EECS 863
Homework

1. Consider a birth-death process with coefficients

\[ \lambda_k = \begin{cases} \lambda & k = 0 \\ 0 & k \neq 0 \end{cases} \quad \mu = \begin{cases} \mu & k = 1 \\ 0 & k \neq 1 \end{cases} \]

this system corresponds to an M/M/1 queue where there is no room for waiting customers.
(a) Give the differential-difference equations for \( P_k(t) \) \((k = 0, 1)\).
(b) Solve these equations in terms of \( P_0(0) \) and \( P_1(0) \).
(c) For \( P_0(0) \) and \( P_1(0) \) equal to 0.50 plot \( P_k(t) \) \((k = 0, 1)\). \( \mu = 1, \lambda = 0.9 \)

2. A leaky bucket flow control scheme works at the entry point of the network to regulate the offered traffic. Packets arriving to the leaky bucket must wait in a queue for a permit before entering the network. Permits are generated independently from the offered traffic. Thus upon arrival a packet will be sent if a permit is available otherwise it waits in the buffer for the arrival of a permit. Permits arriving to a system with no packets ready for transmission will be saved in a queue. The permit queue is limited to \( W \) permits. A permit is discarded if it arrives when the permit queue is full. For this problem assume that packets arrive to the system according to a Poisson process at a rate \( \lambda \). Also assume that the permits arrive according to a Poisson process at a rate \( \mu \). Let \( W = 4 \) for this problem.

a) Find the steady-state probability mass function for the number of packets waiting to be transmitted.
b) Find the steady-state probability mass function for the number of permits. 
   [ Hint: use a M/M/1 formulation with a careful definition of the states.]

3. Plot the probability of blocking for a M/M/1/N system as a function of \( N \) for loads of 0.50, 0.75, 0.95; plot the log(probability of blocking) vs system size \( N \). Consider a range of blocking probability down to \( 10^{-6} \). Comment on the nature of the relationship between log(probability of blocking) vs system size \( N \) for small blocking probability.