The 180° Hybrid Coupler

The 180° Hybrid Coupler (sometimes known as the “ring”, “rat-race”, or “Magic-T” hybrid) is a lossless, matched and reciprocal 4-port device, with a scattering matrix of the anti-symmetric form (D$_1$ symmetry):

\[
S = \begin{bmatrix}
0 & \alpha & \beta & 0 \\
\alpha & 0 & 0 & -\beta \\
\beta & 0 & 0 & \alpha \\
0 & -\beta & \alpha & 0
\end{bmatrix}
\]

Just like the quadrature coupler, however, we find that:

\[
\alpha = \beta = \frac{1}{\sqrt{2}}
\]

So that the scattering matrix for this device is:

\[
S = \begin{bmatrix}
0 & 1/\sqrt{2} & 1/\sqrt{2} & 0 \\
1/\sqrt{2} & 0 & 0 & -1/\sqrt{2} \\
1/\sqrt{2} & 0 & 0 & 1/\sqrt{2} \\
0 & -1/\sqrt{2} & 1/\sqrt{2} & 0
\end{bmatrix}
\]

Hence, this coupler is likewise a 3dB coupler—the power into a given port (with all other ports matched) is equally divided between two of the three output ports.
Note the relative **phase** between the outputs, however, is **dependent** on which port is the input.

For example, if the **input** is port 1 or port 3, the two signals will be **in phase**—no difference in their relative phase!

However, if the input is port 2 or port 4, the output signals will be 180° **out of phase** ($e^{j\pi} = -1$)

An interesting application of this coupler can be seen if we place **two input signals** into the device, at ports 2 and 3 (with ports 1 and 2 terminated in matched loads). Note the signal out of port 1 would therefore be:

$$V_1^- (z) = S_{12} V_2^+ (z) + S_{13} V_3^+ (z)$$

$$= \frac{1}{\sqrt{2}} (V_3^+ (z) + V_2^+ (z))$$

while the signal out of port 4 is:

$$V_4^- (z) = S_{42} V_2^+ (z) + S_{43} V_3^+ (z)$$

$$= \frac{1}{\sqrt{2}} (V_3^+ (z) - V_2^+ (z))$$

Note that the output of port 1 is proportional to the **sum** of the two inputs. Port 1 of a 180° Hybrid Coupler is thus often referred to as the **sum** ($\Sigma$) port.
Likewise, port 4 is proportional to the difference between the two inputs. Port 4 a $180^\circ$ Hybrid Coupler is thus often referred to as the delta ($\Delta$) port.

There are many applications where we wish to take the sum and/or difference between two signals!

The $180^\circ$ Hybrid Coupler can likewise be used in the opposite manner. If we have both the sum and difference of two signals available, we can use this device to separate the signals into their separate components!

**Q:** How is this hybrid coupler constructed?

**A:** Like the quadrature hybrid, it is simply made of lengths of transmission lines. However, unlike the quadrature hybrid, the characteristic impedance of each line is identical ($\sqrt{2}Z_0$), but the lengths of the lines are dissimilar.

**Q:** How can we possibly analyze this mess?

**A:** Note there is one plane of bilateral symmetry ($D_1$) in this circuit—we can use even/odd mode analysis!
However, we must perform **two** separate analysis—one using sources on ports 1 and 3:

![Diagram of ring hybrid](image)

*Figure 7.44 (p. 354)*

Even- and odd-mode decomposition of the ring hybrid when port 1 is excited with a unit amplitude incident wave. (a) Even mode. (b) Odd mode.

While the other uses sources on ports 2 and 4:
Finally, because of the transmission line lengths, we find that the ring hybrid is a narrow-band device:

**Figure 7.45** (p. 356)
Even- and odd-mode decomposition of the ring hybrid when port 4 is excited with a unit amplitude incident wave. (a) Even mode. (b) Odd mode.

**Figure 7.46** (p. 357)
S parameter magnitudes versus frequency for the ring hybrid of Example 7.9