1. Prove the Corollary to Proposition 1 on p. 185. FONC Theory.

2. Consider the linear predictor problem outlined in Section I of Lecture 2.
   a. Derive the first order necessary conditions for a linear predictor of arbitrary order \( p \), assuming that \( \{x_k\} \) is a stationary random process.
   b. What are the FONC for the special case of \( p = 1 \)?
   c. Using the results of part a, find the optimal predictor coefficients for a third order \( (p = 3) \) predictor when the input sequence \( \{x_k\} \) is obtained as follows. Pass white noise through an RC lowpass filter with \( RC = 10^{-3} \), then sample the filter output at an 8 kHz sampling rate to get \( \{x_k\} \). Discuss your result.
   d. Defining the predictor gain as:

   \[
   G_p = \frac{E\left[ x_k^2 \right]}{E\left[ e_k^2 \right]}
   \]

   where \( e_k \) is the predictor error \( x_k - \hat{x}_k \), what is the predictor gain in part c?

3. For Example 2 on p. 187, verify that the summation form of \( f(a) \) is equivalent to the vector form at the bottom of the page. Example Application.

4. Luenberger/Ye, Problem 3, parts a and b only, p. 213. FONC, SOSC.

5. Let \( a \) be a given \( n \)-vector, and \( A \) be a given \( n \times n \) symmetric matrix. Compute the gradient and Hessian of \( f_1(x) = a^T x \) and of \( f_2(x) = x^T Ax \).

6. For the following function, find all points that meet the first order necessary conditions for local minima and determine which, if any, of these are in fact local minima.

   \[
   f(\mathbf{x}) = x_1^3 + x_2^3 + 2x_1^2 + 4x_2^2 + 6
   \]

7. Are there any values of \( x, y \) and \( z \) that maximize:

   \[
   f(x, y, z) = \frac{6xyz}{x + 2y + 2z}
   \]

   subject to \( xyz = 16 \) and \( x \geq 0, y \geq 0, z \geq 0 \)? If so, give the values of the variables and the value of the objective function. If not, justify your answer. Hint: minimizing \(-f\) is not the only way to maximize \( f \).