2.7 - Lossy Transmission Lines

Reading Assignment: pp. 79-82

Recall that we have been approximating low-loss transmission lines as lossless \((R = G = 0)\):

\[
\alpha = 0 \quad \beta = \omega \sqrt{LC}
\]

But, long low-loss lines require a better approximation:

\[
\alpha = \frac{1}{2} \left( \frac{R}{Z_0} + GZ_0 \right) \quad \beta = \omega \sqrt{LC}
\]

Now, if we have really long transmission lines (e.g., long distance communications), we can apply no approximations at all:

\[
\alpha = \text{Re}\{\gamma\} \quad \beta = \text{Im}\{\gamma\}
\]

For these very long transmission lines, we find that \(\beta = \text{Im}\{\gamma\}\) is a function of signal frequency \(\omega\). This results in an extremely serious problem—signal dispersion.

**HO: The Distortionless Line**
The Distortionless Line

Recall that the phase velocity $v_p$ (i.e., propagation velocity) of a wave in a transmission line is:

$$v_p = \frac{\omega}{\beta}$$

where:

$$\beta = \text{Im}\{\gamma\} = \text{Im}\left\{\sqrt{(R + j\omega L)(G + j\omega C)}\right\}$$

Thus, for a lossy line, the phase velocity $v_p$ is a function of frequency $\omega$ (i.e., $v_p(\omega)$)—this is bad!

Any signal that carries significant information must has some non-zero bandwidth. In other words, the signal energy (as well as the information it carries) is spread across many frequencies.

If the different frequencies that comprise a signal travel at different velocities, that signal will arrive at the end of a transmission line distorted. We call this phenomenon signal dispersion.

Recall for lossless lines, however, the phase velocity is independent of frequency—no dispersion will occur!
\[ v_p = \frac{1}{\sqrt{LC}} \quad [R = 0, G = 0] \]

Of course, a perfectly lossless line is impossible, but we find phase velocity is **approximately** constant if the line is low-loss.

Therefore, dispersion distortion on low-loss lines is **most often** not a problem.

**Q:** You say "**most often** not a problem"—that phrase seems to imply that dispersion sometimes is a problem!

**A:** Even for low-loss transmission lines, dispersion can be a problem if the lines are **very** long—just a small difference in phase velocity can result in significant differences in propagation delay if the line is very long!

Modern examples of long transmission lines include phone lines and cable TV. However, the **original** long transmission line problem occurred with the **telegraph**, a device invented and implemented in the 19th century.

Telegraphy was the essentially the **first** electrical engineering technology ever implemented, and as a result, led to the first ever **electrical engineers**!
Early telegraph "engineers" discovered that if they made their telegraph lines too long, the dots and dashes characterizing Morse code turned into a muddled, indecipherable mess. Although they did not realize it, they had fallen victim to the heinous effects of dispersion!

Thus, to send messages over long distances, they were forced to implement a series of intermediate "repeater" stations, wherein a human operator received and then retransmitted a message on to the next station. This really slowed things down!

Q: Is there any way to prevent dispersion from occurring?

A: You bet! Oliver Heaviside figured out how in the 19th Century!

Heaviside found that a transmission line would be distortionless (i.e., no dispersion) if the line parameters exhibited the following ratio:

\[
\frac{R}{L} = \frac{G}{C}
\]
Let’s see **why** this works. Note the complex propagation constant $\gamma$ can be expressed as:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{LC \left(\frac{R}{L} + j\omega\right)\left(\frac{G}{C} + j\omega\right)}$$

Then **IF**:

$$\frac{R}{L} = \frac{G}{C}$$

we find:

$$\gamma = \sqrt{LC \left(\frac{R}{L} + j\omega\right)\left(\frac{G}{C} + j\omega\right)}$$

$$= \sqrt{LC \left(\frac{R}{L} + j\omega\right)^2}$$

$$= (\frac{R}{L} + j\omega)\sqrt{LC}$$

$$= R\sqrt{\frac{C}{L}} + j\omega\sqrt{LC}$$

Thus:

$$\alpha = \text{Re}\{\gamma\} = R\sqrt{\frac{C}{L}}$$

$$\beta = \text{Im}\{\gamma\} = \omega\sqrt{LC}$$

The propagation **velocity** of the wave is thus:

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$
The propagation velocity is **independent** of frequency! This lossy transmission line is **not** dispersive!

Q: Right. All the transmission lines I use have the property that $R/L > G/C$. I've never found a transmission line with this **ideal** property $R/L = G/C$!

A: It is true that typically $R/L > G/C$. But, we can reduce the ratio $R/L$ (until it is equal to $G/C$) by adding series **inductors** periodically along the transmission line.

This was Heaviside's solution—and it worked! Long distance transmission lines were made possible.

Q: Why don't we increase $G$ instead?

A:
“Oliver Heaviside” was born in the same London slums as Dickens. Scarlet fever left him partly deaf. He compensated with shyness and sarcasm. Heaviside finished his only schooling in 1865. He was 16 and a top student, but he’d failed geometry. He loathed all that business of deducing one fact from another. He meant to invent knowledge -- not to compute it.

Heaviside went to work as a telegrapher. That drew him into the study of electricity. Then he read Maxwell’s new *Treatise on Electricity and Magnetism*, and it seemed to have mystical beauty. It changed his life. He quit work and sealed himself in a room in his family’s house. There he reduced Maxwell’s whole field theory into two equations. He gave electric theory its modern shape and form. Hertz got the credit for that. But in the fine print Hertz admits his ideas came from Heaviside.

Next Heaviside picked up the radical new idea of vector analysis. His most important ally was the reclusive American genius J. Willard Gibbs. Vector analysis won out, but only after Heaviside -- this shy man with his acid pen -- had started a war. He brought that war to full pitch a few years later with something called operational calculus.

He invented this strange new math by leaping over logic. It was a powerful tool, but it wasn’t rigorous. Only people like Kelvin, Rayleigh, and Hertz saw the brilliance that was driving Heaviside faster than method could follow. He knew what he was doing. He growled at his detractors, “Shall I refuse my dinner because I do not fully understand ... digestion?”

Like vector analysis, Heaviside’s calculus stood the test of time. So did the rest of his work. He gave us the theory for long distance telephones. His math has served and shaped engineering. Yet his biographer, Paul Nahin, writes a sad ending. Heaviside grew sick of fighting and faded off to Torquay in Southwest England. There he lived out his last 25 years in a bitter retreat. You don’t see much of Heaviside’s name today. But his magnificent works have been woven into the fabric of our textbooks. He deserved a better end. Yet his huge accomplishments force a happy ending on a sad life.” (Taken from [http://www.uh.edu/engines/epi426.htm](http://www.uh.edu/engines/epi426.htm))