2.1 The Ideal Op-Amp

Reading Assignment: pp. 63-66

The transistor is the fundamental circuit element of modern electronics. We can use transistors to form very complex circuits that do all sorts of useful and wonderful things.

Yet, with respect to analog circuits, we might argue that the fundamental electronic circuit element is the operational amplifier, otherwise known as the op-amp.

Now, an op-amp is actually an integrated circuit that implements dozens, or even hundreds of transistors.
Q: So, how could such a complex circuit be considered to be a fundamental circuit element?

A: Despite its complexity, the behavior of an op-amp is simple and straightforward. In fact, we will find that the math describing op-amp operation is far simpler than the math describing transistor operation!

Likewise, we will find that (like the transistor) an op-amp by itself is a mostly useless device. Instead we must construct a circuit around it to achieve utility.

From: www.soton.ac.uk/~apm3/diyaudio/DAXO.html
And—like a transistor—the utility that we can achieve with an op-amp circuit is both vast and substantial.

In other words, the applications of op-amps to analog circuit design are nearly limitless!

Let’s start by examining the characteristics of an ideal op-amp!

**HO: THE IDEAL OP-AMP**
The Ideal Operational Amplifier

We begin by considering the equivalent circuit of an ideal op-amp:

Note that output voltage is defined with respect to ground potential, while the input voltage is simply the potential difference between the plus (+) terminal and the minus (-) terminal.
Very large and very small

Of course, we have three parameters in this circuit model: input resistance, output resistance, and open circuit voltage gain.

**Q:** So what are the ideal attributes ($R_{in}$, $R_{out}$, and $A_{vo}$) of an operational amplifier? In other words, what is the perfect op-amp?

**A1:** The input resistance of a perfect op-amp is infinitely large (i.e., $R_{in} = \infty$).

**A2:** The output resistance of a perfect op-amp is zero (i.e., $R_{out} = 0$).

**A3:** The open-circuit voltage gain of a perfect op-amp is very large, approaching infinity ($A_{vo} \approx \infty$)!
The ideal op-amp model

Thus, the equivalent circuit model of an ideal op-amp is:

\[ i_{in} = 0 \quad + \quad i_{out}(t) \]

\[ v_{in}(t) \quad - \quad v_{out}(t) \]

Here we have changed the notation of the open-circuit voltage gain.

The value \( A_{op} \) is used, where:

\[ A_{op} \triangleq \lim_{A_o \to \infty} A_v \]

In other words, the gain value \( A_{op} \) is unfathomably large!
Ideal at all frequencies!

Note then:

1. Since the input resistance is infinite, the input current is zero—always!

2. Since the output resistance is zero, the output voltage is equal to the open-circuit output voltage, even when the output load is not an open circuit! I.E.:

   \[ v_{out}(t) = A_{op} v_{in}(t) \quad \leftrightarrow \text{regardless of } i_{out}! \]

Q: What about the bandwidth of this “ideal” op-amp; is the model only valid for low-frequencies?

A: Not for an ideal op-amp!

The bandwidth of an ideal op-amp is likewise infinite.
It just seems so perfect

Q: Wow! Unfathomably high voltage gain, infinite input resistance (impedance), zero output resistance (impedance), and:

\[ v_{out}(t) = A_{op} v_{in}(t) \]

regardless of the frequency spectrum \( V_{in}(\omega) \).

This sounds like the perfect voltage amplifier!

A: It is! That’s why we refer to it as the ideal op-amp.
Why the output but not the input?

Q: So why isn't the input voltage with respect to ground potential? Why is not the minus (-) input terminal connected to ground?

A: Generally speaking, we find that two different voltages will be connected to the two different input terminals:
The input is a differential voltage

From KVL it is clear (right?) that the input voltage is:

\[ v_{in}(t) = v_2(t) - v_1(t) \]

And so the output voltage is:

\[ v_{out}(t) = A_{op} \left( v_2(t) - v_1(t) \right) \]

Note that the input voltage is simply the difference between the two input signals \( v_2 \) and \( v_1 \).

We call this the differential input signal:

\[ v_d(t) = v_2(t) - v_1(t) \]
It’s called a differential amplifier

Thus, we can likewise express the output as:

\[ v_{out}(t) = A_{op} v_{d}(t) \]

Amplifiers of this type—where the input voltage is not defined with respect to ground—are referred to as differential amplifiers, as they can amplify the differential mode of two distinct signals (e.g., \( v_2(t) \) and \( v_1(t) \)).
An example

For example, say:

\[ \nu_1(t) = 7.0 \cos(10\pi t) + 2.0 \cos(5\pi t) \]

and:

\[ \nu_2(t) = 7.0 \cos(10\pi t) + 5.0 \cos(5\pi t) \]

the ideal op-amp output voltage is therefore:

\[ \nu_{out}(t) = A_{op} (\nu_2(t) - \nu_1(t)) \]
\[ = A_{op} 3.0 \cos(5\pi t) \]

→ What happened to \( \cos(10\pi t) \) ??
The common mode disappears!

Note:

1. The **difference** between $v_2(t)$ and $v_1(t)$ is amplified.

2. The **common** signal ($7 \cos 10\pi t$) is eliminated by the subtraction.

Difference amplifiers **ideally** have perfect common-mode rejection. That is, the common signal between the two inputs has **no effect** on the output signal.

Of course, we can always **connect** a terminal to ground potential (i.e., $v_1(t) = 0$), thus making the input voltage a value with respect to ground:
The ideal op-amp: is it bogus?

Q: I scoff at your so-called “ideal” op-amp.

Although $R_{in} = \infty$ and $R_{out} = 0$ are obviously correct, I deem your assertion that $A_{op}$ should be unfathomably large (approaching $\infty$) to be a silly notion.

After all, a gigantic gain $A_{op}$ would mean that the output voltage $v_{out} = A_{op}(v_2 - v_1)$ would likewise be unfathomably large—the destructive implications are obvious.

A: It is true that the output voltage will be very large—unless the differential voltage is unfathomably small!
Definitely not bogus!

For example, what if the differential voltage is approximately (i.e., almost) zero:

\[ v_d(t) \approx 0 \implies v_2(t) \approx v_1(t) \]

In this case, the output voltage may not be very large at all!

Q: Yes, but what is the likelihood that the two voltages \( v_2(t) \) and \( v_1(t) \) are nearly the same? This seems improbable.

A: Op-amps are generally not implemented by themselves!

Instead, they typically are but one component of many in a feedback amplifier.
Get used to the virtual short!

In these applications, we will indeed find that \( v_2 \approx v_1 \)—but we will also find that this is a desirable condition!

The condition \( v_2 \approx v_1 \) is known as a virtual short.

If this is not true, the output voltage of an ideal op-amp will be unfathomably large.

As a result, the virtual short \( v_2 \approx v_1 \) is almost always the case in useful op-amp circuits.