EECS 562

Homework 1

1. (Concept: complex numbers and functions)
Let

a. $z_1 = 1+j3$, Find $\mathrm{Re}(z_1)$, $\mathrm{Im}(z_1)$, $|z_1|$, $|z_1|^2$ and α and β in $z_1=\alpha e^{j\beta}$

b. Repeat a. for $z_1 = -1 - j3$

c. Find $x(t) = \mathrm{Re}[z_1 e^{-j2\pi f_c t}]$ where $f_c = 200$ Hz

2. (Concept: mathematical model for quadrature phase shift keying - QPSK modulation; for each symbol time sending one of 4 signals to send 2 bits and signal constellations, aka, I-Q diagrams)
Let $z_1 = 0.84+j1.81$, $z_2 = -1.81+j0.84$, $z_3=-0.84-j1.81$, $z_4 = 1.81-j0.84\}$

a. Plot z_i for i=1..4 putting the real part of z_i on the x-axis and the imagery part of z_i on the y-axis.

b. For $f_c = 10$ MHz find $x_i(t) = \mathrm{Re}[z_i e^{-j2\pi f_c t}]$ for i=1..4

3. (Concept: Equivalence between time delay and phase shift)
Let $x_1(t) = 10\cos(2\pi 1000(t-62.5\mu s))$ and $x_2(t) = 10\cos(2\pi 1000t-\frac{\pi}{8})$, $x_1(t) = x_2(t)$ TRUE or FALSE

4. (Concepts: Properties of the sinc function, bandwidth, and inverse time duration-bandwidth relationship)
In this problem $\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$.
For

$x_1(t) = 10\,\mathrm{sinc}(10\,t)$,
$x_2(t) = 20\,\mathrm{sinc}(20\,t)$
$x_3(t) = 50\,\mathrm{sinc}(50\,t)$
With

a. Find the Fourier transform of $x_1(t)$, i.e. find $X_1(f)$ with f in Hz.

b. Plot $x_1(t)$, $x_2(t)$, and $x_3(t)$

c. Rank order the signals from lowest bandwidth to highest bandwidth.

5. (Concept: Signal power and energy)
Find the power and energy in $2\cos(2\pi f_c t)(1+.5\cos(2\pi f_m t))$, where $f_c = 1000$ and $f_m = 100$

6. (Concepts: Energy per bit and Binary Phase Shift Keying-BPSK)
A sequence of bits, 0's and 1's, are transmitted by sending
$x(t) = A\cos(2\pi f_c t)$ if bit = "1" for T_b=bit time
or
$x(t) = -A\cos(2\pi f_c t) = A\cos(2\pi f_c t-\pi)$ if bit = "0" for T_b
For $A = 1\times10^{-4}$ and $T_b=10\mu s$ and $f_c = 1$ GHz

a. Find the energy in x(t) if bit = "1" and compare to in x(t) if bit = "0" .

b. What is the bit rate in Mb/s?

7. (Concept: The orthogonality property of signals)
Let $x_1(t) = \cos(2\pi f_1 t)\,\mathrm{rect}\left(\frac{t-0.005}{0.01}\right)$ and $x_2(t) = \cos(2\pi f_2 t)\,\mathrm{rect}\left(\frac{t-0.005}{0.01}\right)$
For $f_1=2000$ and $f_2=2100$

a. Find $\int_{-\infty}^{\infty}\cos(2\pi f_1 t)\sin(2\pi f_1 t)\,dt$

b. Find $\int_{-\infty}^{\infty} x_1(t)\, x_2(t)\, dt$

c. $x_1(t)$ and $x_2(t)$ are orthogonal. True or False

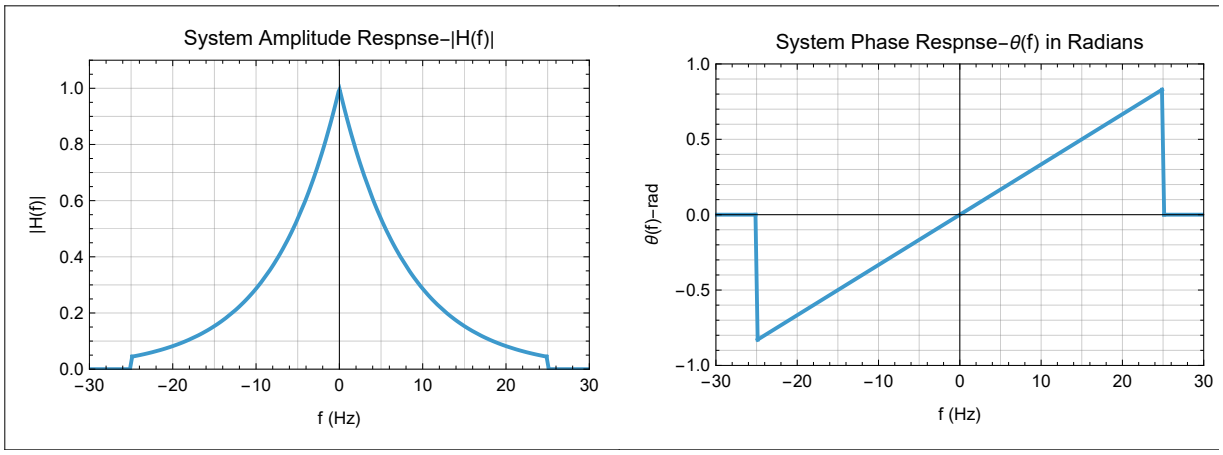
d. Find $\int_{0}^{0.01} x_1(t)\, x_2(t)\, dt$

8. (Concept: double-sided phase and magnitude spectrum)
For x(t) = 2cos(2π2000t) + 4sin(2π4000t)

a. Plot the double-sided phase and magnitude spectrum for x(t).

b. What is the bandwidth of x(t)?

9. (Concept: Sinusoidal response of LTI system and bandwidth)
A system with $H(f)= |H(f)|\, e^{-j\theta(f)}$
where amplitude $|H(f)|= \text{rect}\left(\frac{f}{80}\right) e^{-\left|\frac{f}{8}\right|}$ and phase $\theta(f)=-\frac{f}{30}$ response is given below.



a. The signal x(t)=cos(2π2.8t)+cos(2π5.5t) is the input to the system with the frequency response H(f) given above. Find the system output, y(t).

b. What is the 3dB bandwidth of H(f)?

c. What is the 6dB bandwidth of H(f)

b. A signal g(t) with a bandwidth of B_g is the input to a system with H(f) with a bandwidth of B_{sys}, the system output is y(t). If B_{sys}>>B_g then y(t)≈g(t). TRUE or FALSE.

10. (Concept: Impulse response of LTI system)
A signal x(t)=δ(t-1.0) is the input to a linear time invariant system with an impulse response of h(t) = $10\, e^{-5|t|}$. Find and plot the system output y(t).

11. (Concepts: Bandwidth of ILPF and zero crossing properties of impulse response of ILPF)
A filter has an impulse response of h(t) = 200sinc(200t)

a. Is this an ILPF? Yes or NO

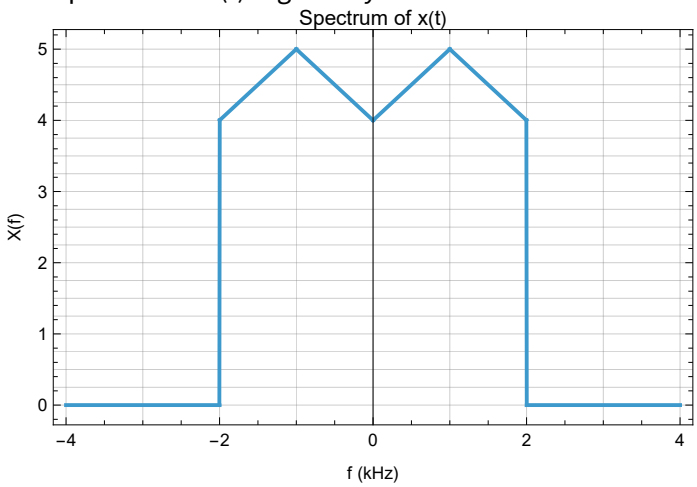
b. What the filter bandwidth?

c. With an input signal x(t)=5δ(t) + δ(t- τ) where τ=5ms input to this filter, plot the output signal y(t) in the time domain.

d. What is y(0) and y(τ)?

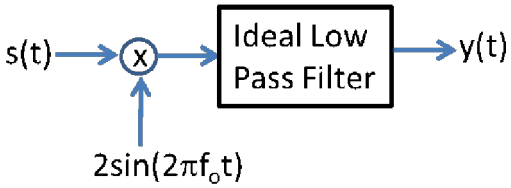
e. Define $y_1(t) = 5\delta(t) * h(t)$ and $y_2(t) = \delta(t-\tau) * h(t)$ what is $y_1(0)$ and $y_2(\tau)$? Compare $y_1(0)$ and $y_2(\tau)$ to y(0) and y(τ)

12. (Concept: Sampling)
The spectrum of x(t) is given by:



a. The signal x(t) is sampled at f_s = 5000 samples/sec to form $x_s(t)$. Plot the spectrum of $x_s(t)$.
b. For x(t) given above, what is the minimum sample rate required to prevent aliasing?
c. With f_s = 5000 samples/sec, describe how x(t) is recovered from $x_s(t)$.

13. (Concept: Quadrature modulation)
Let $s(t) = x(t)\cos(2\pi f_0 t) + z(t)\sin(2\pi f_0 t)$ with $f_0 >> B_x$ and $f_0 >> B_z$



a. Find the output y(t) in terms of x(t) and z(t) of the system above.
b. Change the $2\sin(2\pi f_0 t)$ to $2\cos(2\pi f_0 t)$ and find the output y(t) in terms of x(t) and z(t)
[Hint: use the trigonometry identities for $\sin^2(\theta)$ and $\cos^2(\theta)$]

14. (Concept: DFT and Orthogonal Frequency Division Multiplexing - OFDM)
Mobile cellular systems systems use the DFT. Some system configurations use a DFT of length 2048 with a sample frequency of f_s = 30.72Msamples/s.

a. What is the record length in seconds?
(This is the symbol time.)
b. What is the spacing in kHz between frequency components of this DFT?
(This is the carrier spacing in OFDM.)

15. (Concept: Equivalence between an integrator and a filter)

a. Find the transfer function for a filter with an impulse response $h_R(t)=\frac{1}{T}\text{rect}\left(\frac{t-T/2}{T}\right)$.

b. Show that the impulse response $h_R(t)$ for a system defined as

$$y(t) = \frac{1}{T}\int_{t-T}^{t} x(\lambda)\, d\lambda \text{ is } \frac{1}{T}\text{rect}\left(\frac{t-T/2}{T}\right).$$

16. (Concepts: dB_W and dB_m)

a. Convert 10 Watts to dB_W

b. Convert 10 Watts to dB_m

c. Convert 30 dB_m to watts

d. The path loss between a transmitter and receiver is 30 dB.
The transmitter power is 33 dB_W, what is the receiver power in dB_W and watts.