The Quarter-Wave Transformer

Say the end of a transmission line with characteristic impedance \( Z_0 \) is terminated with a resistive (i.e., real) load. Unless \( R_L = Z_0 \), the resistor is mismatched to the line, and thus some of the incident power will be reflected.

We can of course correct this situation by placing a matching network between the line and the load:

In addition to the designs we have just studied (e.g., L-networks, stub tuners), one of the simplest matching network designs is the quarter-wave transformer.
The quarter-wave transformer is simply a transmission line with characteristic impedance $Z_1$ and length $\ell = \lambda/4$ (i.e., a quarter-wave line).

The $\lambda/4$ line is the matching network!

**Q:** But what about the characteristic impedance $Z_1$; what should its value be??

**A:** Remember, the quarter wavelength case is one of the special cases that we studied. We know that the input impedance of the quarter wavelength line is:

$$Z_{in} = \left( \frac{Z_1}{Z_L} \right)^2 \frac{Z_1}{R_L}$$

Thus, if we wish for $Z_{in}$ to be numerically equal to $Z_0$, we find:

$$Z_{in} = \frac{(Z_1)^2}{R_L} = Z_0$$
Solving for $Z_l$, we find its required value to be:

\[
(Z_l)^2 / R_L = Z_0
\]

\[
(Z_l)^2 = Z_0 R_L
\]

\[
Z_l = \sqrt{Z_0 R_L}
\]

In other words, the characteristic impedance of the quarter wave line is the geometric average of $Z_0$ and $R_L$!

Therefore, a $\lambda/4$ line with characteristic impedance $Z_1 = \sqrt{Z_0 R_L}$ will match a transmission line with characteristic impedance $Z_0$ to a resistive load $R_L$.

Thus, all power is delivered to load $R_L$!

Alas, the quarter-wave transformer (like all our designs) has a few problems!
Problem #1

The matching bandwidth is narrow!

In other words, we obtain a perfect match at precisely the frequency where the length of the matching transmission line is a quarter-wavelength.

But remember, this length can be a quarter-wavelength at just one frequency!

Remember, wavelength is related to frequency as:

\[ \lambda = \frac{v_p}{f} = \frac{1}{f \sqrt{LC}} \]

where \( v_p \) is the propagation velocity of the wave.

For example, assuming that \( v_p = c \) (\( c \) = the speed of light in a vacuum), one wavelength at 1 GHz is 30 cm (\( \lambda = 0.3 \text{ m} \)), while one wavelength at 3 GHz is 10 cm (\( \lambda = 0.1 \text{ m} \)). As a result, a transmission line length \( \ell = 7.5 \text{ cm} \) is a quarter wavelength for a signal at 1 GHz only.

Thus, a quarter-wave transformer provides a perfect match (\( \Gamma_{in} = 0 \)) at one and only one signal frequency!
As the signal frequency (i.e., wavelength) changes, the electrical length of the matching transmission line changes. It will no longer be a quarter wavelength, and thus we no longer will have a perfect match.

We find that the closer $R_L (R_{in})$ is to characteristic impedance $Z_0$, the wider the bandwidth of the quarter wavelength transformer.

We will find that the bandwidth can be increased by adding multiple $\lambda/4$ sections!

**Figure 5.12 (p. 243)** Reflection coefficient magnitude versus frequency for a single-section quarter-wave matching transformer with various load mismatches.
Problem #2

Recall the matching solution was limited to loads that were purely real! I.E.:

\[ Z_L = R_L + j0 \]

Of course, this is a BIG problem, as most loads will have a reactive component!

Fortunately, we have a relatively easy solution to this problem, as we can always add some length \( \ell \) of transmission line to the load to make the impedance completely real:

\[ Z_L' = Z_L + j \frac{Z_L}{Z_0} \]

However, remember that the input impedance will be purely real at only one frequency!

We can then build a quarter-wave transformer to match the line \( Z_0 \) to resistance \( R_{in} \):
Again, since the transmission lines are lossless, all of the incident power is delivered to the load $Z_L$. 