Improved Semi-Online Makespan Scheduling with a Reordering Buffer

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Joint work with *Rui Fan*

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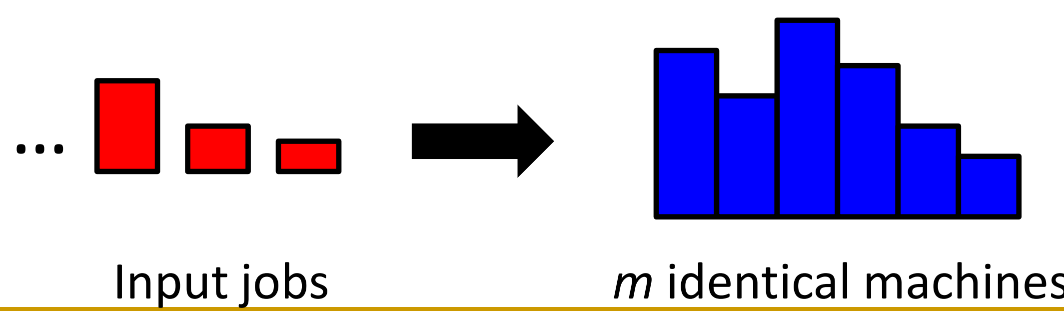
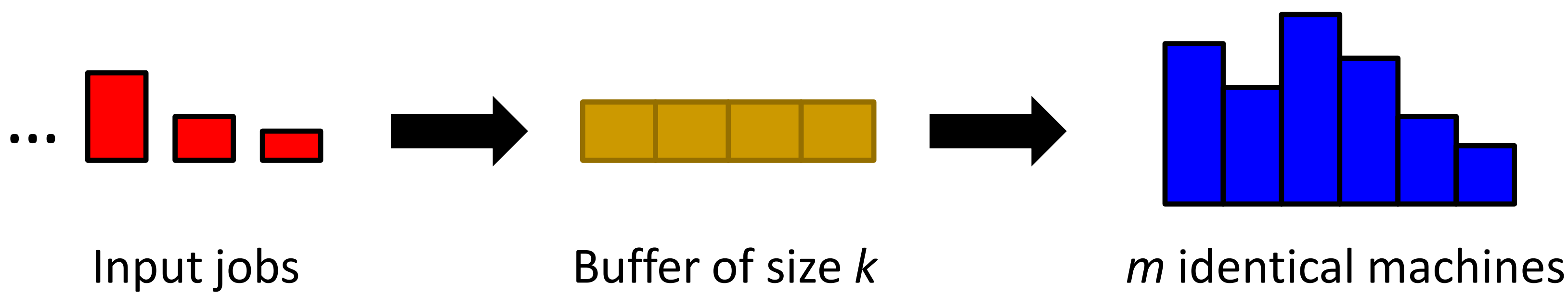
Background

- **Classical online scheduling**
  - Schedule a sequence of jobs arriving one by one on m identical machines to minimize makespan
  - List scheduling algorithm (Graham 1966)
    - Assign arriving job on a machine with least load
    - $(2-1/m)$-competitive
  - Best known competitive ratio of deterministic algorithm [1.88, 1.9201] for large m



Background

- **Semi-online scheduling with reordering buffer**
  - A buffer of limited size *k* (independent of input size) can be used to store and reorder jobs
    - Store: If buffer is not full, we can admit a new job into the buffer without assigning any job to any machine
    - Reorder: If buffer is full, we can select any job from the buffer or the arriving job and assign it to a machine



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  - For general m, optimal comp. ratio (lower bound) = r_m (Englert, Ozmen and Westermann 2008)
    - r_m is the solution to the following equation
      $\lceil (r_m - 1)m/r_m \rceil \cdot r_m/m + (r_m - 1) \cdot \sum_{i=\lceil (r_m-1)m/r_m \rceil}^{m-1} 1/i = 1$

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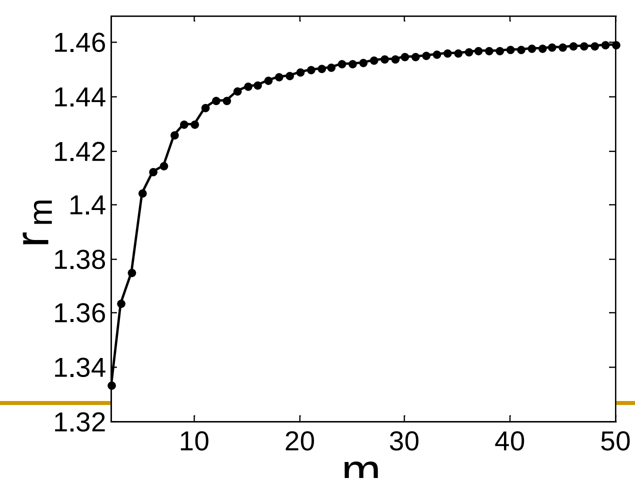
    - r_m is a monotonically increasing function of m
    - $r_2 = 4/3$, and $\lim_{m\to\infty} r_m \approx 1.4659$



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  - For general m, optimal comp. ratio (lower bound) = r_m (Englert, Ozmen and Westermann 2008)
    - $\boldsymbol{k = \Theta(m)}$ is necessary and sufficient to achieve r_m
    - A r_m-comp. algorithm was proposed with a reordering buffer of size $\boldsymbol{k = (1+2/r_m)m \approx 2.364m}$ for large m
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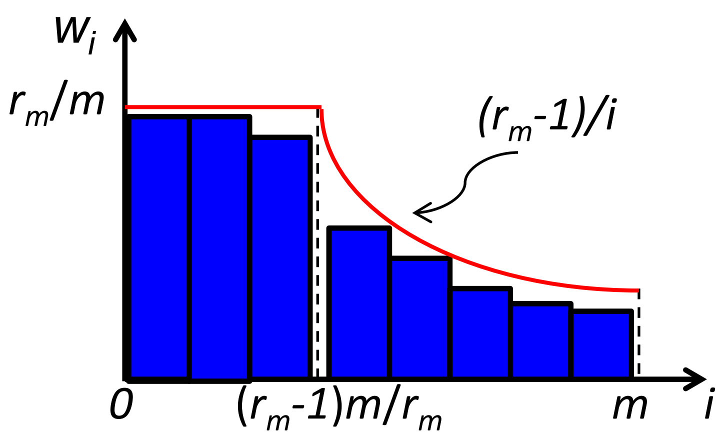
  - What is the exact buffer size required to achieve r_m?
  - **Our result improves the required buffer size**
    - A r_m-comp. algo. with a buffer size $\boldsymbol{k = (5-2r_m)m \approx 2.068m}$ for large m, improving the previous result by $\boldsymbol{\approx 0.296m}$

Outline

- Lower bound $\boldsymbol{r_m}$
- A scheduling framework to get optimal comp. ratio
  - Buffer size $\boldsymbol{k = (1+2/r_m)m \approx 2.364m}$ (Englert et al.)
  - Buffer size $\boldsymbol{k = (5-2r_m)m \approx 2.068m}$ (Our result)
- Tradeoff between comp. ratio and buffer size
- Remarks

Lower bound r_m

- Consider the following load profile (weight w_i)
  - $\boldsymbol{w_i = \min\{r_m/m, (r_m-1)/i\}}$, where $\boldsymbol{\sum w_i = 1}$



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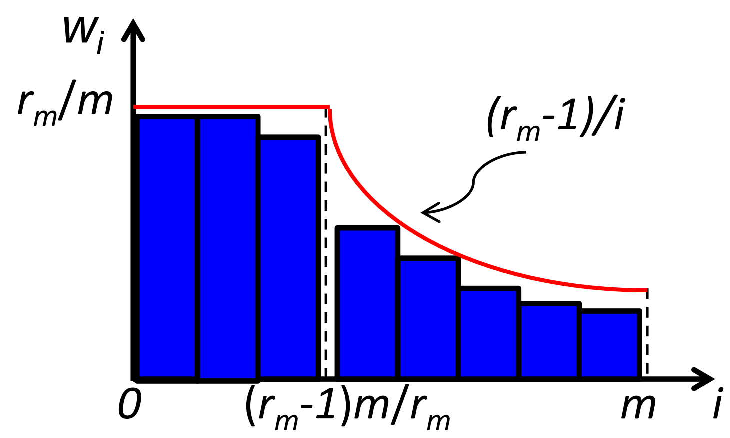

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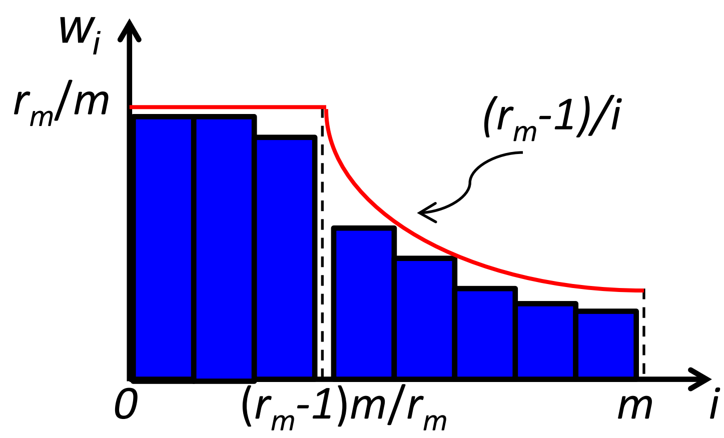

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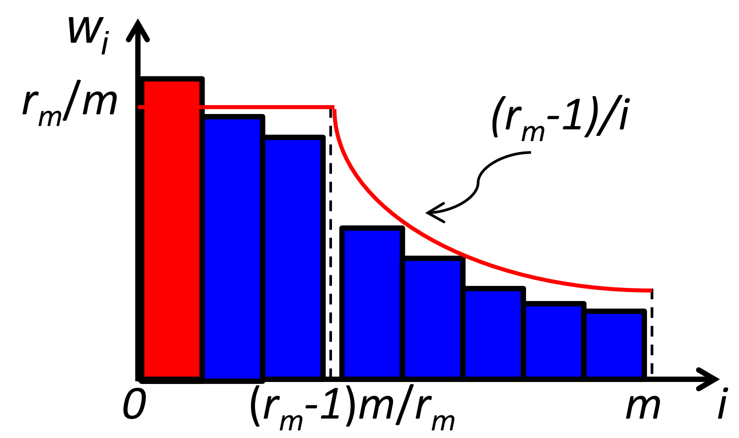

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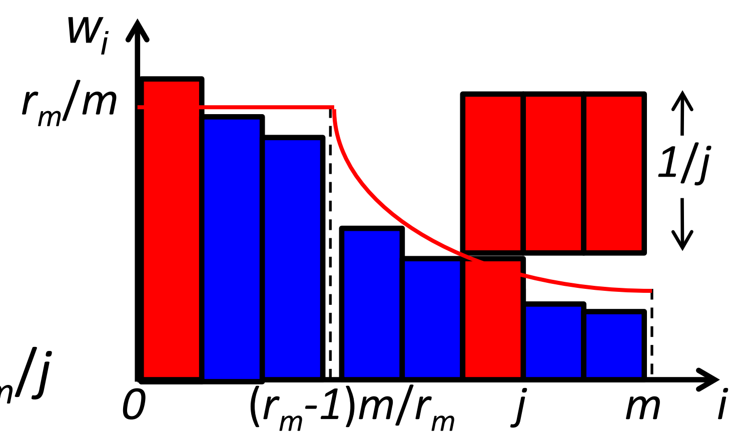

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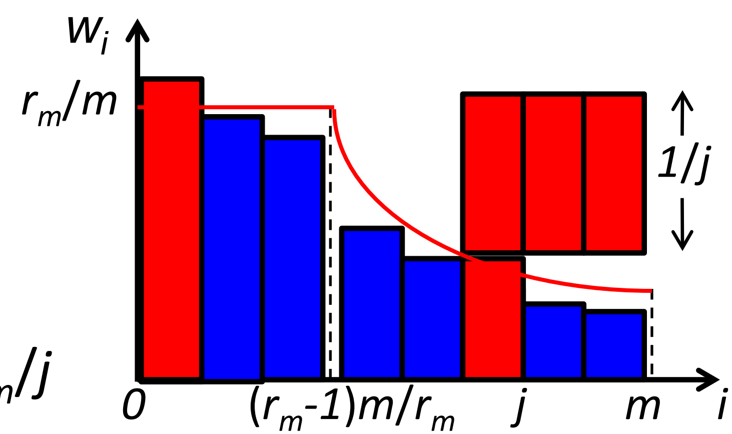
- In both cases, competitive ratio $\geq r_m$



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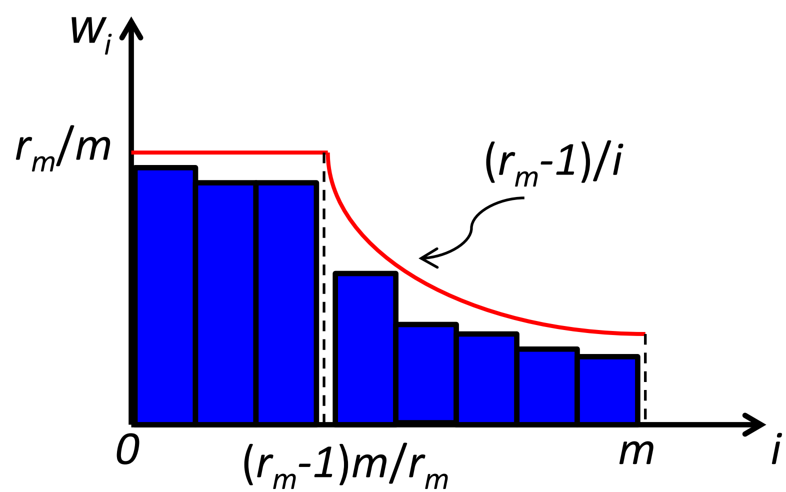
- **Three phases:**
  - **(1) Initial phase:** Admit *k* jobs into buffer w/o assignment
  - **(2) Iterative phase**: Admit a new job, and pick a smallest job and assign it to some machine *j*
    - Choice of the machine depends on the algorithm
    - Maintain a profile of machine loads related to $\boldsymbol{w_i = \min\{r_m/m, (r_m\text{-}1)/i\}}$, where normalized total area = 1, i.e., $\boldsymbol{\sum w_i = 1}$



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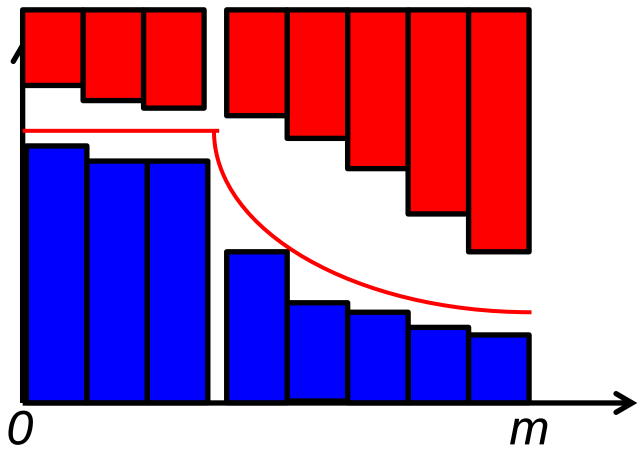

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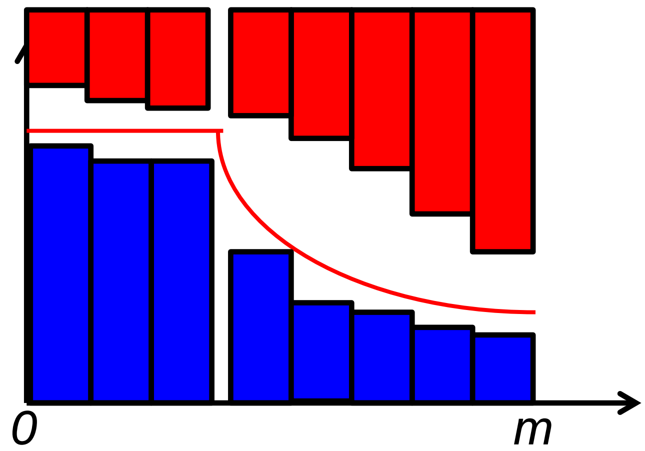

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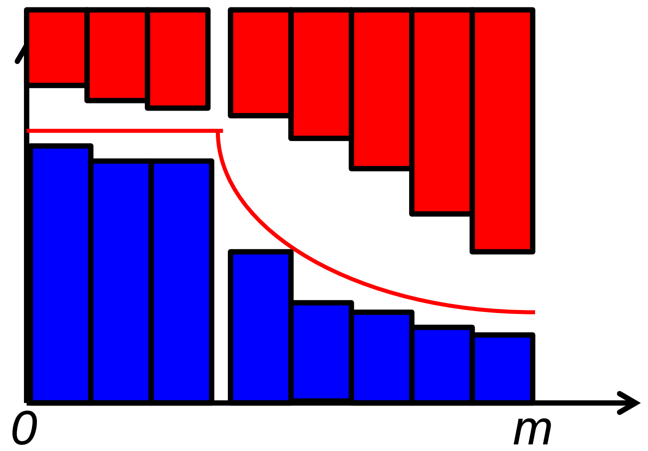
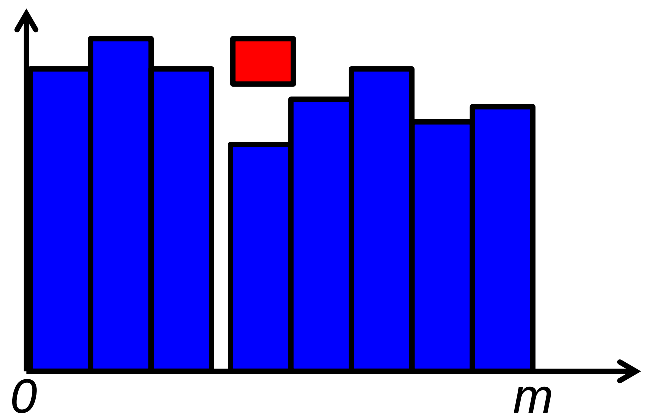
    → Mathematics can ensure the optimal comp. ratio r_m
  - **2nd Step**: Small jobs (size ≤ $1/3 \cdot$OPT)
    - Place one by one greedily
    - ≤ OPT + $1/3 \cdot$OPT ≤ $r_m \cdot$OPT

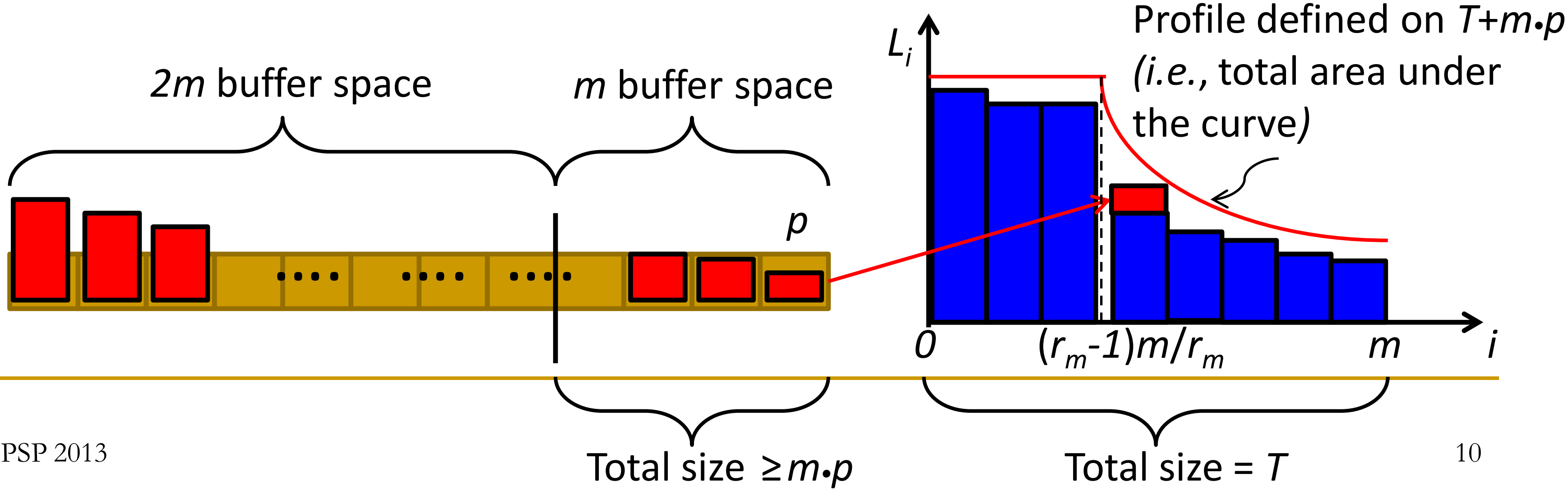
    → Still optimal comp. ratio





Buffer size ***k* = *3m*** (Englert et al.)

- **Iterative phase**: Assign smallest job of size p to a machine $\boldsymbol{j}$ with load $\boldsymbol{L_j \leq w_j \cdot (T+m \cdot p) - p}$



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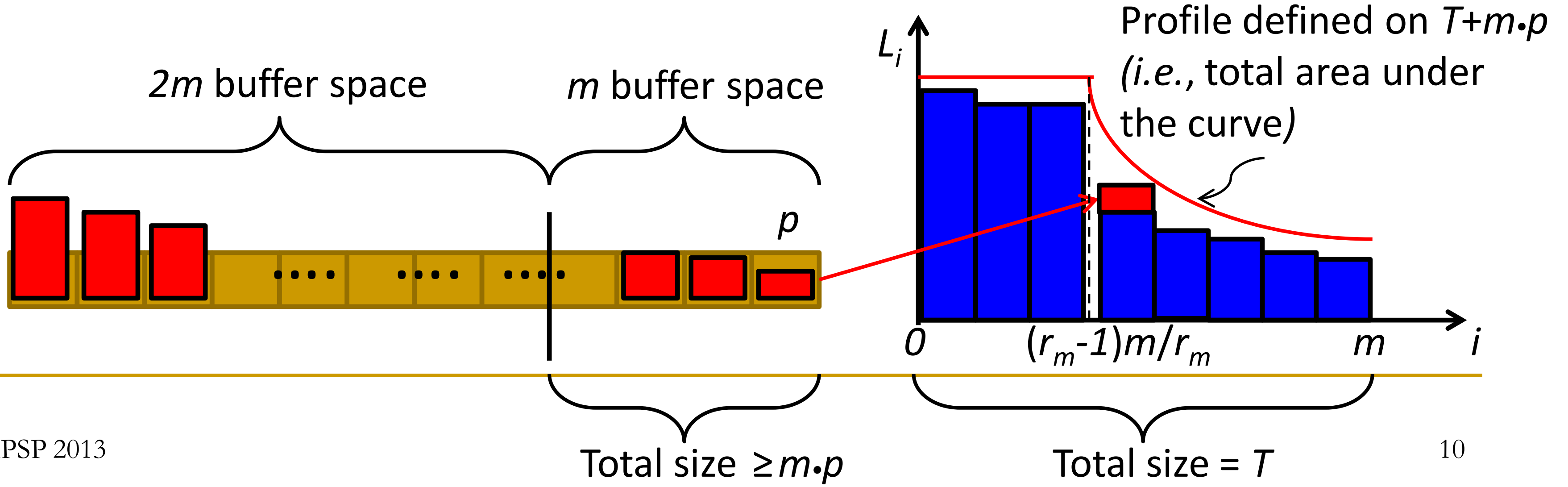

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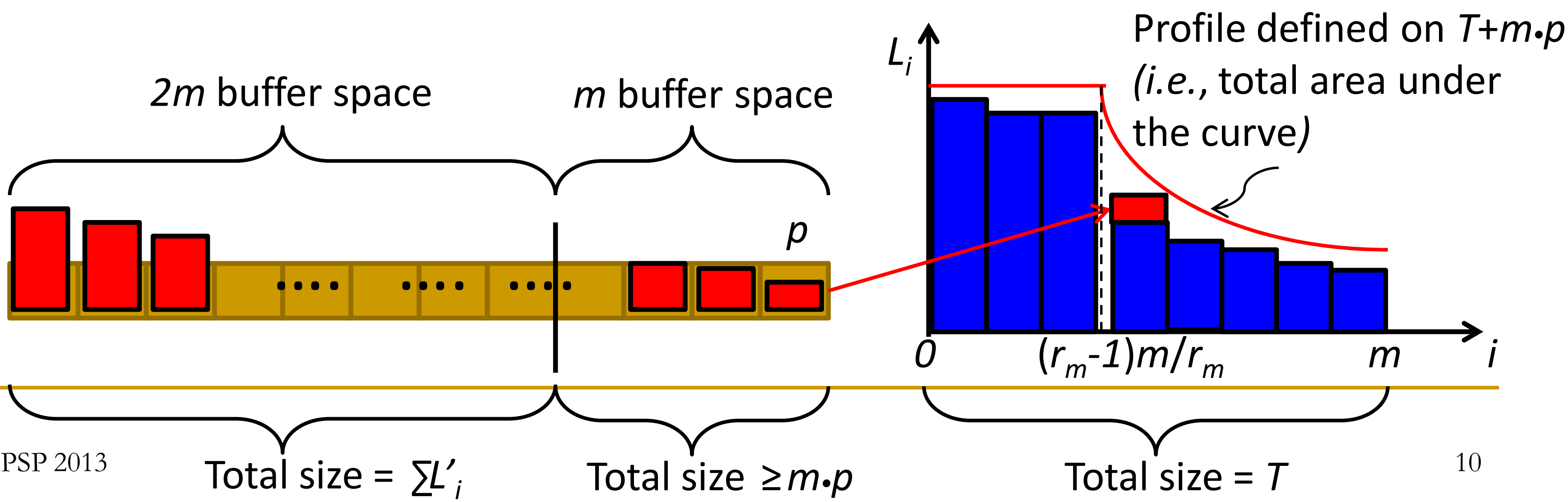

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For all $0 \le j \le m-1$

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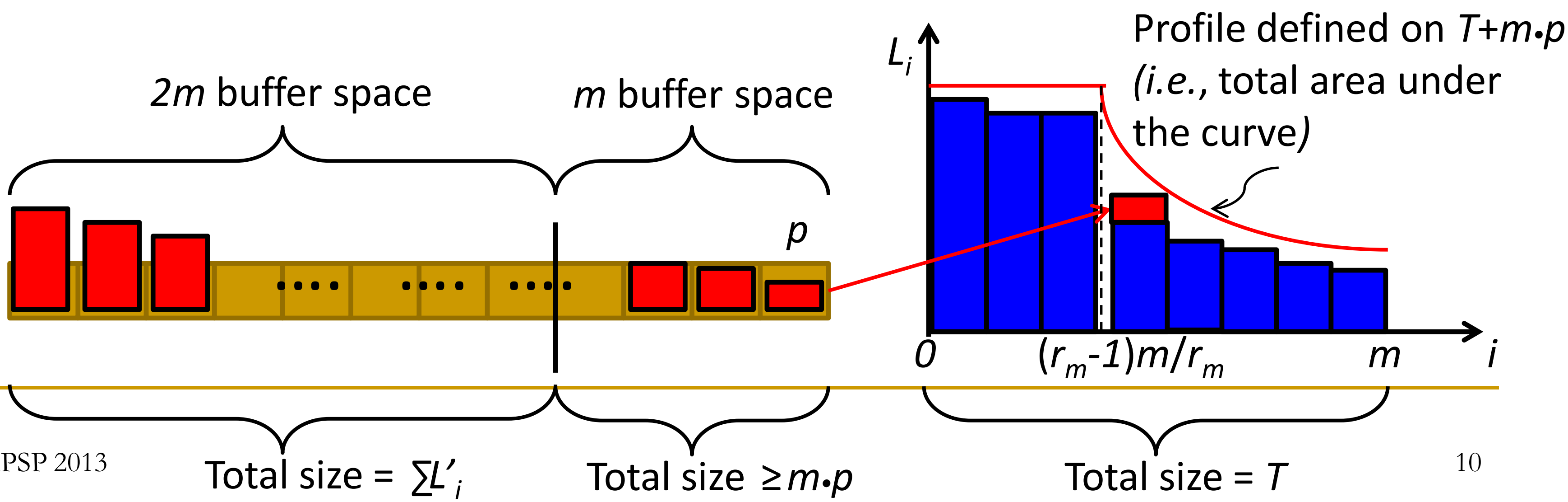

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Similar derivations carry over to the other algorithms

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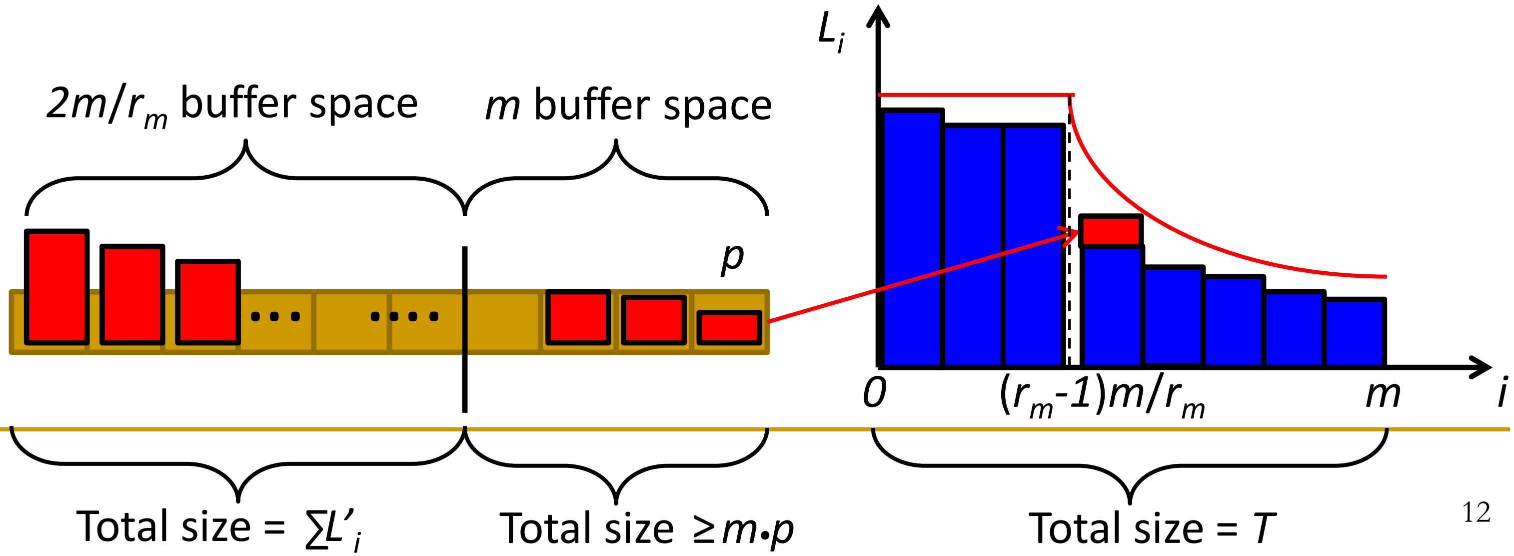

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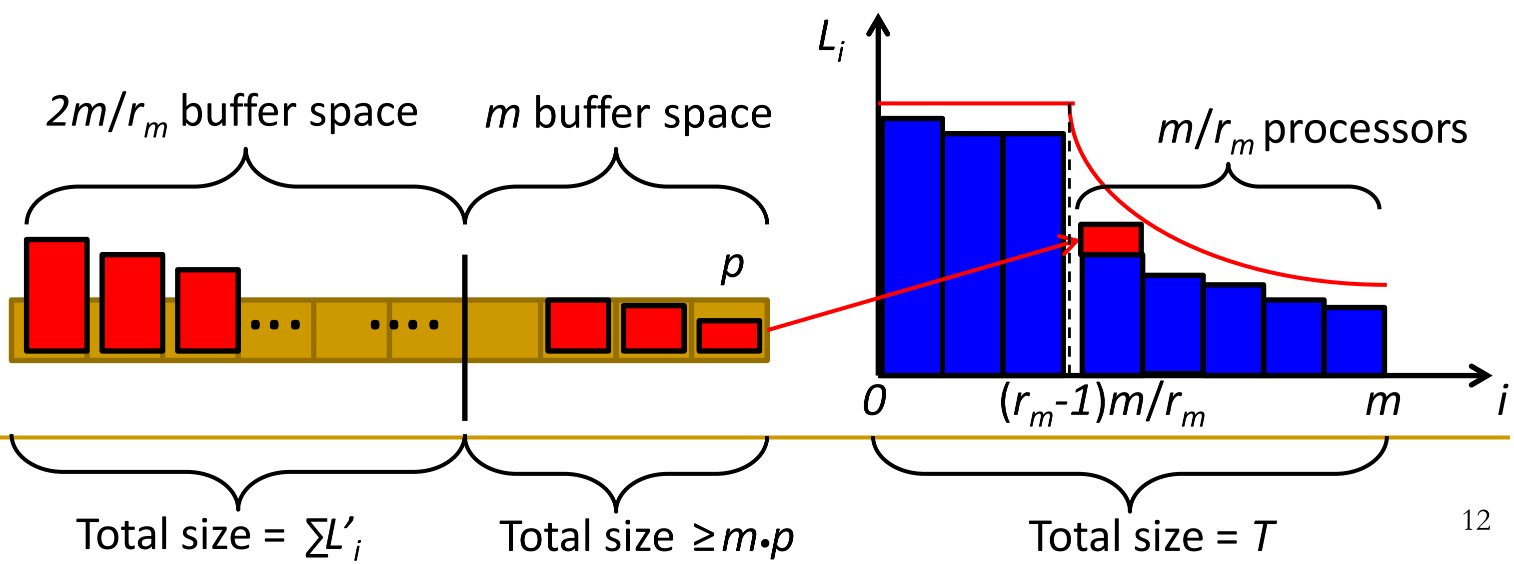

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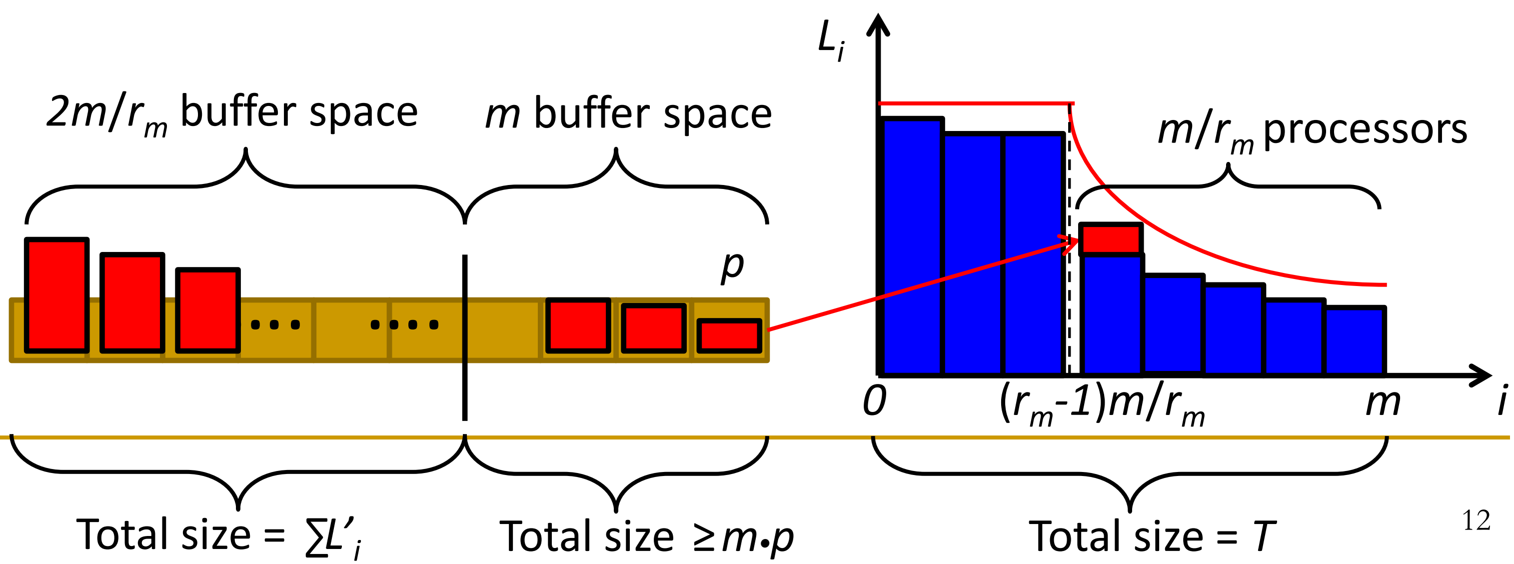
These processors get at most *2* jobs each according to the optimal LPT rule, so total extra buffer space required is $2m/r_m$



Buffer size $k = (5\text{-}2r_m)m$ (Our Result)

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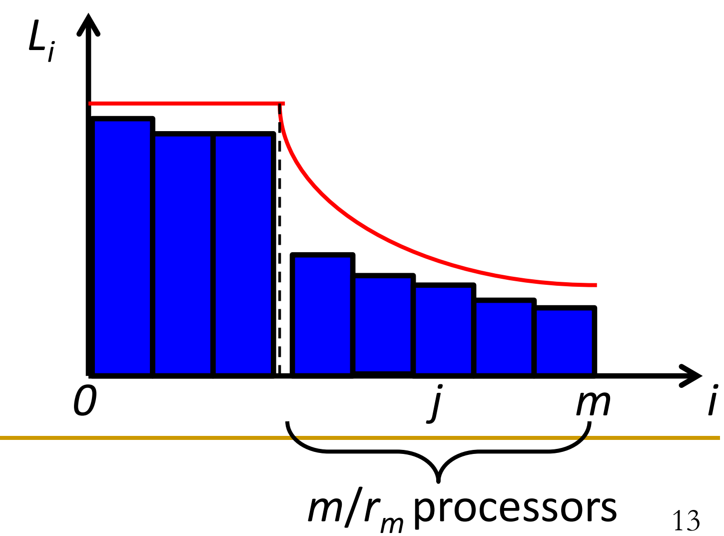

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- **Design a non-uniform profile**: Observe from the proof of the final phase

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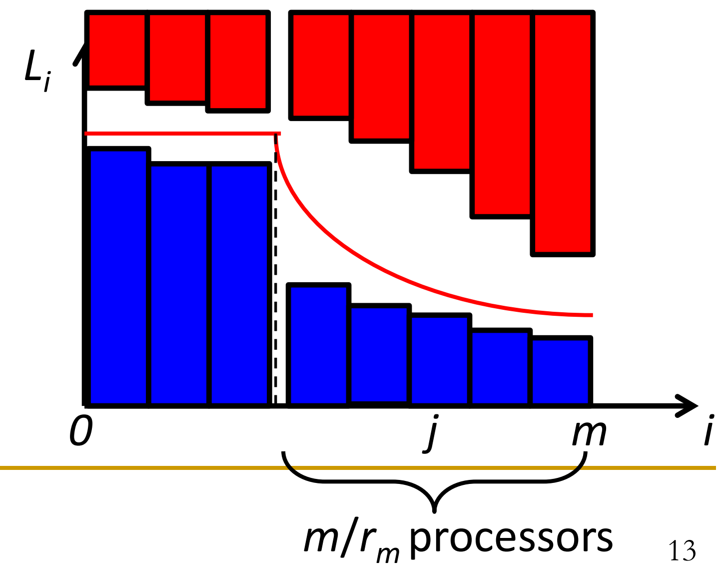

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- **Observation**: In the *iterative phase*, it is **not** necessary to maintain a **uniform** load profile for all processors, or more precisely for the m/r_m processors on the right, in order to satisfy the following in the **final phase**

$\boxed{\text{For } (r_m-1)m/r_m \le j \le m-1}$

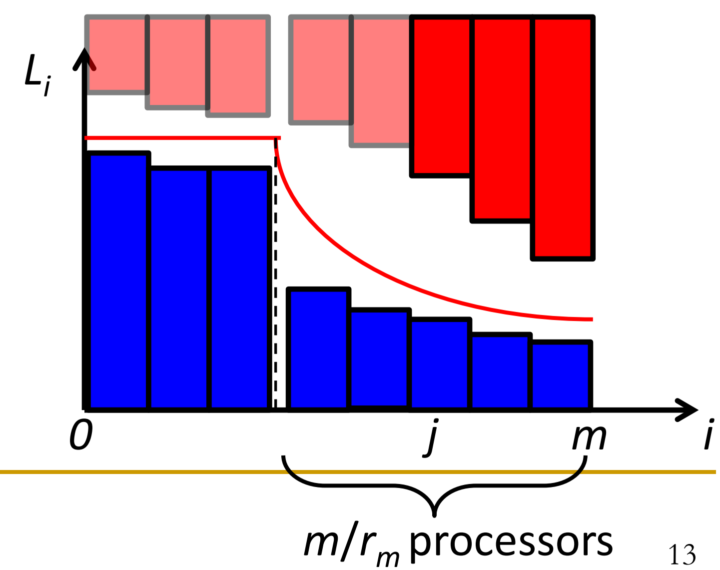
$$\left.\begin{aligned} OPT &\ge (T_{\text{final}} + m{\cdot}p + \textstyle\sum L'_i)/m \\ L_j &\le w_j \cdot (T_{\text{final}} + m{\cdot}p) + L'_j \end{aligned}\right\} \Rightarrow L_j \le r_m \cdot OPT$$

- **Design a non-uniform profile**: Observe from the proof of the final phase

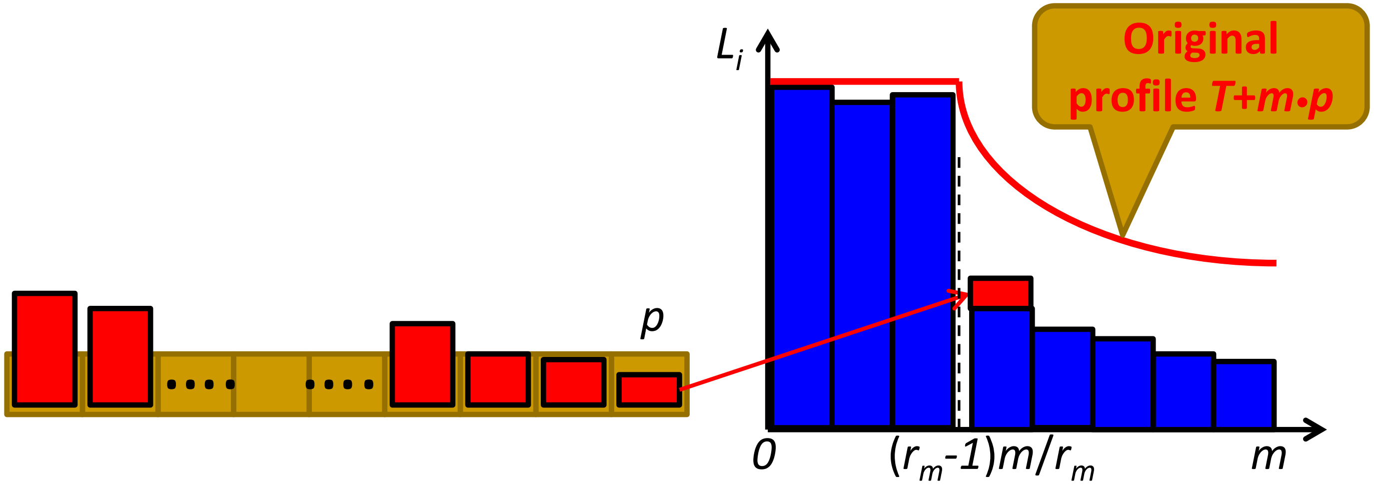
For $(r_m-1)m/r_m \le j \le m-1$, $w_j = (r_m-1)/j$

$$\begin{aligned} L_j &\le w_j \cdot (T_{\text{final}} + m{\cdot}p) + L'_j \\ &= (r_m-1)/j \cdot (m\,OPT - \textstyle\sum L'_i) + L'_j \\ &\le (r_m-1)/j \cdot (m\,OPT - (m-j)\,L'_j) + L'_j \\ &= (r_m-1)/j \cdot (m\,OPT - m\,L'_j + j\,L'_j) + L'_j \\ &= (r_m-1)/j \cdot (m\,OPT - m\,L'_j) + r_m\,L'_j \\ &\le r_m/m \cdot (m\,OPT - m\,L'_j) + r_m\,L'_j \\ &= r_m \cdot OPT \end{aligned}$$

$\boxed{\textstyle\sum L'_i \ge (m-j)L'_j,\ \text{giving up } L'_1 \text{ to } L'_{j-1}}$



Buffer size $k = (5-2r_m)m$ (Our Result)



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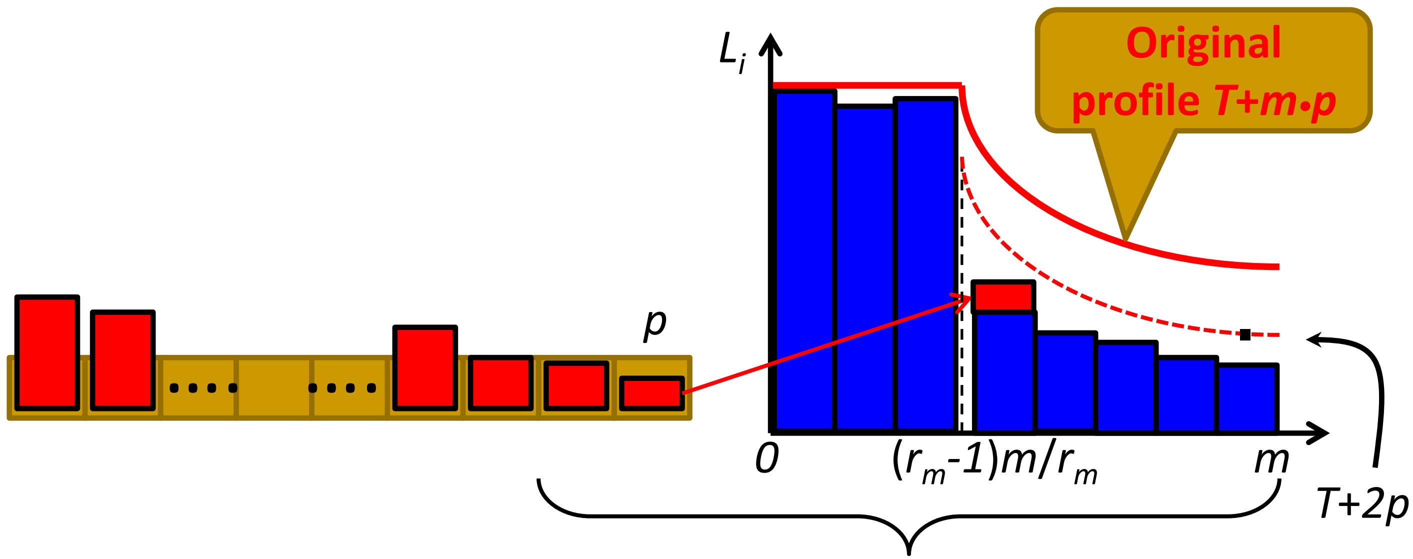

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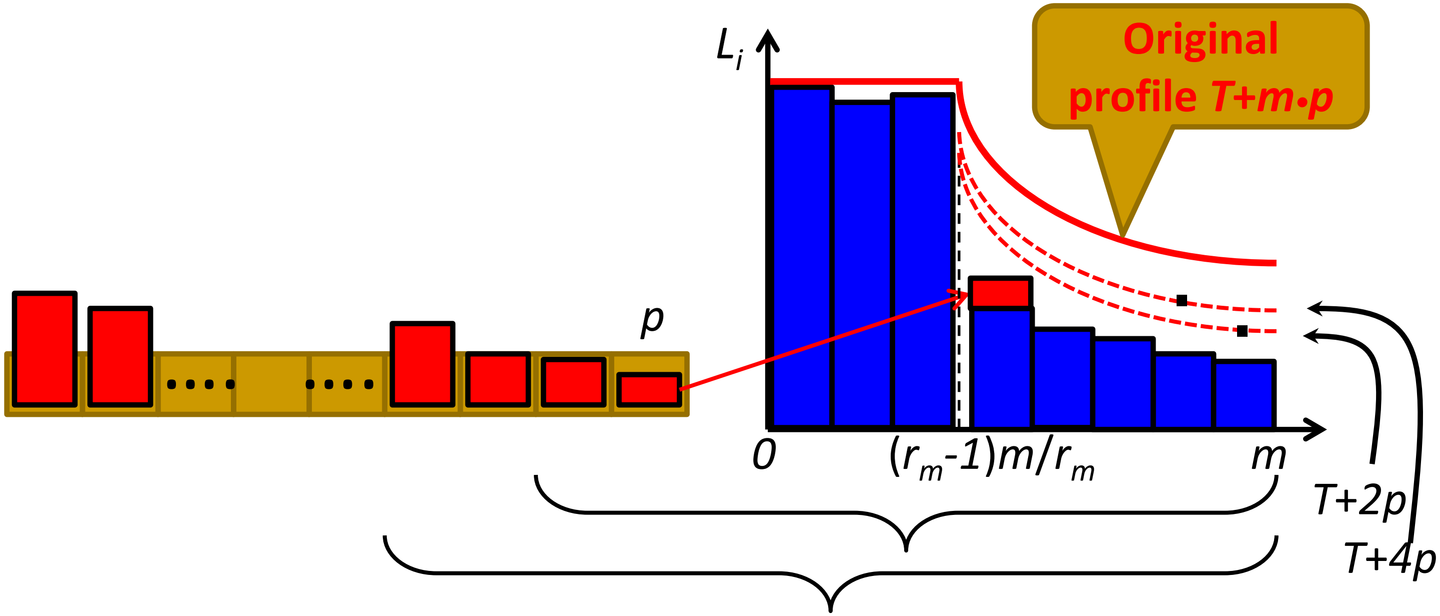

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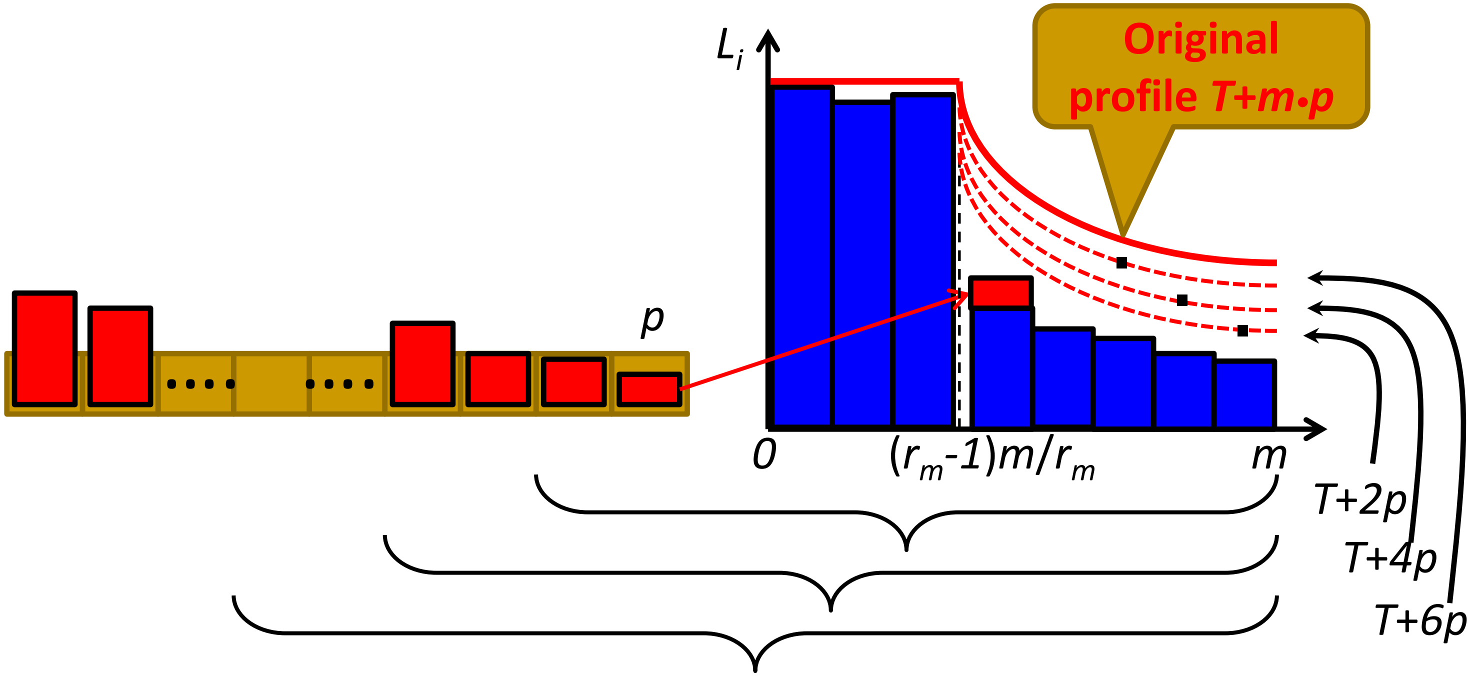

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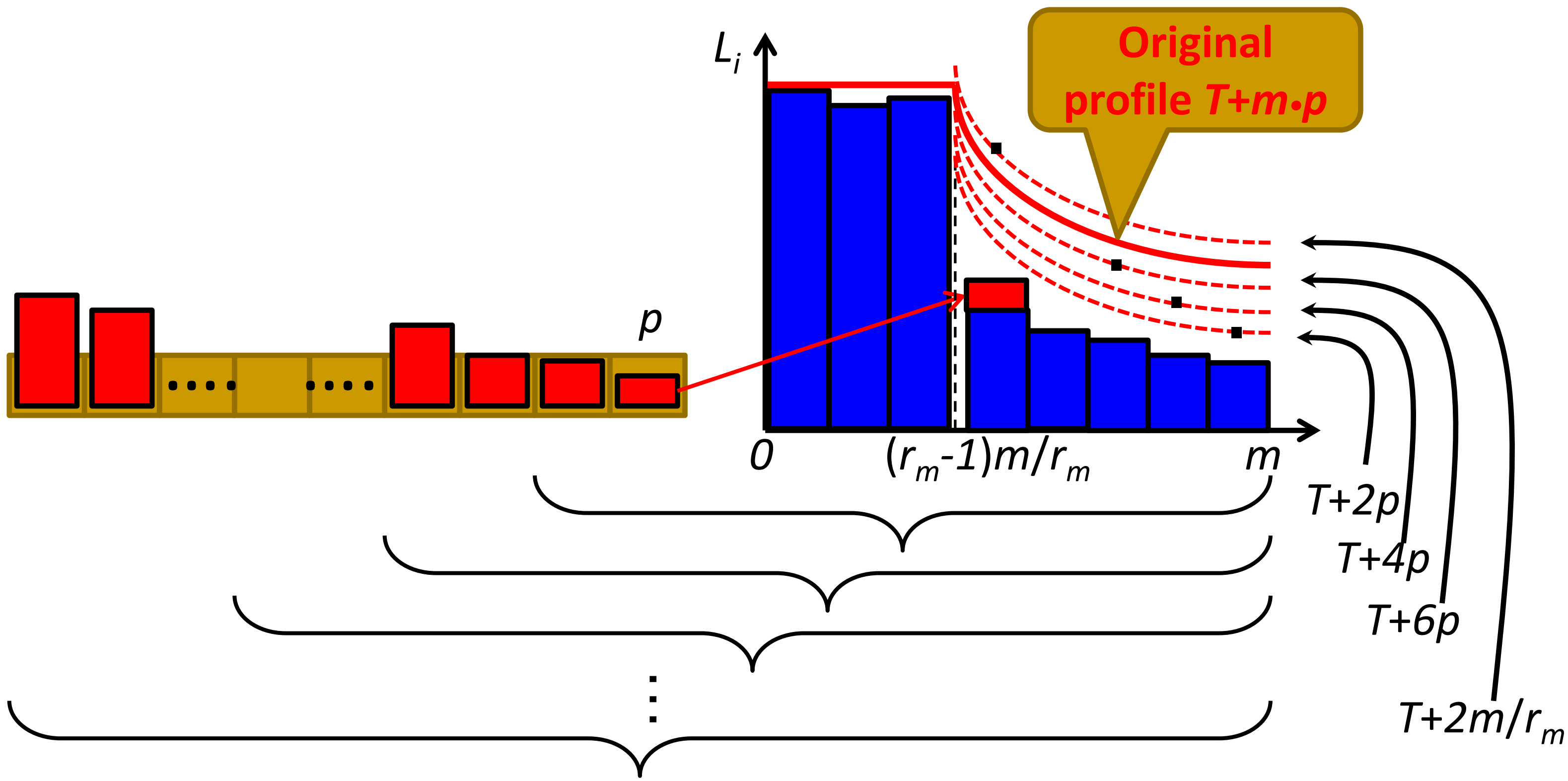

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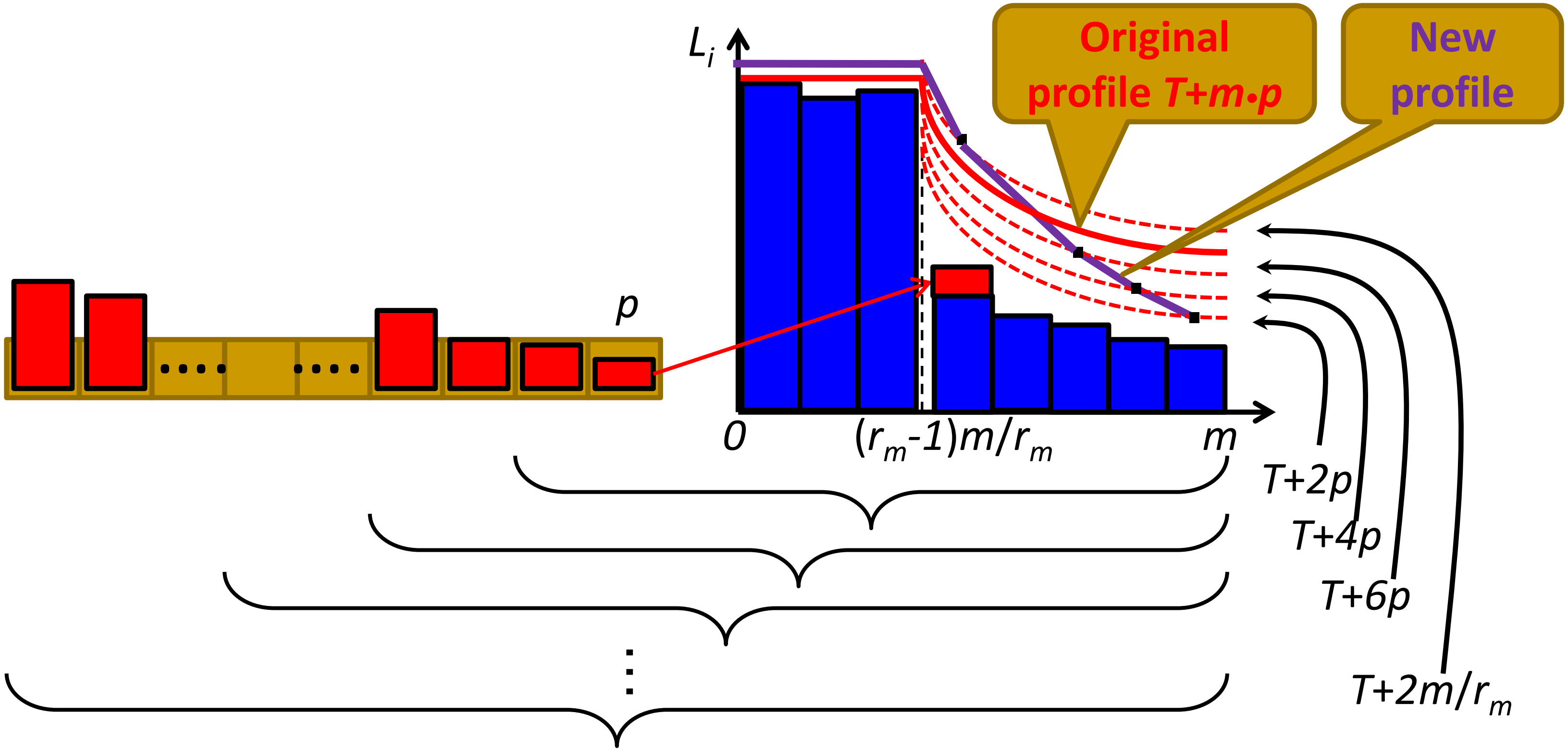

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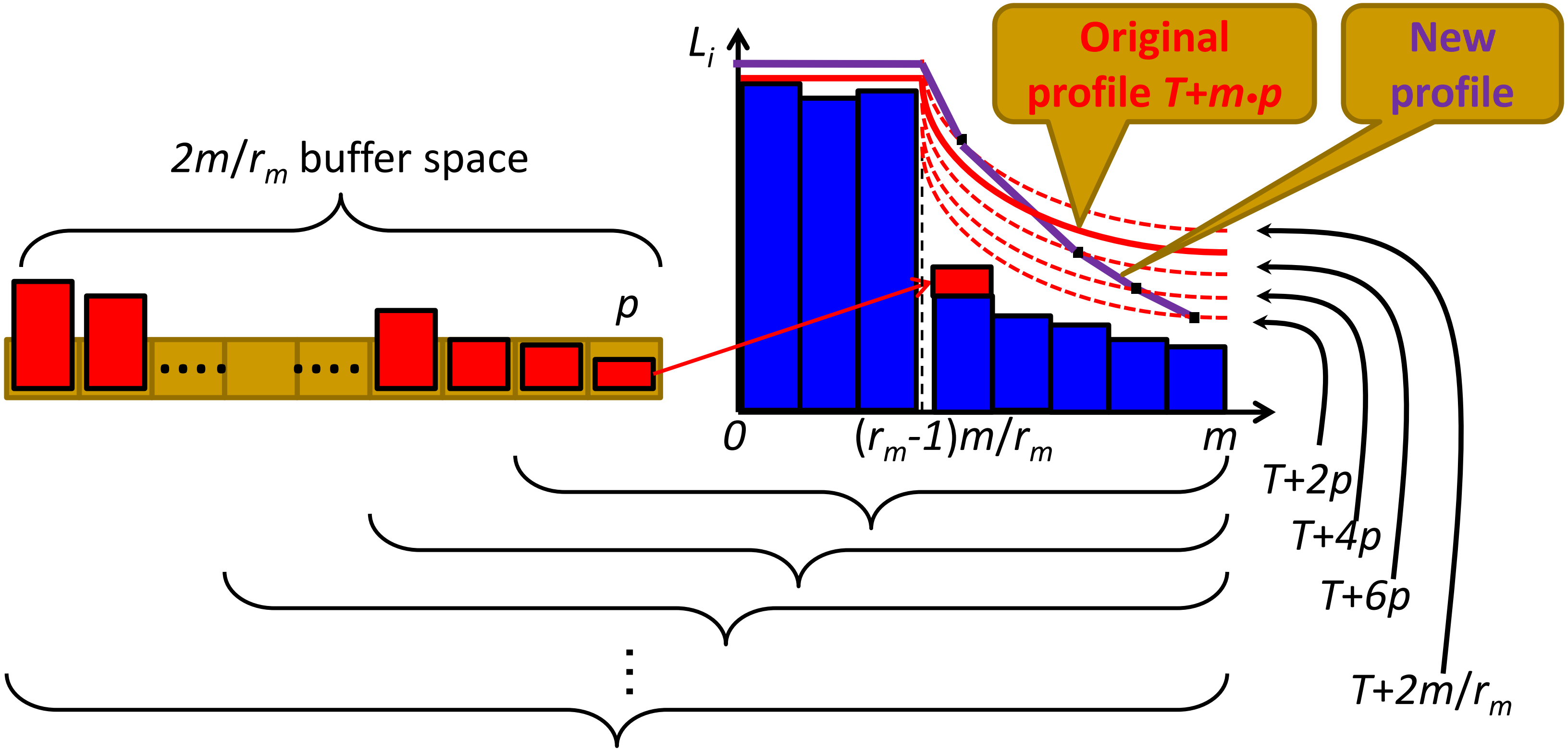

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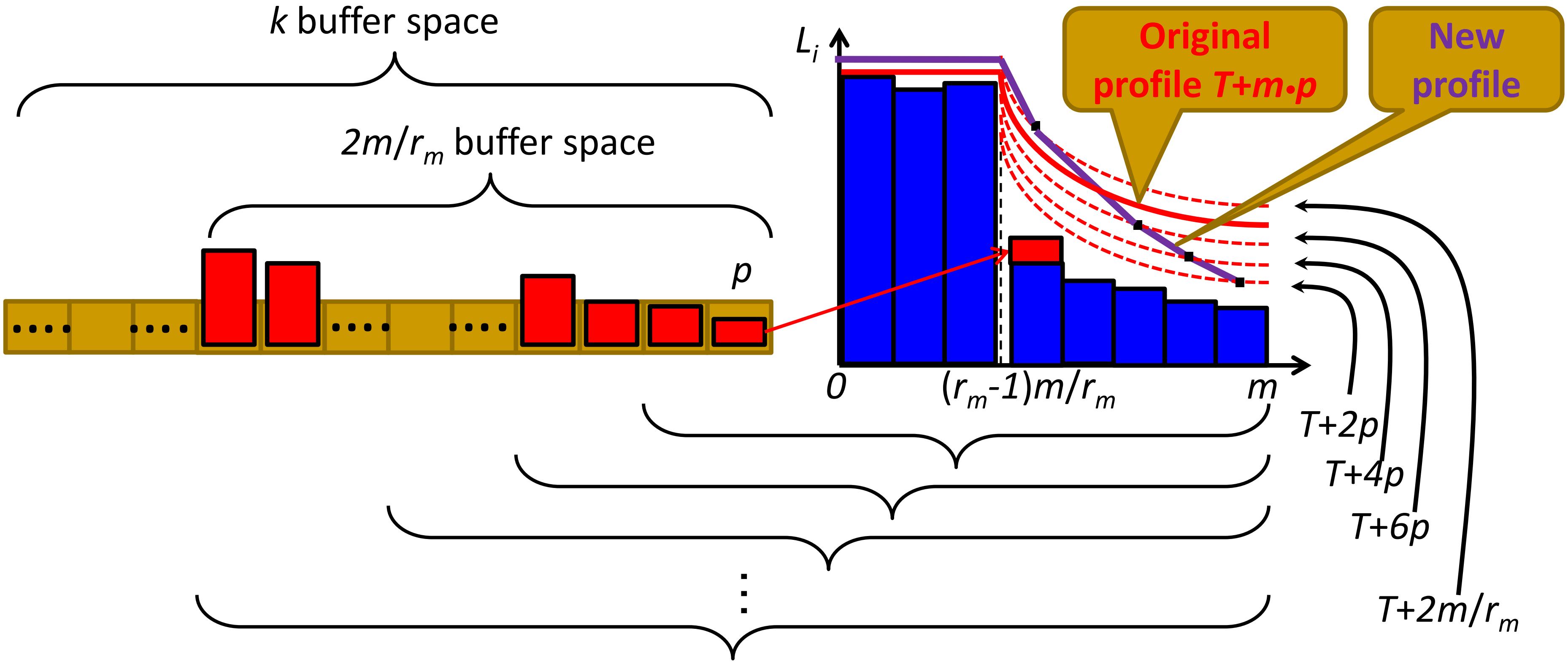

Iterative phase: Assign smallest job of size p to a machine j with load $\leq w_j \cdot (T+k-2(m-j')p) - p$, where $j' = \max\{j, (r_m-1)m/r_m\}$

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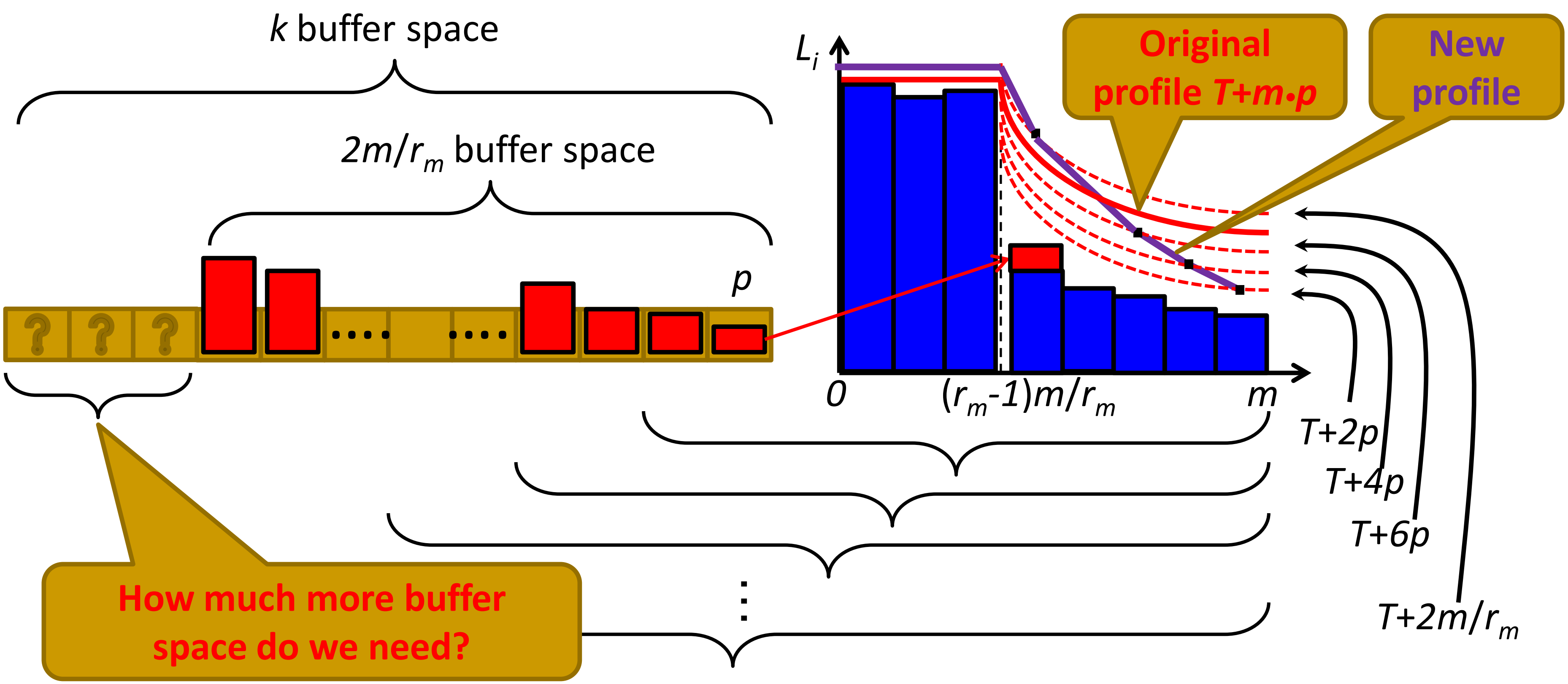

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- **Determine total buffer size *k*:** Apply the ***feasibility condition*** in the ***iterative phase***
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$$\sum w_j \cdot (T+k-2(m-j')p) \ge T + m \cdot p$$

$$\Leftarrow T + k - 2m \cdot p + 2\sum w_j \cdot j' \ge T + m \cdot p \qquad (\textstyle\sum w_j = 1)$$

$$\Leftarrow T + k - 2m \cdot p + 2(r_m-1)m \ge T + m \cdot p \qquad (\textstyle\sum w_j \cdot j' \ge (r_m-1)m)$$

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m	*2*	*3*	*4*	…	*10*	*20*	*30*	…	*m→∞*
Old *k*	*6*	*9*	*11*	…	*2.5m*	*2.455m*	*2.406m*	…	*2.364m*
New *k*	*6*	*8*	*10*	…	*2.25m*	*2.182m*	*2.125m*	…	*2.068m*

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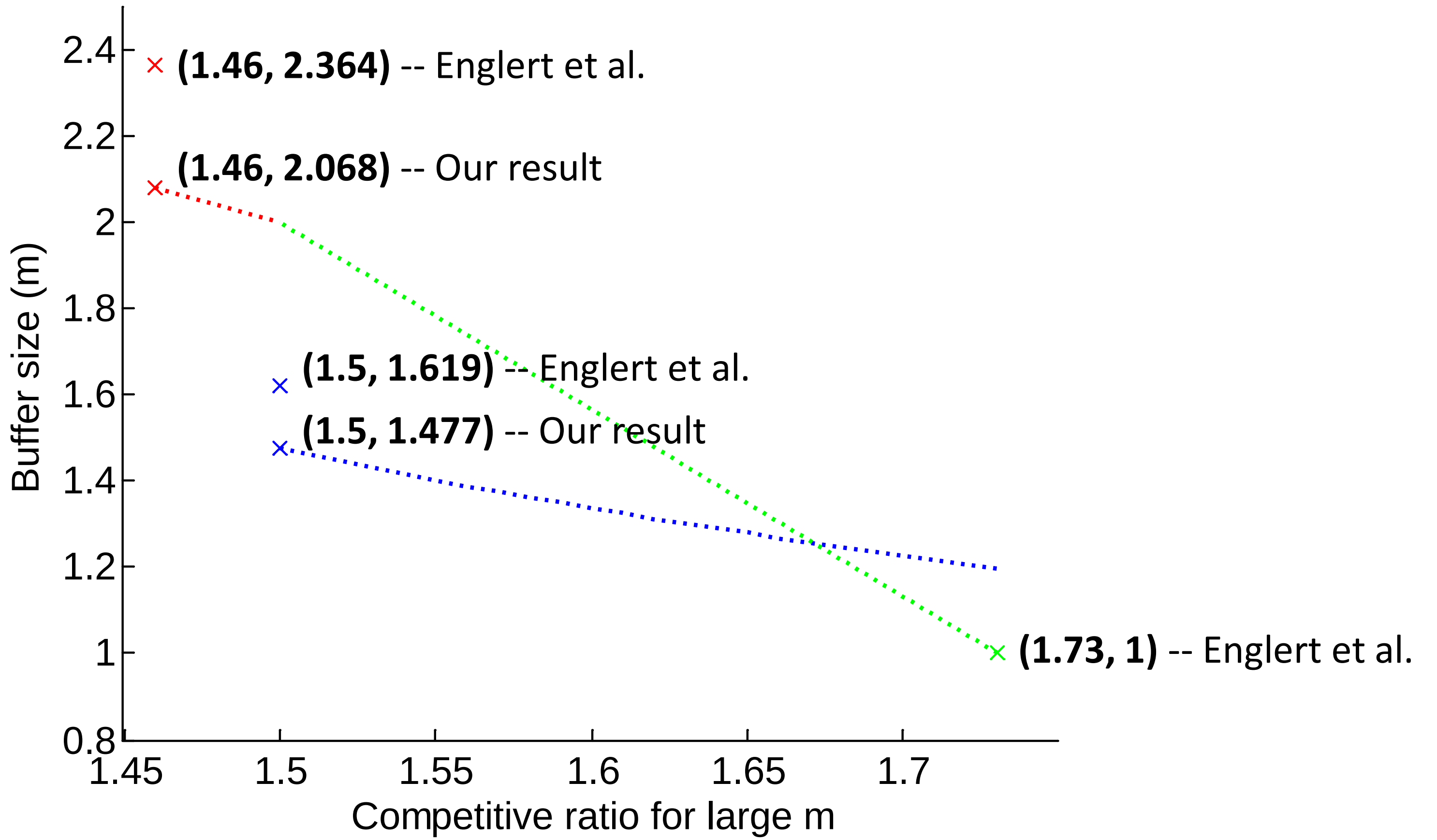
	1	6							

⇐ Lan et al. 2012

Tradeoff: Comp. ratio vs Buffer size



Combining and extending our results with the ones from Englert et al. 2008

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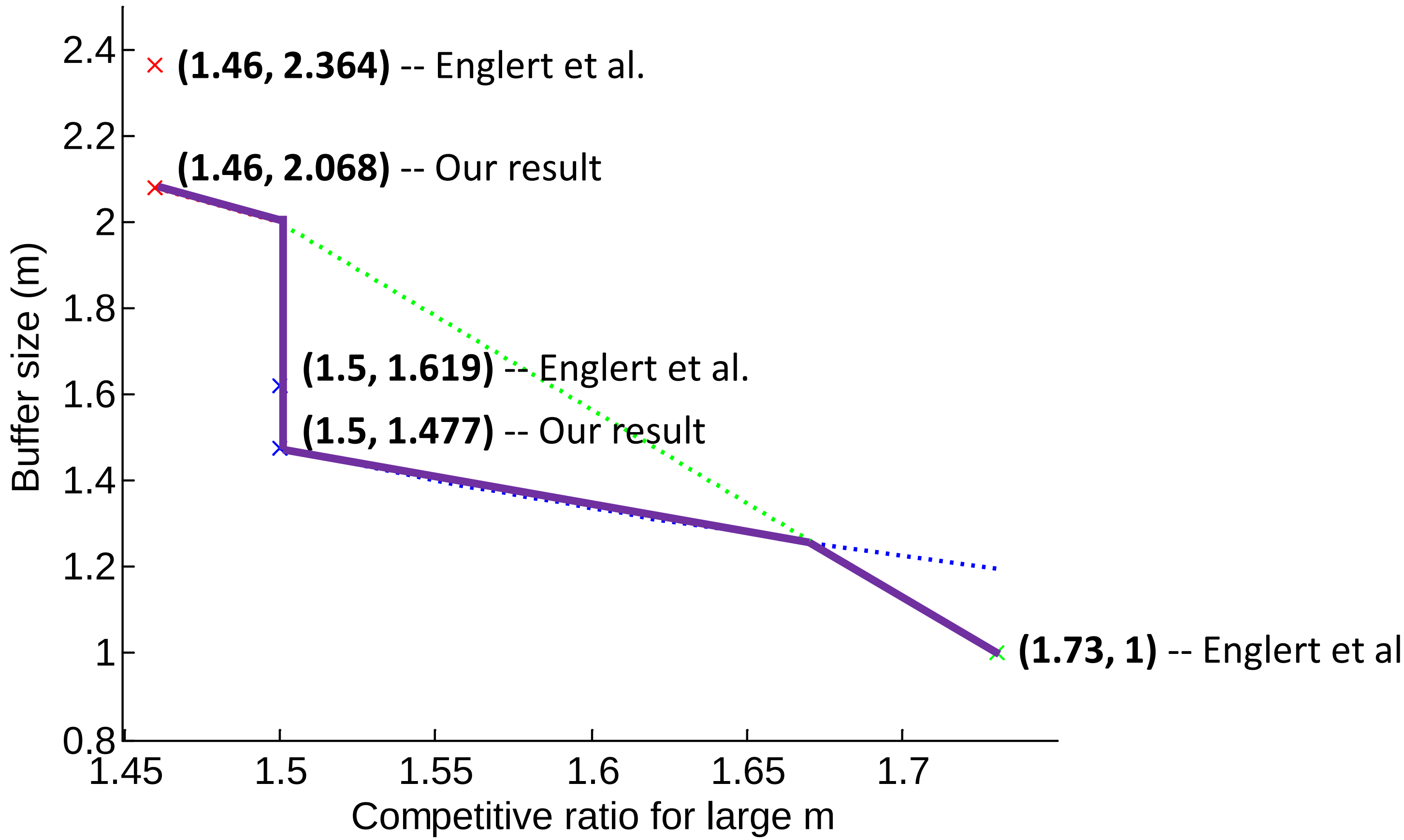

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Remarks

- **Classical online makespan scheduling**
  - **Comp. ratio for large *m***

Upper Bound:

2	(Graham 1966)
1.986	(Bartal et al. 1995)
1.945	(Karger et al. 1996)
1.923	(Albers 1999)
1.9201	(Fleischer et al. 2000)
….	

Lower Bound:

1.88	(Rudin III 2001)
1.854	(Gormley et al. 2000)
1.852	(Albers 1999)
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- **Semi-online scheduling with reordering buffer**
  - **Buffer size needed to get opt. comp. ratio for large *m***

2.364m	(Englert et al. 2008)
2.068m	(Our result)
….	
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m?	(Our work in progress)
0.5m	(Englert et al. 2008)

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A similar problem

- **Semi-online scheduling with job migrations**
  - **Problem**
    - Online algorithm is allowed to perform a limited number (independent of input size) of job migrations
  - **Results** (Albers and Hellwig 2012)
    - For general m, optimal comp. ratio (lower bound) = r_m **(Identical to the case with reordering buffer)**
    - An algorithm that achieves r_m-comp. with $[(2-r_m)/(r_m-1)^2+4]m \approx 7m$ migrations for large m
  - Results with migration can be transformed into results with reordering buffer, but not vise versa

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  - Results with migration can be transformed into results with reordering buffer, but not vise versa
  - What is min. no. of migrations required? Can the existing result be improved to maintain the optimal comp. ratio?

Thanks for your attention!