Closed-Loop Bandwidth

Say we build in the lab (i.e., the op-amp is not ideal) this amplifier:

\[ A_v(\omega) = \frac{v_{out}(\omega)}{v_{in}(\omega)} \]

We know that the open-circuit voltage gain (i.e., the closed-loop gain) of this amplifier should be:

\[ A_v(\omega) = \frac{v_{out}(\omega)}{v_{in}(\omega)} = 1 + \frac{R_2}{R_1} \]

This gain will certainly be accurate for input signals \( v_{in}(\omega) \) at low frequencies \( \omega \).
As the signal frequency increases

But remember, the Op-amp (i.e., open-loop gain) gain $A_{op}(\omega)$ decreases with frequency.

If the signal frequency $\omega$ becomes too large, the open-loop gain $A_{op}(\omega)$ will become less than the ideal closed-loop gain!
The amp gain cannot exceed the op-amp gain

Note as some sufficiently high frequency (ω′ say), the open-loop (op-amp) gain will become equal to the ideal closed-loop (non-inverting amplifier) gain:

$$|A_{op}(\omega = \omega')| = 1 + \frac{R_2}{R_1}$$

Moreover, if the input signal frequency is greater than frequency ω′, the op-amp (open-loop) gain will in fact be smaller than the ideal non-inverting (closed-loop) amplifier gain:

$$|A_{op}(\omega > \omega')| < 1 + \frac{R_2}{R_1}$$

Q: If the signal frequency is greater than ω′, will the non-inverting amplifier still exhibit an open-circuit voltage (closed-loop) gain of $A_v(\omega) = 1 + \frac{R_2}{R_1}$?

A: Allow my response to be both direct and succinct—NEVER!
Closed-loop gain \(<\ \text{or}\ =\ \text{open-loop gain}\)

The gain \(A_v(\omega)\) of any amplifier constructed with an op-amp can never exceed the gain \(A_{op}(\omega)\) of the op-amp itself.

In other words, the closed-loop gain of any amplifier can never exceed its open-loop gain.

* We find that if the input signal frequency exceeds \(\omega'\), then the amplifier (closed-loop) gain \(A_v(\omega)\) will equal the op-amp (open-loop) gain \(A_{op}(\omega)\).

* Of course, if the signal frequency is less than \(\omega'\), the closed-loop gain will be equal to its ideal value \(A_v(\omega) = 1 + \frac{R_2}{R_1}\), since the op-amp (open-loop) gain is much larger than this ideal value \(|A_{op}(\omega < \omega')| \gg 1 + \frac{R_2}{R_1}\).

* We now refer to the value \(1 + \frac{R_2}{R_1}\) as the mid-band gain of the amplifier.
$1 + \frac{R_2}{R_1}$ is the midband gain

Therefore, we find for this non-inverting amplifier that:

$$|A_{vo}(\omega)| \approx \begin{cases} 1 + \frac{R_2}{R_1} & \omega < \omega' \\ |A_{op}(\omega)| & \omega > \omega' \end{cases}$$
Can we determine this bandwidth?

Now for one very important fact: the transition frequency $\omega'$ is the break frequency of the amplifier closed-loop gain $|A_{\omega}(\omega)|$.

Thus, we come to conclusion that $\omega'$ is the 3dB bandwidth of this non-inverting amplifier (i.e., $\omega' = \omega_{3dB}$)!

Q: Is there some way to numerically determine this value?

A: Of course!

Recall we defined frequency $\omega'$ as the value where the open-loop (op-amp) gain and the ideal closed-loop (non-inverting amplifier) gains were equal:

$$|A_{op}(\omega = \omega')| = 1 + \frac{R_2}{R_1}$$

Recall also that for $\omega > \omega_b$, we can approximate the op-amp (open-loop) gain as:

$$|A_{op}(\omega)| \approx \frac{A_0 \omega_b}{\omega}$$
Divide the gain-bandwidth product by gain, and you have determined the bandwidth!

Combining these results, we find:

\[ |A_{op}(\omega = \omega')| = 1 + \frac{R_2}{R_1} = \frac{A_0\omega_b}{\omega'} \]

and thus:

\[ \omega' = \left(1 + \frac{R_2}{R_1}\right)^{-1} (A_0\omega_b) \]

But remember, we found that this frequency is equal to the breakpoint of the non-inverting amplifier (closed-loop) gain \(A_v(\omega)\).

Therefore, the 3dB, closed-loop bandwidth of this amplifier is:

\[ \omega_{3dB} = \left(1 + \frac{R_2}{R_1}\right)^{-1} (A_0\omega_b) \]
This is not rocket science

Recall also that $A_0 \omega_b = \omega_r$, so that:

$$\omega_{3dB} \approx \left(1 + \frac{R_2}{R_1}\right)^{-1} \omega_r$$

If we rewrite this equation, we find something interesting:

$$\omega_{3dB} \left(1 + \frac{R_2}{R_1}\right) = \omega_r$$

Look what this says: the **PRODUCT** of the amplifier (mid-band) **GAIN** and the amplifier **BANDWIDTH** is equal to the **GAIN-BANDWIDTH PRODUCT**.

This result should not be difficult to remember!
The gain-bandwidth product is an op-amp parameter

The above approximation is valid for virtually all amplifiers built using operational amplifiers, i.e.:

\[ \left| A_v(\omega_m) \right| \omega_{3dB} = \omega_t \]

where:

\[ \left| A_v(\omega_m) \right| \text{mid-band gain} \]

In other words, \( \omega_m \) is some frequency within the bandwidth of the amplifier (e.g., \( 0 < \omega_m < \omega_{3dB} \)). We of course can equivalently say:

\[ \left| A_v(f_m) \right| f_{3dB} = f_t \]

The product of the amplifier gain and the amplifier bandwidth is equal to the op-amp gain-bandwidth product!