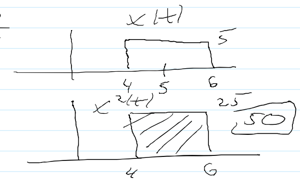
- Special function $\delta_{T_0}(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0)$

$x_p(t) = \sum_{\forall k} x(t - kT_0)$

- $\Pi(t)$
- $\delta(t)$
- $E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$x(t) = 5\,\mathrm{rect}\left(\frac{t-5}{2}\right)$



- $P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt$

if periodic $P_x = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt$ T_0 = period

$x(t) = 5\cos(\omega_0 t)$ $P_x = 25/2 = 12.5$ $\frac{A^2}{2}$

$x_p(t) = \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$

$P_{x_p} = \sum_{n=1}^{\infty} \frac{c_n^2}{2}$

- Energy signal E_x finite $P=0$ Power Signal P finite $E_x = \infty$

- Linear System $x(t) \to \boxed{} \to y(t)$

$\alpha x_1(t) \to y_1(t)$ $\beta x_2(t) \to y_2(t)$

$\alpha x_1 + \beta x_2 \to y_3(t)$

If $y_1(t) + y_2(t) = y_3(t)$ Then Linear

Time invariance $x(t) \to \boxed{} \to y(t)$

$x(t-\tau) \to \boxed{} \to y(t-\tau)$

LTI

let System be LTI $\delta(t) \to \boxed{} \to h(t)$

$h(t)$ = impulse response

$u(t) \to \boxed{} \to y_{step}(t)$ Step response

$y_{step}(t) = \int_{-\infty}^{t} h(\lambda)\,d\lambda$

$y(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)\,d\lambda$

$y(0) = \int_{-\infty}^{\infty} x(\lambda)h(-\lambda)\,d\lambda$

$h(\lambda) \to h(-\lambda)$ flip

multiply $x(\lambda)h(-\lambda)$

integrate $\to y(0)$

$y(1) = \int_{-\infty}^{\infty} x(\lambda)h(1-\lambda)\,d\lambda$

$h(\lambda)$ flip $h(-\lambda)$

$h(-\lambda)$ shift $\to h(1-\lambda)$

multiply

$x(\lambda)h(1-\lambda)$

integrate $\to y(1)$

$x(t)$ is causal then $x(t) = 0$ $t < 0$

or $x(t) = x(t)u(t)$

$y(t) = \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)\,d\lambda$

[illegible]

or $x(t) = x(t)u(t)$

$y(t) = \int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)d\lambda$

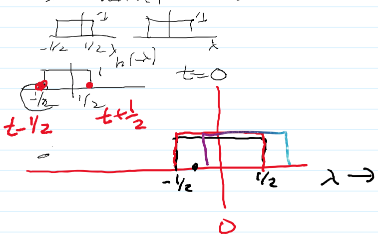
$\int_{-\infty}^{\infty} h(\lambda)\, u(t-\lambda)\, x(t-\lambda)\, d\lambda$

$u(t-\lambda) = 1 \quad t > \lambda$

$\int_{-\infty}^{t} h(\lambda)\, x(t-\lambda)\, d\lambda$

$h(t) = u(t)\, h(t)$

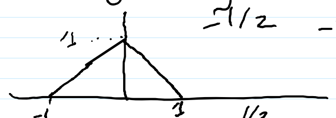
$x(t) = rect(t) \qquad h(t) = rect(t)$



Case 1 $y(t) = 0 \qquad -\infty < t < -1$

$t + 1/2 = -1/2 \qquad t = -1$

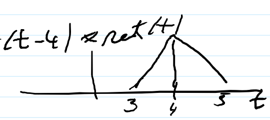
Case 2 $y(t) = \int_{-1/2}^{t+1/2} (1)(1)\, d\lambda \quad d\lambda \qquad -1 < t < 0$

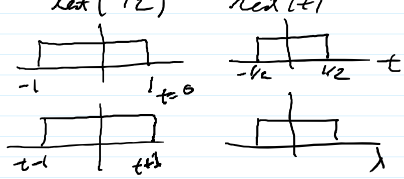


Case 3 $y(t) = \int_{t-1/2}^{1/2} (1)(1)\, d\lambda \qquad 0 < t < 1$

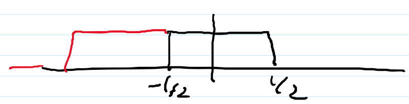
Case 4 $y(t) = 0 \qquad t > 1$

$y(t) = rect(t-4) * rect(t)$



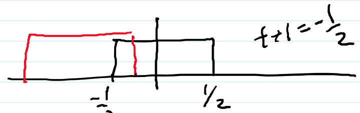


Case 1 $y(t) = 0 \qquad -\infty < t < -3/2$

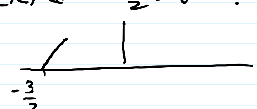


$t+1 = -1/2 \qquad t = -3/2$

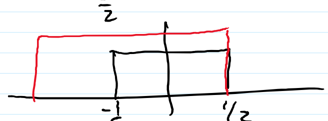
Case 2



$y(t) = \int_{-\frac{1}{2}}^{t+1} (1)(1)\, d\lambda \qquad -\frac{3}{2} < t < ?$



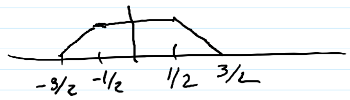
Case 3



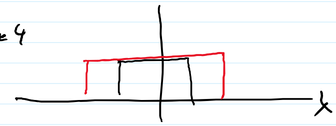
$y(t) = \int_{-1/2}^{1/2} (1)(1)\, d\lambda = 1 \qquad -\frac{3}{2} < t < -1/2$

$t+1 = \frac{1}{2} \qquad t = -\frac{1}{2}$

$y(t) = \int_{-1/2} (1)(1)\, d\lambda = \ldots$ $\quad t+1 = \frac{1}{2} \quad t = -\frac{1}{2}$

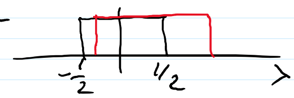


Case 4



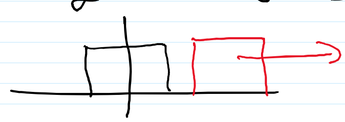
$y(t) = 1 \quad -\frac{1}{2} < t < \frac{1}{2}$

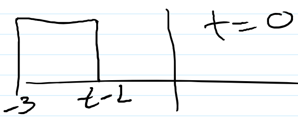
Case 5



$\int_{t-1}^{1/2} (1)(1)\, d\lambda$

Case 6 $\quad y(t) = 0 \quad t > 5/2$





$A^2/2 \qquad \frac{(\sqrt{2})^2}{2} + \frac{2^2}{2} = 3$

$A\cos(\omega_1 t) + B\cos(\omega_2 t)$ powers add

$\sqrt{2}\cos(\omega_1 t + 0.7)$ phase change

HW 3 #4 $\quad 100\times10^{-3}\cos(2\pi f_c t)$ "1" $\quad T_b = \frac{1}{10\times10^{9}}$

BPSK $\quad -100\times10^{-3}\cos(2\pi f_c t)$ "0" $\quad r_b = 10\times10^{9}$

$$\int_0^{T_b} (100\times10^{-3})^2 \cos^2(2\pi f_c t)\, dt \qquad \cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos(2\theta)$$

$\approx 5\times10^{-13}$

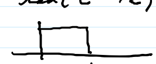
5. $\quad 10\cos(2\pi f_m t)\cos(2\pi f_c t) \qquad$ m = message, c = carrier

$$\left.\begin{array}{l} 5\cos(2\pi(f_c+f_m)t) \\ +5\cos(2\pi(f_c-f_m)t) \end{array}\right\} \quad \frac{5^2}{2} + \frac{5^2}{2}$$

$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = \int_{-\infty}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda$

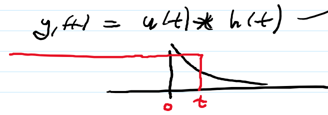
Example $\quad x(t) = \text{rect}(t - 1/2) \qquad h(t) = u(t)\, e^{-t}$

$y(t) = x * h$



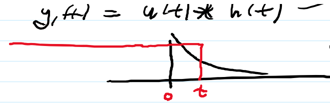
$x(t) = u(t) - u(t-1) \qquad y(t) = y_1(t) - y_1(t-1)$

$y_1(t) = u(t) * h(t)$



$y(t) = \int_0^t e^{-t}\, dt = u(t)(1 - e^{-t})$

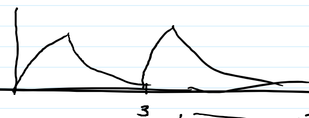
$y_1(t) = u(t) * h(t)$ — $y(t) = \int_0^{t} e^{-t}\, dt = u(t)(1-e^{-t})$

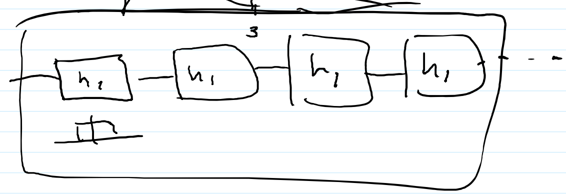


$y(t) = u(t)(1-e^{-t}) + u(t-1)(1-e^{-(t-1)})$

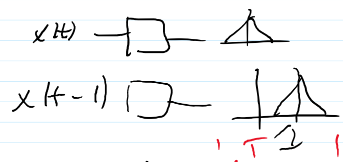
$(1-e^{-1})$

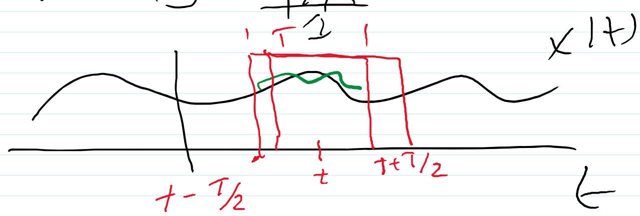
$x_1(t) = rect(t - 1/2) + rect(t-3)$





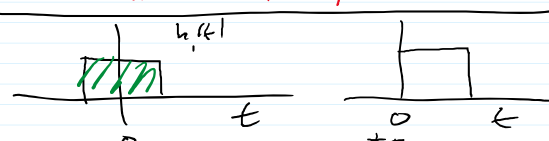
$h(t) = h_1 * h_1 * h_1 \ldots$





If $h(t) = 0$ $t<0$ then causal

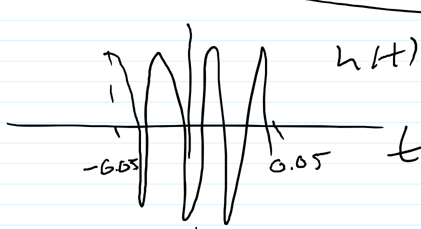
$h(t) = u(t)\, h(t)$



$y(t) = u(t)h(t) * x(t) = \int_{-\infty}^{t} x(\lambda) h(t-\lambda)$

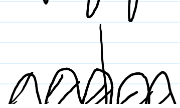
$\int_{-\infty} |h|\, d\lambda$

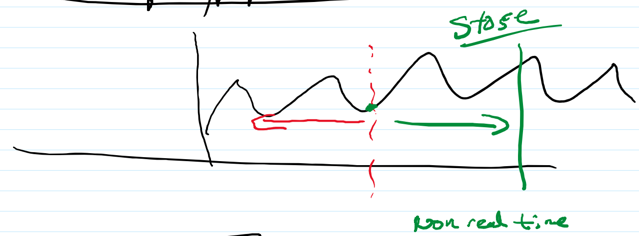
If $\int_{-\infty}^{\infty} |h(\lambda)|\, d\lambda$ is finite then BIBO stable

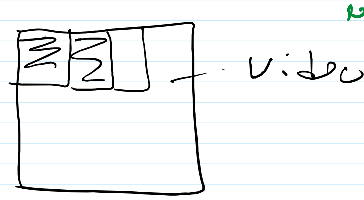


Not causal

Is stable







$$\boxed{x(t) = e^{j\omega t}} \qquad \boxed{y(t) = H(\omega)\, e^{j\omega t}}$$

$$\frac{dy}{dt} = j\omega H(\omega) e^{j\omega t}$$

$$j\omega H(\omega) \cancel{e^{j\omega t}} + \frac{1}{RC} H(\omega) \cancel{e^{j\omega t}} = \frac{1}{RC} \cancel{e^{j\omega t}}$$

no fnd in of t

solve for $H(\omega)$

$$H(\omega)\left(j\omega + \frac{1}{RC}\right) = \frac{1}{RC}$$

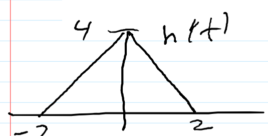
$$H(\omega) = \frac{\frac{1}{RC}}{j\omega + \frac{1}{RC}} = \frac{1}{j\omega RC + 1}$$

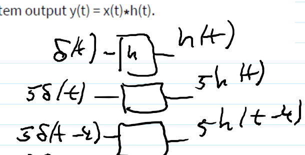
$$x(t) = \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$$

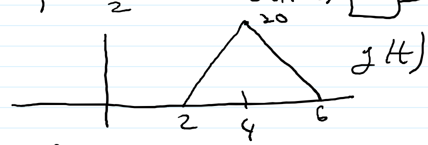
$H(\omega)$

$$y(t) = \sum_{n=1}^{\infty} c_n \left|H(n\omega_0)\right| \cos(n\omega_0 t + \theta_n + \phi_n)$$

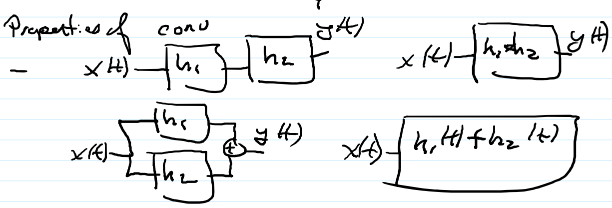
1. Let the system input be $x(t) = 5\delta(t-4)$ and the system impulse response of an LTI system be $h(t) = 4\text{tri}(\frac{t}{2})$. Find the system output $y(t) = x(t) * h(t)$.





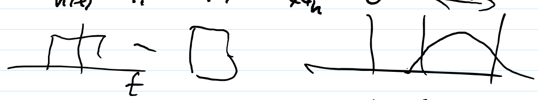


Properties of conv



$x(t)$ has width W_x $\quad y \to W_y = W_x + W_h$

$h(t)$ " " W_h



– causality LTI If $h(t) = u(t)h(t)$ then causal

– BIBO stability If $\int_{-\infty}^{\infty} |h(t)|$ is finite then stable

- BIBO Stability: If $\int_{-\infty}^{\infty} |h(t)|$ is finite then stable

- $x(t) = e^{j\omega t}$ → [ ] → $H(\omega) e^{j\omega t} = y(t)$

- Find $H(\omega)$ for LCCDE

- $x(t) = A\cos(\omega_1 t + \theta)$

$y(t) = A|H(\omega_1)| \cos(\omega_1 t + \theta + \angle H(\omega_1))$

$x(t) = \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t + \theta_n)$

$y(t) = \sum_{n=1}^{\infty} C_n |H(n\omega_0)| \cos(n\omega_0 t + \theta_n + \angle H(n\omega_0))$

Example: $\frac{C\,dy(t)}{dt} + \frac{y(t)}{R} = C\frac{dx(t)}{dt}$

$Cj\omega H(\omega) \cancel{e^{j\omega t}} + \frac{H(\omega)\cancel{e^{j\omega t}}}{R} = Cj\omega \cancel{e^{j\omega t}}$

Solve for $H(\omega)$ Not function of $\underline{t}$

$H(\omega)$ is complex $\quad H(-\omega) = H^*(\omega)$

$|H(\omega)|$ even $\quad \angle H(\omega)$ is odd

$H(\omega) = \frac{j\omega}{j\omega + \frac{1}{RC}}$ $\quad RC = \frac{1}{2\pi}$

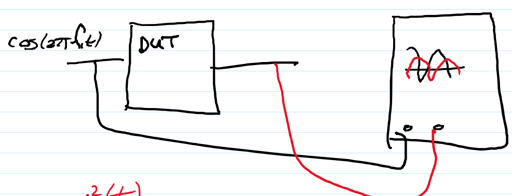
$x(t) = \cos(2\pi t)$ find $y(t)$ $\quad \omega = 1$

$|H(1)| = \frac{1}{\sqrt{2}}$ $\quad \angle H(1) = \pi/4$

$y(t) = \frac{1}{\sqrt{2}}\cos(2\pi t + \pi/4)$

$P_x = \frac{1}{2}$ $\quad P_y = \frac{(1/\sqrt{2})^2}{2} = 1/4$

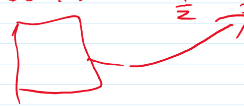
$P_y/P_x = \frac{1}{2}$ $\quad 10\log(P_y/P_x) = -3\text{dB}$



$x^2(t)$

$\cos^2(t) = \frac{1}{2} + \frac{1}{2}\cos(2\omega t)$

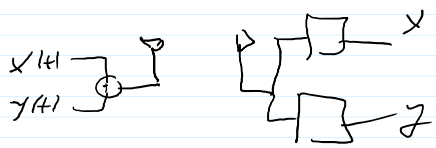
$\underline{\cos()}$

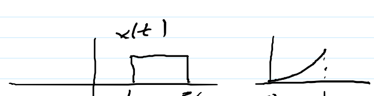


$x_p(t)$ $\quad T_0$ $\quad f_0 = \frac{1}{T_0}$ Hz

$\rightarrow \omega_0 = \frac{2\pi}{T_0}$

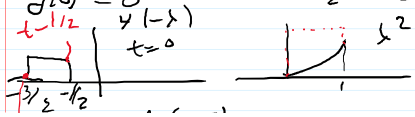
$n\omega_0$ $\quad n > 1$ harmonics

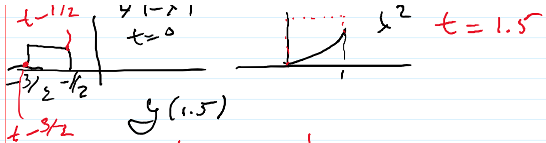




$y(0) = 0$

$t - 1/2$ $\quad y(-\lambda)$ $\quad t = 0$ $\quad \lambda^2$ $\quad t = 1.5$





$y(1.5)$

$y(1.5) = \int_0^1 \lambda^2 d\lambda = \frac{1}{3}$

– $H(\omega) \rightarrow$ given LCCDE

$x(t) = \sum C_n \cos(n\omega_0 t + \theta_n)$ $\quad \omega_0 =$ fundamental freq

$y(t) = \sum |H(n\omega_0)| c_n \cos(n\omega_0 t + \theta_n + \angle H(n\omega_0))$

– Phasor $\bar{X} = |X| e^{j\phi}$ $\quad$ LCCDE

solve for $\bar{Y}$ @ ω_1 $\quad y(t) = Re[\bar{Y} e^{j\omega t}]$

– Orthogonality $\quad x$ & y are real

$$\int_0^T x(t) y(t) dt = 0$$

Example: $f_c = 100 MHz$ $\quad \Delta f = 20 kHz$

$T = \frac{1}{\Delta f} = 50 \mu s$

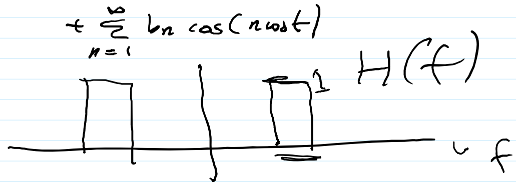
$$\int_0^T \cos(2\pi f_c t) \cos(2\pi(f_c + \Delta f)t) = 0$$

$\frac{f_c}{\Delta f} =$ integer

– $x_p(t)$ periodic T_0 $\quad f_0 (Hz) = \frac{1}{T_0}$

$\omega_0 = \frac{2\pi}{T_0}$

$$x_p(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \cos(n\omega_0 t)$$



$$y(t) = c_0 |H(0)| + \sum_{n=1}^{\infty} c_n |H(nf_0)| \cos(2\pi n f_0 + \theta_n + \angle H(nf_0))$$

Feb 12, 2026

Fourier Series $\quad x_p(t) = \sum_{k=-\infty}^{\infty} p(t - kT_0)$ $\quad p(t)$ $\quad -\tau/2 \quad t\tau/2$

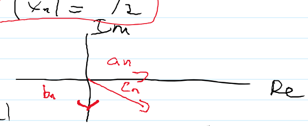
$x_p(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$

$c_n \geq 0 \quad n \neq 0$

$a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$

$\sum_{n=-\infty}^{\infty} x_n e^{jn\omega_0 t}$ $\quad x_n = |x_n| e^{j\phi_n}$

$x_n = \frac{1}{2}(a_n + j b_n)$ $\quad \boxed{|x_n| = c_n/2}$



– $\text{sinc}(x) = \frac{\sin(x)}{x}$

– $p(t) = \text{rect}(t/\tau)$

$$\frac{\tau}{T_0} + \sum_{n=-\infty}^{\infty} (\tau/T_0) \text{sinc}\left(\frac{n\pi\tau}{T_0}\right)$$

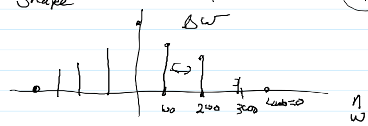
- $p(t) = \text{rect}(t/\tau)$

$$\underbrace{\frac{\tau}{T_0}}_{n=0} + \underbrace{\sum_{n=-\infty}^{\infty}}_{n\neq 0} (\tau/T_0)\,\text{sinc}\left(\frac{n\pi\tau}{T_0}\right)$$

Line spacing $= 1/T_0$ (Hz) $\quad \frac{2\pi}{T_0}$ rad/sec

Shape $\rightarrow \tau$ $\quad$ $\tau/T_0 = 1/4$

BW

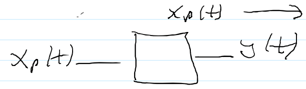


$\frac{n\pi}{4} \quad n = \frac{4}{\pi}$

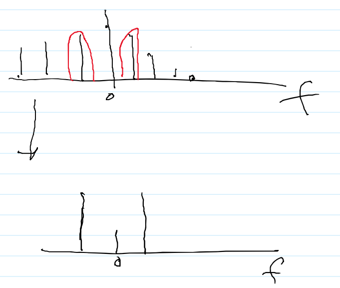
- If $p(t)$ is real then $|X_n|$ real ϕ_n odd

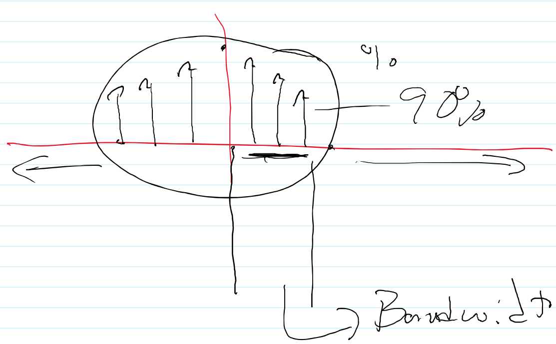
- Magnitude plots
- Given LCCDE $\rightarrow H(\omega)$

$x_p(t) \longrightarrow$ F.S

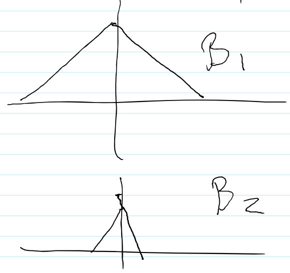


$= c_0 |H(0)| + \sum_{n=1}^{\infty} c_n |H(n\omega_0)| \cos(n\omega_0 t + \phi_n)$





$$B \propto \frac{1}{\tau}$$



$B_2 > B_1$

Feb 17, 2026 HW 6 #9

$h(t) = u(t)e^{-t} - 16e^{-2t}u(t) + 13e^{-3t}u(t)$

$x(t) = \cos(2t) \quad \omega_1 = 2$

$h(t) = u(t)e^{-t} - 16e^{-2t}u(t) + 13e^{-3t}u(t)$

$x(t) = \cos(2t) \quad \omega_1 = 2$

Section 3.7 $\quad H(\omega) = \int_{-\infty}^{\infty} h(\tau)e^{-j\omega\tau}\,d\tau$

Ex 3.7.1 $\quad \int_0^{\infty} e^{-a\tau}e^{-j\omega\tau}\,d\tau = \frac{1}{a+j\omega}$

$H(\omega) = \frac{5}{1+j\omega} - \frac{16}{2+j\omega} + \frac{13}{3+j\omega}$

$$\boxed{y(t) = |H(\omega_1)|\cos(2t + \angle H(\omega_1))}$$

$H(\omega_1) = \underbrace{\frac{5}{1+2j}}_{1-2j} - \underbrace{\frac{16}{2+2j}}_{-4+4j} + \underbrace{\frac{13}{3+2j}}_{3-2j} = 0$

$\omega_1 = 2$

HW 6 #7 $\quad \frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 7y = 5\frac{dx}{dy}$

$H(\omega) \quad x(t) = e^{j\omega t} \quad y(t) = H(\omega)e^{j\omega t}$

$H(\omega) = \frac{5j\omega}{(j\omega)^2 + 2j\omega + 7} = \frac{5j\omega}{(7-\omega^2) + 2j\omega}$

b. Find ω_1 such that $\angle H(\omega_1) = 0$

$7 - \omega_1^2 = 0 \quad H(\omega_1) = 5/2$

$\omega_1 = \sqrt{7}$

— F.S. $\quad x_p(t) = \Sigma X_n e^{jn\omega_0 t}$

$H(\omega) \quad y(t) = |X_n||H(n\omega_0)|$
$\cos(n\omega_0 t + \angle X_n + \angle H(n\omega_0))$

$H(\omega)$ weighting on each ω

— Parseval's Theorem $\quad \sum_{n=-\infty}^{\infty} |X_n|^2 = \frac{1}{T_0}\int_0^{T_0} |x(t)|^2\,dt$

— Example! $\quad x(t) = 10\,\text{rect}(t/0.1) \quad x_p(t) = \sum_{k=-\infty}^{\infty} x(t-kT_0)$

$T_0 = .5 \quad$ Find power between 1Hz – 5Hz

$x_p(t) = \frac{A\tau}{T_0} + \sum_{n=1}^{\infty} \frac{A\tau}{T_0}\,\text{sinc}\left(\frac{n\pi\tau}{T_0}\right)\cos(n\omega_0 t)$

$A = 10 \quad \tau = .1 \quad T_0 = .5 \quad \frac{\tau}{T_0} = .2 \quad f_0 = 2\text{Hz}$

$\text{sinc}\left(\frac{5\pi}{.2}\right)$

$\text{sinc}(\pi) = 0$

$50\,\text{sinc}\left(\frac{\pi}{5}\right)\cos(2\pi 2t) + 50\,\text{sinc}\left(\frac{2\pi}{5}\right)\cos(2\pi 4t)$

$P = 1803.$ $\quad$ End for Test 1

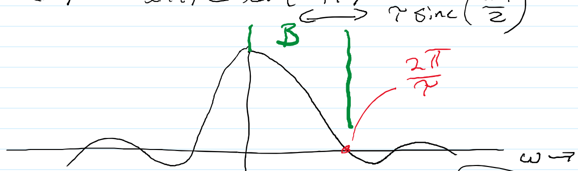
Fourier Transform $\quad X(\omega) = \int_{-\infty}^{\infty} x(\tau)e^{-j\omega\tau}\,d\tau$

$H(\omega) = \int_{-\infty}^{\infty} h(\tau)e^{-j\omega\tau}\,d\tau$

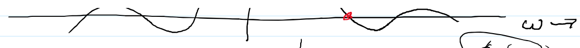
$H(\omega) = \mathcal{F}\{h(t)\}$

$x(t) = \mathcal{F}^{-1}(X(\omega)) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega$

Example $\quad x(t) = \text{rect}(t/\tau) \longleftrightarrow \tau\,\text{sinc}\left(\frac{\omega\tau}{2}\right)$



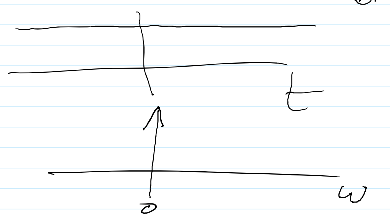
$B \propto \frac{1}{\tau} \quad$ Hz $\quad$ $\boxed{\$/\text{Hz}}$



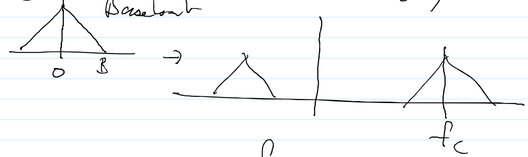
$B \propto \frac{1}{\tau}$ Hz $\boxed{\$/Hz}$

bits/sec $\neq$ Hz

D.C.

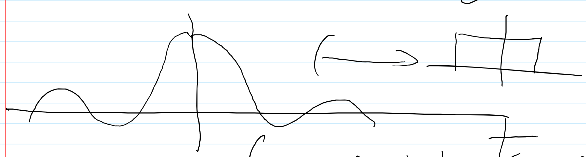


$X(f)$ Baseband $\quad x(t)\cos(2\pi f_c t)$

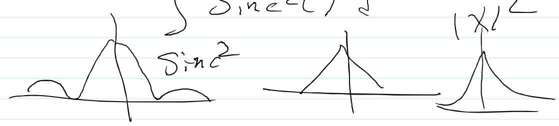


C = carrier freq.

$B = 3$ kHz AM $f_c = 610$ kHz $\quad$ 91.5 MHz



$\int \text{Sinc}^2(\,)\,d$ $\quad |X|^2$



February 19, 2026

$\frac{dy}{dt} + y(t) = x(t) \qquad x(t) = e^{j\omega t}$

$y(t) = H(\omega)e^{j\omega t}$

$j\omega H(\omega)\cancel{e^{j\omega t}} + H(\omega)\cancel{e^{j\omega t}} = \cancel{e^{j\omega t}}$

$H(\omega) = \frac{1}{1+j\omega} \qquad x(t) = 1 \rightarrow 1$

$\omega = 0$

$\cos(t) \rightarrow \omega = 1 \quad H(1) = \frac{1}{1+j\omega} = \frac{1}{\sqrt{2}e^{j\pi/4}}$

$\sqrt{2}\cos(t - \pi/4)$

Fourier Transform — properties

- Additive — Time scaling $B \propto \frac{1}{\tau}$ — time shift $x(t-t_0)$ linear phase
- freq shift — modulation — duality $x(t) \leftrightarrow X(\omega)$, $X(t) \leftrightarrow x(-\omega)$
- conv in time is multiply freq

$x(t) * h(t) \longleftrightarrow X(\omega)H(\omega)$

- multiply time

$x(t)\cdot y(t) \longleftrightarrow \frac{1}{2\pi} X(\omega) * Y(\omega)$?

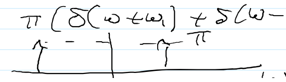
- If $x(t)$ is real then $|X(\omega)|$ is even, $\angle X(\omega)$ is odd

FT pairs

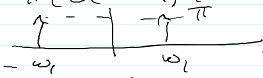
$\delta(t) \leftrightarrow 1 \qquad 1 \leftrightarrow 2\pi\delta(\omega)$

$A\,\text{rect}(t/\tau) \longleftrightarrow A\tau\,\text{sinc}\left(\frac{\omega\tau}{2}\right)$

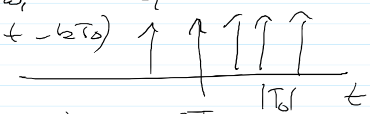
$\cos(\omega_0 t) \longleftrightarrow \pi\left(\delta(\omega+\omega_0) + \delta(\omega-\omega_0)\right)$



$$\cos(\omega_1 t) \longleftrightarrow \pi[\delta(\omega+\omega_1)+\delta(\omega-\omega_1)]$$



$$\delta_{T_0}(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT_0)$$



$$\frac{1}{T_0}\sum_{n=-\infty}^{\infty} \delta(\omega-n\omega_0) \qquad \omega_0 = \frac{2\pi}{T_0}$$

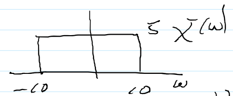
Example $x(t) = \frac{50}{\pi}\text{sinc}(10t)$

a) E_x

b) % energy $B = 2.5$ rad/sec

$$\int_{-\infty}^{\infty}\left(\frac{50}{\pi}\text{sinc}(10t)\right)^2 dt$$

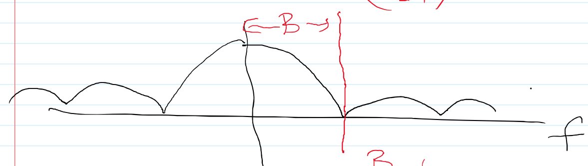
$$X(\omega) = 5\,\text{rect}\left(\frac{\omega}{20}\right)$$

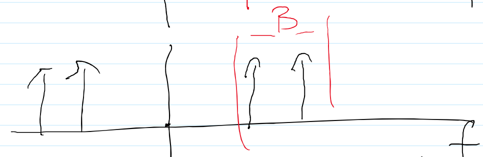


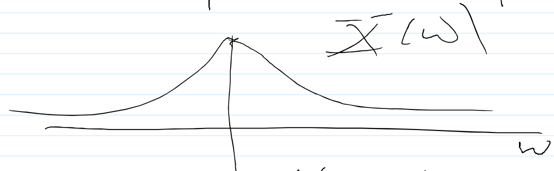
$$\frac{1}{2\pi}\int |X(\omega)|^2 d\omega$$

$$\frac{20\cdot 25}{2\pi} = 79.6$$

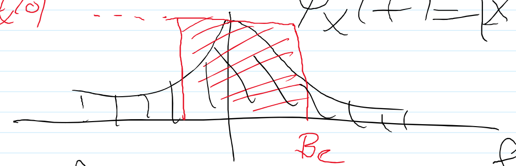
$$\% = 100\left(\frac{\frac{5\cdot 25}{2\pi}}{\frac{20\cdot 25}{2\pi}}\right) = 25\%$$







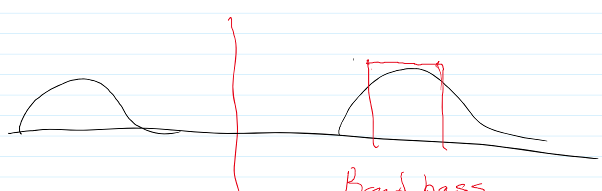
$\psi_x(0)$ $\qquad \psi_x(f) = |X(f)|^2$



$$\int_{-\infty}^{\infty}\psi_x(f)\,df = \psi_x(0)\,2B_e$$

$$B_e = \frac{\int_{-\infty}^{\infty}\psi_x(f)\,df}{2\psi_x(0)}$$

$$= \frac{\int_{0}^{\infty}\psi_x(f)\,df}{\psi_x(0)}$$



Band pass case

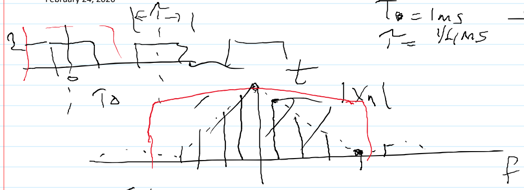
$$B \propto \frac{1}{\tau}$$

$$B \propto \frac{1}{\tau}$$

February 24, 2026

$T_0 = 1ms \quad f_0 = 1000$

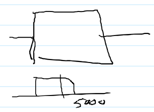
$\tau = 1/4 \, ms$



$1/\tau = 4000 Hz$

$$P_x = \int_{-T_0/2}^{T_0/2} (2\,rect(t/\tau))\,dt\, f_0/2 = \int_{-\tau/2} 4\, d\tau = 1$$

$x(t)$



$y(t) = \quad n = -5 \ldots 5$

$$P_y = \sum_{n=-5}^{5} |X_n|^2$$

$x_0 = 1/2$

$$x_n = A\frac{\tau}{T_0}\,sinc\left(\frac{n\pi\tau}{T_0}\right)$$

$x_1 = 0.45 \quad x_4 = 0$

$x_2 = .35 \quad x_5 = .09$

$x_3 = .15$

$P_y = 0.91 \quad 91\%$

$$x_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\, e^{jn\omega_0 t}\, dt$$

$$\frac{1}{T_0}\int_{-\tau/2}^{\tau/2} 2\, e^{jn\omega_0 t}\, dt$$

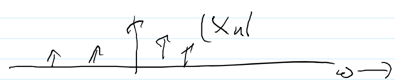
$x(t) = \cos(2\pi 500 t) + \frac{1}{2} + \frac{1}{2}\cos(2\pi 1000 t)$

$a_0 = \frac{1}{2} \quad a_1 = 1 \quad a_2 = 1/2$

- $E_x = \int_{-\infty}^{\infty} |x(t)|^2\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\, d\omega$

$x(t) \longleftrightarrow X(\omega)$

- $x_p(t) = \sum X_n e^{jn\omega_0 t} \longleftrightarrow \sum 2\pi X_n \delta(\omega - n\omega_0)$

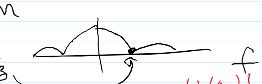


- $x_p(t) = \sum x(t - kT_0)$

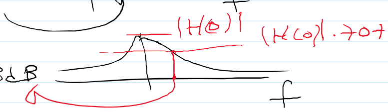
$\omega_0 = \frac{2\pi}{T_0}$

$x_n = \frac{1}{T_0} X(n\omega_0)$

- Bandwidth
  + B_{f_3}



  + B_{3dB}



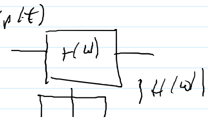
  + B_e

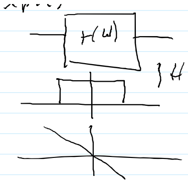
  + B_{rms}

+ $B\tau \to$ constant

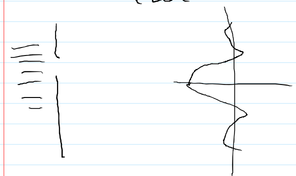
$B \propto \frac{1}{\tau}$

$x_p(t)$



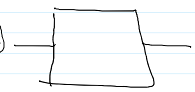


$y(t) = A|H(\omega)| \cos(\omega t + \theta \angle H(\omega))$

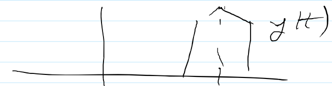


Ideal filter

$\delta(t)$ $\quad K\delta(t-t_0)$ $\quad K \neq 0$

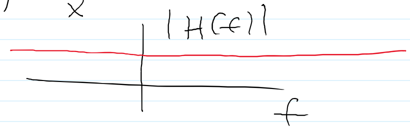


$Kx(t-t_0)$



$X(f)$ $\qquad Y(f) = K e^{-j2\pi f t_0} X(f)$

$H(f) = \frac{Y}{X} = K e^{j2\pi f t_0}$



Constant Magnitude

Linear Phase

$h(t) = K\delta(t - t_0)$

dB $\qquad$ dB$_m$ $\qquad$ Power wrt 1mS

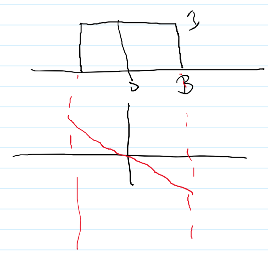
1mW	0dB$_m$
10mW	=10dBm

dBw $\qquad$ 1watt

dB$_{\text{stuff}}$

ILPF $\quad K e^{-j2\pi f t_0}$ $\quad |f| < B$

$= 0$ $\quad |f| > B$



February 26, 2026

HW 7 $\quad$ 15 $\quad 5\cos(2\pi 300 t) - 2\sin(2\pi 600 t)$

$5\cos(2\pi 300 t) - 2\cos(2\pi 600 t - \pi/2)$

$5\cos(2\pi 300 t) + 2\cos(2\pi 600 t - \pi/2 - \pi)$ $\quad -\frac{3\pi}{2}$

$\underbrace{\frac{2}{2} e^{-j3\pi/2}}_{X_{-2}} e^{-j2\pi 600 t} + \underbrace{\frac{5}{2}}_{X_{-1}} e^{-j2\pi 300 t} + \underbrace{\frac{5}{2}}_{X_1} e^{+j2\pi 300 t} + \underbrace{\frac{2}{2} e^{+j\frac{3\pi}{2}}}_{X_2} e^{+j2\pi 600 t}$

$P = \sum |X_i|^2 = 1^2 + \left(\frac{5}{2}\right)^2 + \left(\frac{5}{2}\right)^2 + 1^2$

$= 5^2/2 + 2^2/2$

- $H(\omega) = |H(\omega)| e^{j\phi(\omega)} \rightarrow \phi(\omega)$ is important
- LPF, BPF, BRF, HPF → passband → BW

$h(t)$ $\quad$ If $B_{system} >> B_{signal}$

- LPF, BPF, BRF, HPF → passband → BW
- $x(t)$ B_{signal} — $\boxed{B_{system}}$ — $y(t)$

  If $B_{system} >> B_{signal}$ then $y(t) \approx x(t)$

- RLC bandpass & bandreject filter

  tune by change $\underline{C}$

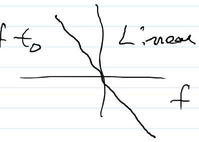
  $\omega_0 = \frac{1}{\sqrt{LC}}$

  $\boxed{Q\text{-factor}}$

- Distortionless transmission   Ideal

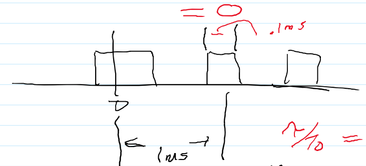
  $x(t)$ —$\boxed{}$— $y(t) = K\,x(t - t_0)$ $\forall f$

  $H(f) = K e^{-j2\pi f t_0}$ $\phi(f) = -2\pi f t_0$

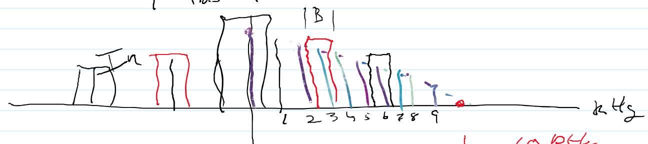


ILPF $H(f) = K e^{-j2\pi f t_0}$ $|f| < B$

$= 0$



$\tau/T = .1$ 10%



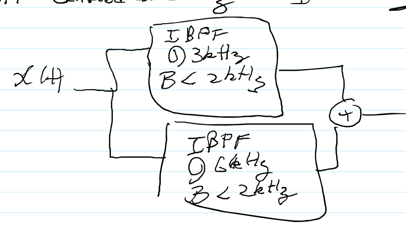
$\frac{1}{\tau} = 10\text{ kHz}$

$f_0 = 1\text{ kHz}$

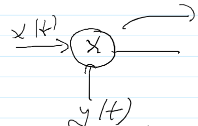
Out

a) IBPF centered at 3 kHz $B < 2$ Hz

b)



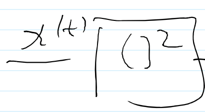
c) ILPF $B < 1$ kHz



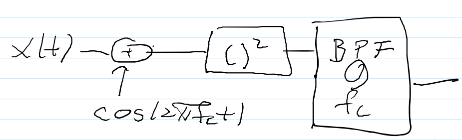
→ mixer   product multiplier

typically $y(t) = \cos(2\pi f_c t)$

C = Carrier



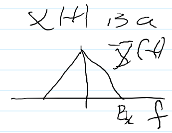
$y(t) = x^2(t)$

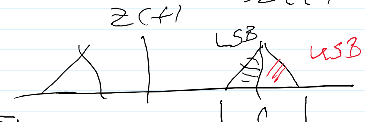


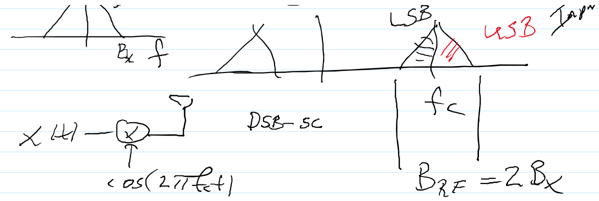
→ $K\,x(t)\cos(2\pi f_c t)$

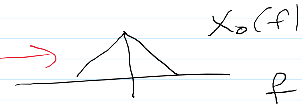
→ $z(t)$

$x(t)$ is a LP signal





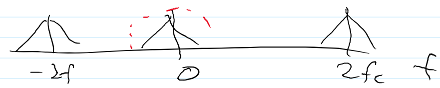




won't

$$z(t)\cos(2\pi f_c t) = [x(t)\cos(2\pi f_c t)]\cos(2\pi f_c t)$$

$$x(t)\cos^2(2\pi f_c t) = \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos(2\pi 2 f_c t)$$



$z(t) \to \otimes \to$ LPF B_x $\to \frac{x(t)}{2}$

$\cos(2\pi f_c t)$

March 3, 2026

$x_p(t)$ $P = \frac{1}{T_0}\int_{T_0} |x(t)|^2 dt$

$E_x = \int |x(t)|^2 dt$

$x(t)$

$x\left(\frac{t - t_0}{\tau}\right)$

$A^2/2$

$A_n \cos(n\omega_0 t + \theta_n)$

LTI

$\delta(t) \to \square \to h(t)$

$u(t)$ $y_{step}(t)$

$$y_{step}(t) = \int_{-\infty}^{t} h(\lambda)\, d\lambda$$

series

$\to h_1 \to h_2 \to$

$\boxed{h = h_1 * h_2}$

h_1, h_2, $+$

$\boxed{h_1 + h_2}$

BIBO

$x(t) \to \square \to y(t)$

Bounded Bounded

If $\int |h(t)|\, dt$ finite

then BIBO

If $h(t) = 0$ $t < 0$

then causal

If $h(t) = h(t)u(t)$

$x(t) = e^{j\omega t}$ LTI

$H(\omega)e^{j\omega t}$

If $h(t) = h(t)u(t)$

$x(t) = e^{j\omega t}$ LTI

$H(\omega) e^{j\omega t}$

$H(\omega)$

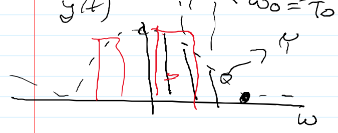
If $x(t) = A\cos(\omega_1 t + \theta)$

$y(t) = A\,|H(\omega_1)| \cos(\omega_1 t + \theta + \angle H(\omega_1))$

Find $H(\omega)$

Find c_n & ϕ_n

$y(t)$ $\quad \omega_0 = \frac{2\pi}{T_0}$



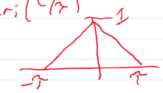
$\sum A_n \cos(n\omega_0 t + \phi_n)$

$P = \sum_{n=0}^{\infty} A_n^2/2$

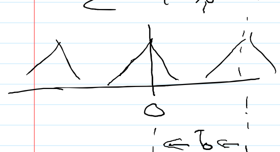
$\sum_{n=-\infty}^{\infty} |X_n|^2$

% power in Band of frequencies

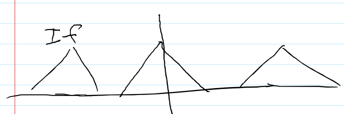
$tri(t/\tau)$



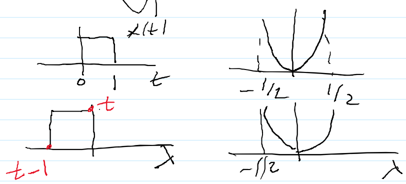
$\sum tri\left(\frac{t - nT_0}{\tau}\right)$



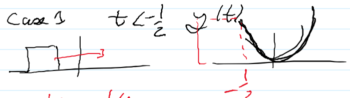
If



$b_n = 0$ $\quad a_n = 0$

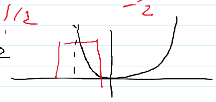


Case 1 $\quad t < -\frac{1}{2}$ $\quad y(t)$



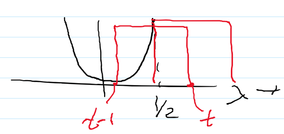
$t = -1/2$

case 2



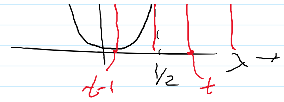
$\int_{-1/2}^{t} \lambda^2 \, d\lambda \qquad -\frac{1}{2} < t \; \frac{1}{2}$

Case 3



$\int^{1/2}$

Case 3



$$\int_{t-1}^{1/2} \lambda^2 \, d\lambda \qquad \tfrac{1}{2} < t < \tfrac{3}{2} \qquad t-1 = 1/2$$

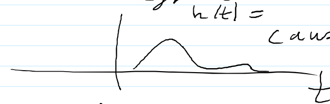
Dynamic $y(t) = x(t) - 5x(t+3)$

$y(3) = x(3) - 5x(8)$

Causal $\rightarrow$ not causal

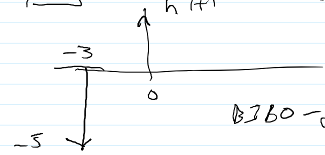
LTI for $h(t) \neq K$ dynamic

$h(t) =$ causal



Find $h(t)$

$\delta(t) \rightarrow \boxed{} \rightarrow h(t) \qquad h(t) = \delta(t) - 5\delta(t+3)$



BIBO - Jost

$a x_1(t) \rightarrow a x_1(t) - 5a x_1(t+3) = y_1$

$b x_2(t) \rightarrow b x_2(t) - 5b x_2(t+3) = y_2$

$a x_1 + b x_2 \rightarrow y_1 + y_2$ so Linear

$x(t-\tau) = x_1(t-\tau) - 5x_1(t+3-\tau)$

$= y_1(t-\tau)$ TI

$y(t) = x(t) - 5x(t+3) + 3$

NL

$\mathrm{sinc}(x) = \dfrac{\sin(x)}{x}$

$c_1 \dfrac{dy}{dt} + c_2 y(t) = x(t)$

$c_1 j\omega H(\omega) \cancel{e^{j\omega t}} + c_2 H(\omega) \cancel{e^{j\omega t}} = \cancel{e^{j\omega t}}$

$H(\omega) = \dfrac{1}{c_1 j\omega + c_2}$

$= |H(\omega)| e^{j\phi(\omega)}$

$x_1(t) = 4\cos(300t - \pi/6)$

$y_1(t) = 4|H(300)| \cos(300t - \frac{\pi}{6} + \phi(300))$

$x_2(t) = -8\cos(600t - \pi/4)$

$y_2(t) = -8|H(600)| \cos(600t - \frac{\pi}{4} + \phi(600))$

$y(t) = y_1(t) + y_2(t)$

HW 7 #7

$x(t) = \underset{n=1}{8\cos(300t)} - \underset{n=2}{3\sin(600t)}$

$\omega_0 = 300$

$a_0 = 0 \quad a_1 = 8 \quad b_1 = 0$

$a_2 = 0 \quad b_2 = -3$

$c_n \quad c_0 = 0$

$8\cos(300t) - 3\cos(600t - \frac{\pi}{2})$

$8\cos(300t) + 3\cos(600t - \frac{\pi}{2} - \pi)$

$c_1 = 8 \quad \phi_1 = 0 \quad c_2 = 3 \quad \phi_2 = -\frac{3\pi}{2}$

$8\cos(300t) + 3\cos(600t - \frac{\pi}{2} - \pi)$

$c_1 = 8 \quad \phi_1 = 0 \qquad c_2 = 3 \quad \phi_2 = -\frac{3\pi}{2}$

$\frac{3}{2} e^{+j\frac{3\pi}{2}} e^{-j600t}$) $n = -2$

$+ \frac{8}{2} e^{-j300t}$) $n = -1$

$+ \frac{8}{2} e^{+j300t}$) $n = +1$

$3/2\, e^{-j\frac{3\pi}{2}} e^{+j600t}$) $n = +2$

$X_{-2} = 3/2\, e^{+j3\pi/2} \qquad X_{-1} = 8/2$

$X_1 = 8/2 \qquad X_2 = \frac{3}{2} e^{-j\frac{3\pi}{2}}$

$X_0 = 0$

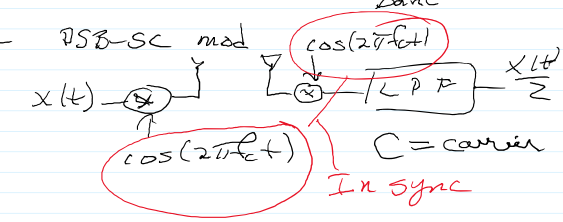
March 10, 2026

- Ideal filters $\quad H(\omega) = K e^{-j\omega t_0}$ over some Band

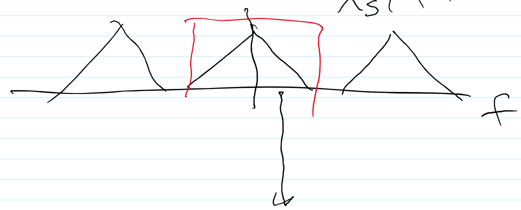
ILPF $\quad$ IHPF

IBPF $\quad$ IBRF

- DSB-SC mod

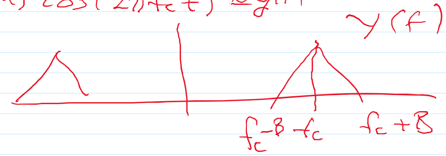


- DSB-LC AM

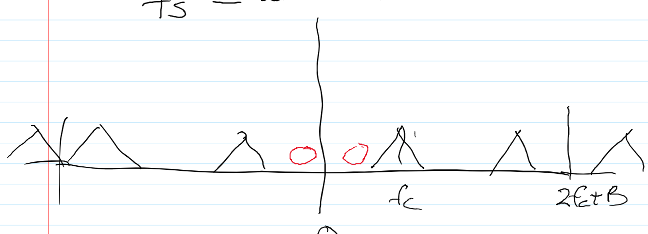


want Recover $X(f)$

$x(t)\cos(2\pi f_c t) = y(t)$



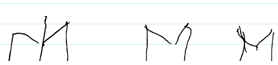
$f_s = 2(f_c + B)$



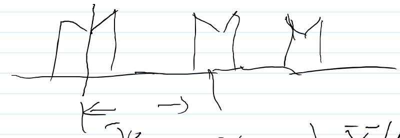
March 12, 2026

$x(t) = rect(t/2) - \frac{1}{2} tri(t) \quad \tau = 1$

$X(\omega) = 2\,sinc(\omega) - \frac{1}{2} sinc^2(\frac{\omega}{2})$



$\bar{X}(\omega) = 2\,\text{sinc}(\omega) - \frac{1}{2}\,\text{sinc}(\frac{\omega}{2})$



T_0

$x_n = \frac{1}{T_0}\bar{X}(n\omega_0)$

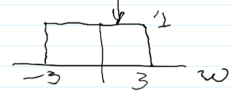
$\omega_0 = 2\pi/T_0$

HW 9 #7

$x(t) = u(t)e^{-.5t}$ → $y(t)$

$|X(\omega)|^2 = \frac{16}{\frac{3}{4}+\omega^2}$

$\frac{3}{\pi}\,\text{sinc}(3f)$



$Y(\omega) = \bar{X}(\omega)\,\text{rect}\left(\frac{\omega}{6}\right)$

$E_y = \int_{3}^{3} |\bar{X}(\omega)|^2\, d\omega = 14.31$

– FDM

– Frequency conversion

$x(t)\cos(2\pi(f_c+f_{LO})t) + x(t)\cos(2\pi(f_c-f_{LO})t)$

$x(t)\cos(2\pi f_c t)$ → ($\times$) → BPF $B = B_x$ → $x(t)$

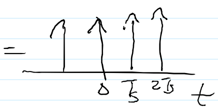
$\cos(2\pi f_{LO} t)$

$f_c - f_{LO}$

– Superhet

– Sampling theorem

$\delta_{T_s}(t) =$



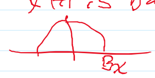
$x_s(t) = x(t)\,\delta_{T_s}(t)$

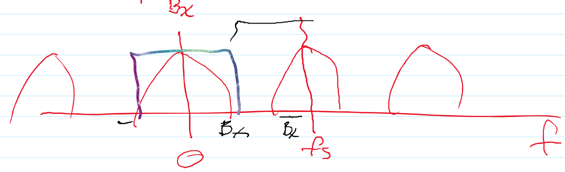
$X(f)$

$\mathcal{F}[\delta_{T_s}(t)] = f_s \sum_{n=-\infty}^{\infty} \delta(\omega - nf_s)$

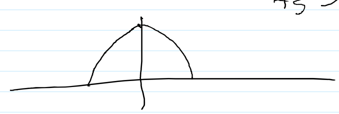
Assume $x(t)$ is band limited

$f_s = \frac{1}{T_s}$



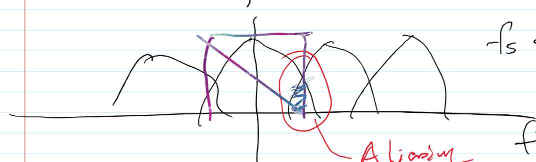


$f_s > 2B_x$



$2B_x$ = Nyquist rate

$f_s < 2B_x$



– $x(t)$ not band limited → Aliasing

① Combat aliasing

→ Anti Aliasing filter
ILPF
$B = f_s/2$ → Sample

② $f_s \uparrow$

$x_s(kT_s) \rightarrow x[k]$ discrete time

$x(t) = \cos(\omega_0 t)$
$= \cos(2\pi f_0 t)$

as $f_0 \uparrow$ $T_0 \downarrow$

$T_0 = 1/f_0$

March 24, 2026

$rect\left(\frac{t-0.05}{0.25}\right)$ $rect(t/\tau) \Longleftrightarrow \tau Sinc\left(\frac{\omega\tau}{2}\right)$

$x(t-t_0) \Longleftrightarrow X(\omega) e^{-j\omega t_0}$

$0.25\, sinc(0.125\omega) e^{-j0.05\omega}$

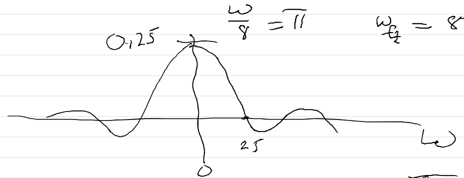
$x(t) = \underbrace{rect(t/0.25)}\ \underbrace{\cos(200t)}$

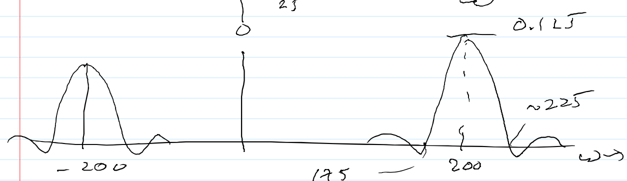
$\pi\delta(\omega+200) + \pi\delta(\omega-200)$

$0.125\,(sinc(0.125(\omega+200))$
$+ sinc(0.125(\omega-200)))$

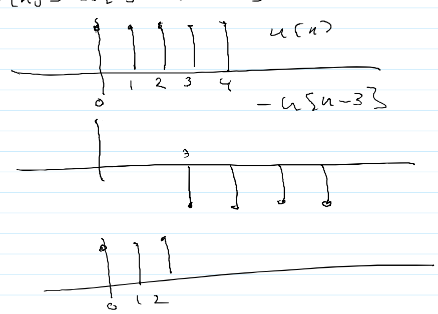
$0.25\,Sinc(0.125\omega)$ $0.125 = 1/8$

$\frac{\omega}{8} = \pi$ $\omega_{f} = 8\pi \approx 25$





- Sampling $f_s > 2B$
- $x(kT_s) = x[n] = \{3\ \underline{7}\ 9\ 0\ 1\}$
  $x[-1] = 3$ $x[0] = 7$ $x[1] = 9$
- $x[-n]$ $x[n-k]$ $x[\frac{n}{k}]$
- $\delta[n]$ $-u[n]$
- $x[n] = u[n] - u[n-3]$



- $u[n]\rho^n$
- $\cos(\omega_0 t + \theta)$ sample @ f_s $T_s = \frac{1}{f_s}$

$x[n] = \cos(\omega_0 n T_s + \theta)$
$= \cos(\Omega_0 n + \theta)$
$\underline{\underline{\Omega_0}} = \omega_0 T_s = \frac{2\pi f_0}{f_s}$ $\left(\frac{f_0}{f_s}\right)$

Period $N_0 = \frac{k T_0}{T_s}$ $T_0 = \frac{2\pi}{\omega}$

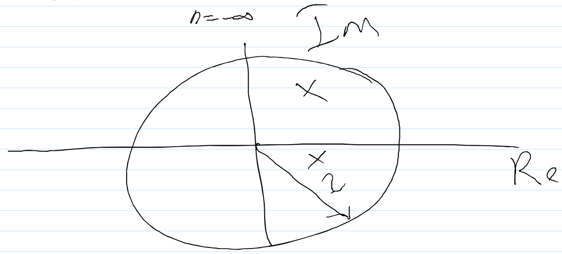
Find smallest k such that
N_0 is integer

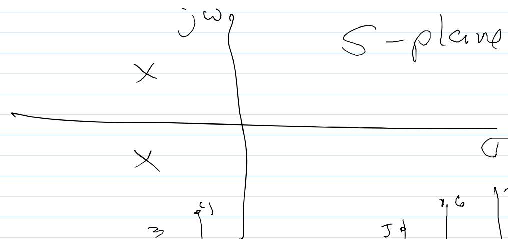
$$y[n] = \frac{1}{3}x[n] + \frac{1}{3}x[n-1] + \frac{1}{3}x[n-2]$$

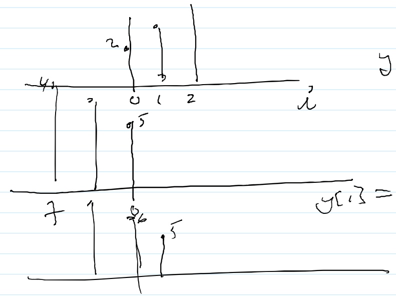
$$h[n] = \frac{1}{3}\delta[n] + \frac{1}{3}\delta[n-1] + \frac{1}{3}\delta[n-2]$$

Causal $h[n] = u[n]h[n]$

BIBO Stable $\sum_{n=-\infty}^{\infty} |h[n]| < \infty$







$y[0] = 10$

$y[1] = 6 \cdot 2 + 5 \cdot 3$

March 26, 2026

$$y(t) = 1 + \cos(.5t) + \cos(t)$$

$$H(\omega) = \text{rect}(\omega) + 4\,\text{rect}\left(\frac{\omega}{4}\right)$$

$$h(t) = \frac{1}{2\pi}\text{sinc}\left(\frac{t}{2}\right) + \frac{8}{\pi}\text{sinc}(2t)$$

HW 10 #9 $x(t) =$

$x(t) \rightarrow \boxed{\text{ILPF } B} \rightarrow y(t)$ $B > 5000$

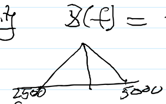
$H(\omega) = Ke^{j\omega t_0}$ over B_x

$5000\,\text{sinc}^2(5000\pi f)$

$\text{tri}(t/\tau) \longleftrightarrow \tau\,\text{sinc}^2\left(\frac{\omega\tau}{2}\right)$

Duality $X(f) = \text{tri}\left(\frac{f}{B}\right)$

$B = 5000$



$$\int_{-2500}^{2500} |X(f)|^2\, df$$

$$= 2\int_0^{2500} \left(1 - \frac{f}{5000}\right)^2 df$$

$x(t) = \cos(\omega_0 t)$ Sample at f_s

$= \alpha \int_0 (1 - \frac{\ }{5000}) \ldots$

$x(t) = \cos(\omega_0 t)$ Sample at f_s

$T_s = 1/f_s$

$x[n] = \cos(\Omega_0 n)$ $\Omega_0 = \frac{2\pi f_0}{f_s}$

$2\pi f_0 = \omega_0$ $\left(\frac{f_0}{f_s}\right)$

- DT LTI Systems
- Causal if $h[n] = h[n]u(n)$ then
- BIBO Stable $\sum_{n=-\infty}^{\infty} |h[n]| < \text{finite}$
- ARMA

$$y[n] = \sum_{i=0}^{M} b_i x[n-i] - \sum_{k=1}^{N} a_k y[n-k]$$

MA AR

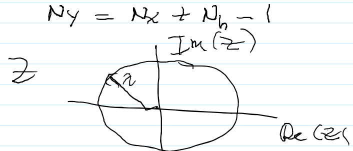
- Block diagram
- $\delta[n]$ —[ ]— $h[n]$

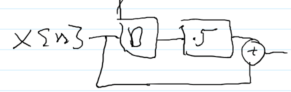
- MA $h[n] = \{b_0, b_1, \ldots b_M\}$
- Case $h[n] = \sum C_i p_i^n u[n]$

BIBO stable if all $|p_i| < 1$

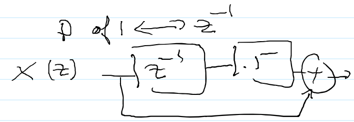
- D.T conv $y[n] = \sum_{i=-\infty}^{\infty} x[i] h[n-i]$
- N_x length of x N_h length of h

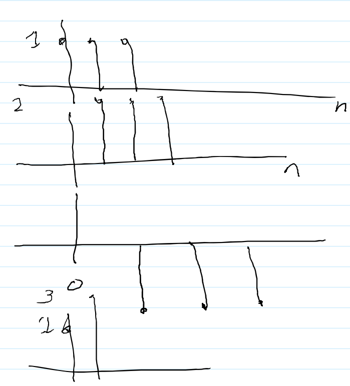
$N_y = N_x + N_h - 1$





D of $1 \longleftrightarrow z^{-1}$



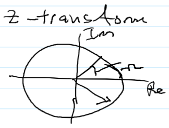


March 31, 2026

— DT conv

— z-transform $X(z) = \sum x[n] z^{-n}$

z complex



FT of sampled signal

$X(z)\big|_{e^{j\Omega}}$

$\delta[n] \leftrightarrow 1$ $\quad u[n] \leftrightarrow \frac{z}{1-z}$ $\quad a^n u[n] \leftrightarrow \frac{z}{z-a}$

Table

$\delta[n-k] \leftrightarrow z^{-k}$

$— \boxed{D} —$

z^{-1}

— Properties

+ Add + Associative $x*h = h*x$

$x_1+x_2 \leftrightarrow X_1(z) + X_2(z)$

$x[n-k] \leftrightarrow z^{-k} X(z)$

LTI $x[n]*h[n] \longleftrightarrow X(z)\, H(z)$

— Finite Impulse Response filter or MA (FIR)

$y[n] = \sum_{k=0}^{M} b_k x[n-k]$

$h[n] = \{b_0, b_1, b_2 \cdots b_M\}$

$H(z) = \sum_{k=0}^{M} b_k z^{-k} = b_0 + b_1 z^{-1} + b_2 z^{-2}$

$= \frac{b_0 z^M + b_1 z^{M-1} \cdots b_M}{z^M}$

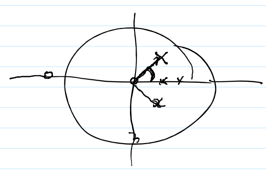
— Example $x[n] = \{\underline{0}, 1, 1\}$ $\quad h[n] = \{\underline{0}, 0, 2, 1\}$

$y[n]$ $\quad X(z) = z^{-1} + z^{-2}$ $\quad H(z) = 2z^{-2} + z^{-3}$

$Y(z) = 2z^{-3} + z^{-4} + 2z^{-4} + z^{-5}$

$2z^{-3} + 3z^{-4} + z^{-5}$

Inverse Z-T $Y(z)$ $\quad y[n] = \{\underline{0}, 0, 0, 2, 3, 1\}$



April 2, 2026

δ —[ ]— $h[n]$

$h[n] = \cancel{x}\,\delta(n+2)$

$+ 2\delta(n+1)$

$+ 3\delta[n]$

$+ 2\delta[n-1]$

$+ 1\delta[n-2]$

$\sum |h[n]| = 9 < \infty$

Stable

$h[n] \neq h[n]u[n]$ $\quad$ non causal

$y(n+2)$ $\quad n=0$ $\quad x[2]$

— Inverse Z-transform

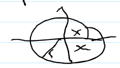
+ match terms + polynomial divide + from table + PFE

— $H(z) = \frac{Y(z)}{X(z)} = \frac{N(z)}{D(z)}$ $\quad N(z) \rightarrow$ zeros, $D(z) \rightarrow$ poles

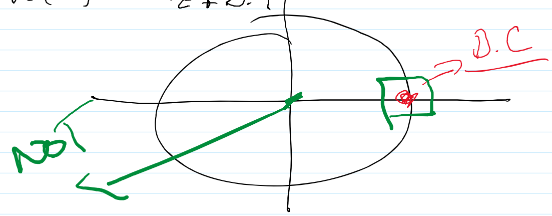
— BIBO stable poles are inside unit circle

$X(z)$ $D(z) \longrightarrow$ pole

- BIBO Stable poles are inside unit circle



$$H(z) = \frac{z}{z+0.9}$$ $h[n] = (-.9)^n u[n]$



April 7, 2026

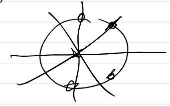
$$\frac{z^2}{z^2+0.81} = \frac{1}{1+0.81z^{-2}}$$ $\frac{Y}{X}$

$X(z) = Y(z) + 0.81z^{-2}Y(z)$

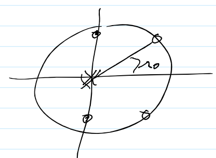
$Y(z) = X(z) - 0.81z^{-2}Y(z)$

$y[n] = x[n] - 0.81y(n-2]$

BRF



$$\frac{z^2+0.81}{z^2}$$



HW13 11c

$y[n] = x[n] + 1.66y[n-1] - 0.81y[n-2]$

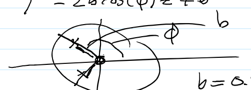
$Y(z) = X(z) + 1.66z^{-1}Y(z) - 0.81z^{-2}Y(z)$

$$H(z) = \frac{1}{1-1.66z^{-1}+0.81z^{-2}}$$

$$= \frac{z^2}{z^2-1.66z+0.81}$$

12. $y[n] = x[n] + 2b\cos(\phi)y[n-1] - b^2$

$$\frac{z^2}{1-2b\cos(\phi)z+b^2}$$



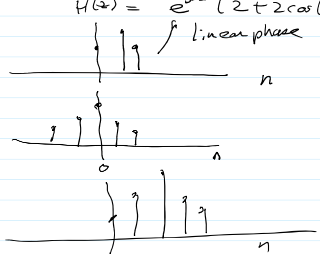
$b = 0.9$
$\phi = \pi/8$

#13 $h[n] = \{1\ \underline{2}\ 1\} = \delta(n+1) + 2\delta[n] + \delta[n-1]$

$z^{-1} + 2 + z^{-1}$

$z = e^{j\Omega}$ $H(z) = e^{j\Omega} + 2 + e^{-j\Omega}$
$2 + 2\cos(\Omega)$

$h[n] = \{\underline{1}\ 2\ 1\}$
$H(z) = e^{j\Omega}(2+2\cos(\Omega))$ linear phase



- $H(z)$ Freq responce $H(z)\Big|_{e^{j\Omega}}$

- $x[n] = A\cos(n\Omega_0+\theta) \rightarrow \boxed{LTI}$

$y[n] = A|H(e^{j\Omega_0})|\cos(n\Omega_0+\theta+\angle H(e^{j\Omega_0}))$

$x(t) = \cos(2\pi f_0 t)$ sample(T) f_s

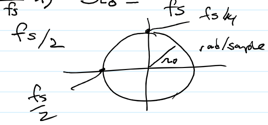
- $x[n] = A\cos(n\Omega_0 + \theta) \rightarrow \boxed{LTI}$

$y[n] = A|H(e^{j\Omega_0})|\cos(n\Omega_0 + \theta + \angle H(e^{j\Omega_0}))$

- $x(t)\cos(2\pi f_0 t)$ sampled @ f_s

$x[n] = \cos\left(\frac{2\pi f_0}{f_s} n\right) \quad \Omega_0 = \frac{2\pi f_0}{f_s}$

Highest freq $= f_s/2$

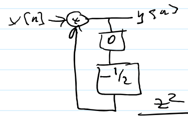


- $|H(e^{j\Omega})|$ even $\quad \angle H(e^{j\Omega})$ odd

- Representations
  + D.E.eq + Block diagram + $H(z)$ + $H(e^{j\Omega})$
  + $h[n]$

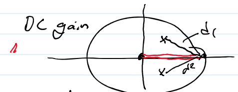
Ex: $H(e^{j\Omega}) = \frac{e^{j\Omega}}{e^{j\Omega} - 1/2} \qquad H(z) = \frac{z}{z - 1/2}$

$h[n] = (\frac{1}{2})^n u[n] \qquad y[n] = x[n] + \frac{1}{2}y[n-1]$



$H(z) = \frac{z^2}{(z+.5e^{j\pi/4})(z+.5e^{-j\pi/4})}$

DC gain



$|H(e^{j0})| = \frac{(1)(1)}{d_1 d_2}$

April 9, 2026

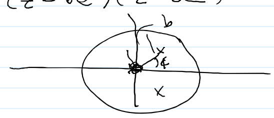
April 14, 2026

HW14 #4 $\quad y[n] = x[n] + 2b\cos(\phi)y[n-1] - b^2 y[n-2]$

$\frac{z^2}{z^2 - 2b\cos(\phi)z + b^2}$ $\qquad$ 2 zeros @ 0

$= \frac{z^2}{(z - be^{j\phi})(z - be^{-j\phi})}$ $\qquad$ poles are at $be^{\pm j\phi}$

BIBO $b < 1$



HW 14 #6 $\quad y[n] = x[n] + 2x[n-1] + x[n-2]$

$H(z) = \frac{z^2 + 2z + 1}{z^2}$

$x(t) = 2\cos(2\pi 750 t) \quad f_s = 4000 \rightarrow x[n]$

$y[n] \qquad \Omega_1 = \frac{2\pi 750}{4000} = 0.375\pi = 1.17$

$|H(e^{j\Omega_1})| = \left| \frac{e^{j2(1.17)} + 2e^{j1.17} + 1}{e^{j(2\cdot 1.17)}} \right| = 2.76$

$\angle H(e^{j\Omega_1}) = -1.17$

$y[n] = (2 \cdot 2.76)\cos(n\,1.17 - 1.17)$

- $H(z) \quad H(e^{j\Omega})$

- LP, HP, BP, BR, notch, comb

April 16, 2026

$y[n] = x[n] + 2x[n-1] + x[n-2]$

$Y(z) = X(z) + 2z^{-1}X(z) + z^{-2}$

$\frac{Y}{X} = 1 + 2z^{-1} + z^{-2} = \frac{z^2 + 2z + 1}{z^2}$

$f_1 = 1000 \quad f_s = 2000$

$\Omega_1 = \frac{2\pi f_1}{f_s} = \pi$

$H(e^{j\pi}) = e^{j2\pi} + 2e^{j\pi} + 1$

$f_1 = 1000 \quad f_s = 2000$

$\Omega_1 = \frac{2\pi f_1}{f_s} = \pi$

$z = e^{j\pi} \qquad H(e^{j\pi}) = \frac{e^{j2\pi} + 3e^{j\pi} + 1}{e^{j2\pi}}$

$e^{j\pi} = -1 \quad e^{j2\pi} = \qquad = \frac{1 + (-3) + 1}{1} = -1$

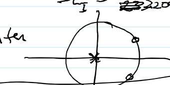
$H(e^{j0}) = \frac{e^{j0} + 3e^{0} + 1}{e^{j0}} = 5$

$y[n] = \frac{1}{5}x[n] + \frac{3}{5}x[n-1] + \frac{1}{5}x[n-2]$

HW 14 #13 $y(t) = x(t) + \cos(2\pi 2000 t)$ ← Interfere

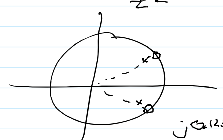
$f_s = 32000 \qquad \Omega_I = \frac{2\pi\, 2000}{32000} = 0.125\pi$

notch filter



$H(z) = \frac{1 - 1.8z + z^2}{z^2}$

$\frac{(z - e^{j(0.125\pi)})(z - e^{-j(0.125\pi)})}{z^2}$



$H(z) = \frac{(z - e^{j0.125\pi})(z - e^{-j0.125\pi})}{(z - 0.92e^{j0.125\pi})(z - 0.92e^{-j0.125\pi})}$

DFT (FFT) FS of a period DT Signal

$x[0], \ldots, x[N_0-1]$ — $\boxed{\text{DFT}}$ — $\{X[0], \ldots, X[N_0-1]\}$ complex

Parameters $\Delta f = \frac{1}{L}$ L (sec)

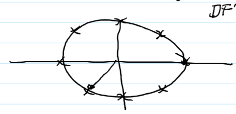
$f_s \quad \Delta t = 1/f_s$

$L\uparrow \quad \Delta f \downarrow$

Picket Fence   Discrete frequency Δf

Spectral Leakage

- $x[n] \longleftrightarrow X(z)$



$DFT\{x[n]\} = X(z)\Big|_{e^{j\frac{2\pi k}{N_0}}}$

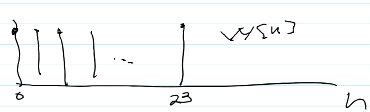
Properties — Linear — Time Shift — freq shift — Energy

$y[n] = x[n] * h[n] \qquad N_x = \text{length of } x[n]$

$N_h = \text{〃 〃 } h[n]$

$N_y = N_x + N_h - 1$

$DFT^{-1}\left\{\left[DFT\{x\} \cdot DFT\{h\}\right]\right\} = y[n]$



$W(z)$