

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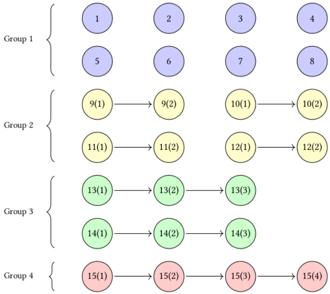

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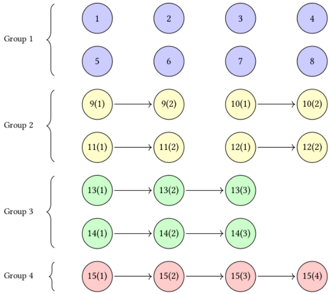
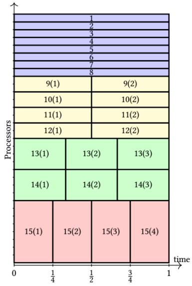
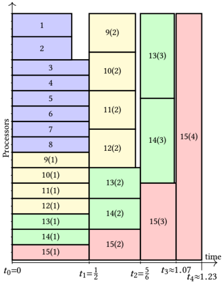


- $D = 2^\ell$ groups of identical tasks with execution time function $t(p) = \frac{1}{\lg(p)+1}$
- **For optimal algorithm** (a): 2^{i-1} processors for tasks in group $i \Rightarrow$ makespan of 1
- **For online algorithm** (b): same processors for all tasks (best online strategy) $\Rightarrow$ makespan of $\Omega(\ln(D))$



(a)



(b)

Outline

Conclusion

- A new algorithm for online scheduling of moldable task graphs
- Almost tight competitive ratios for several common speedup models
- No constant competitive ratio for arbitrary speedup model by any deterministic online algorithm

Future work:

- Consider other speedup models or special task graphs
- Improve the ratios for upper and/or lower bounds
- Experimental evaluation of the algorithm's performance

Latest Results

Model	Roofline	Comm.	Amdahl	General
Old Results	≈ 2.62	≈ 3.61	≈ 4.74	≈ 5.72
New Results[1]	≈ 2.62	≈ 3.39	≈ 4.55	≈ 4.63

with matching lower bounds

- New upper bounds benefit from a tighter (α, β) analysis: worst-case time and area bounds don't happen simultaneously
- New lower bounds also apply to a class of algorithms with deterministic local processor allocation (i.e., stronger)

[1] Lucas Perotin, Hongyang Sun. Improved Online Scheduling of Moldable Task Graphs under Common Speedup Models. 2023. https://arxiv.org/abs/2304.14127