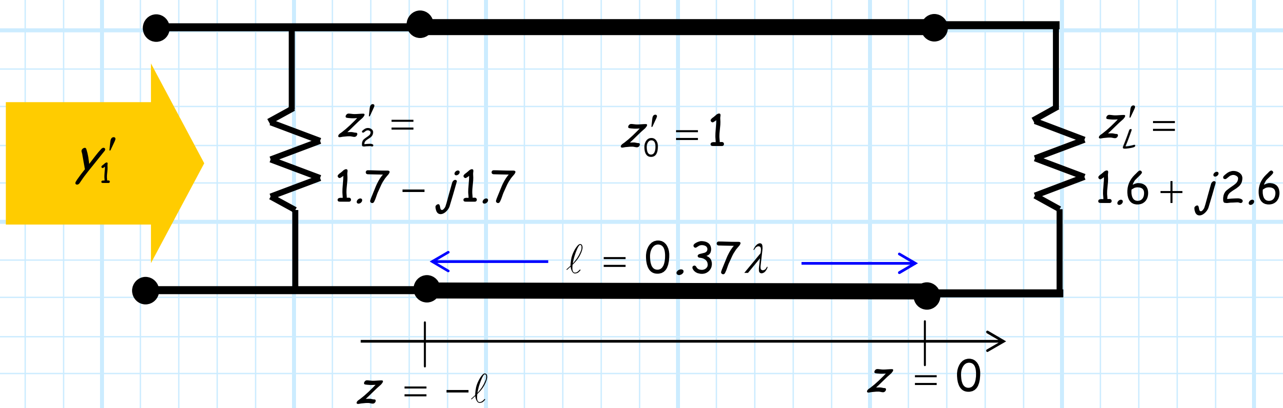
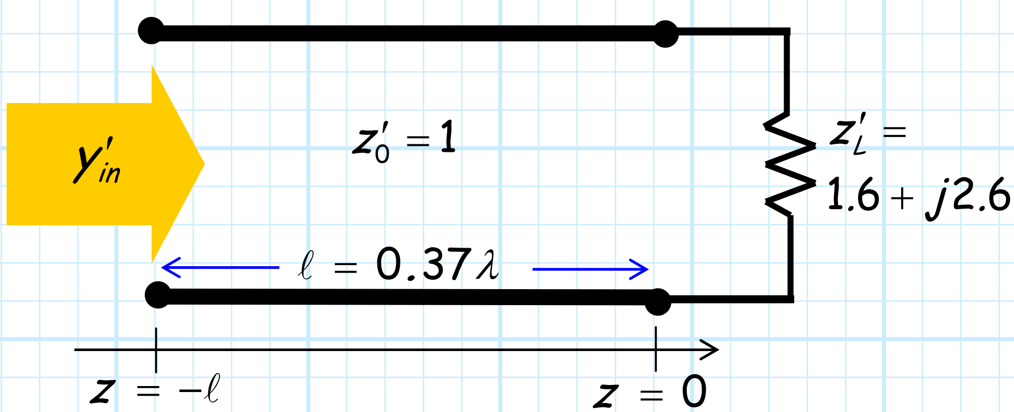
Example: Admittance Calculations with the Smith Chart

Say we wish to determine the **normalized admittance** y_1' of the network below:



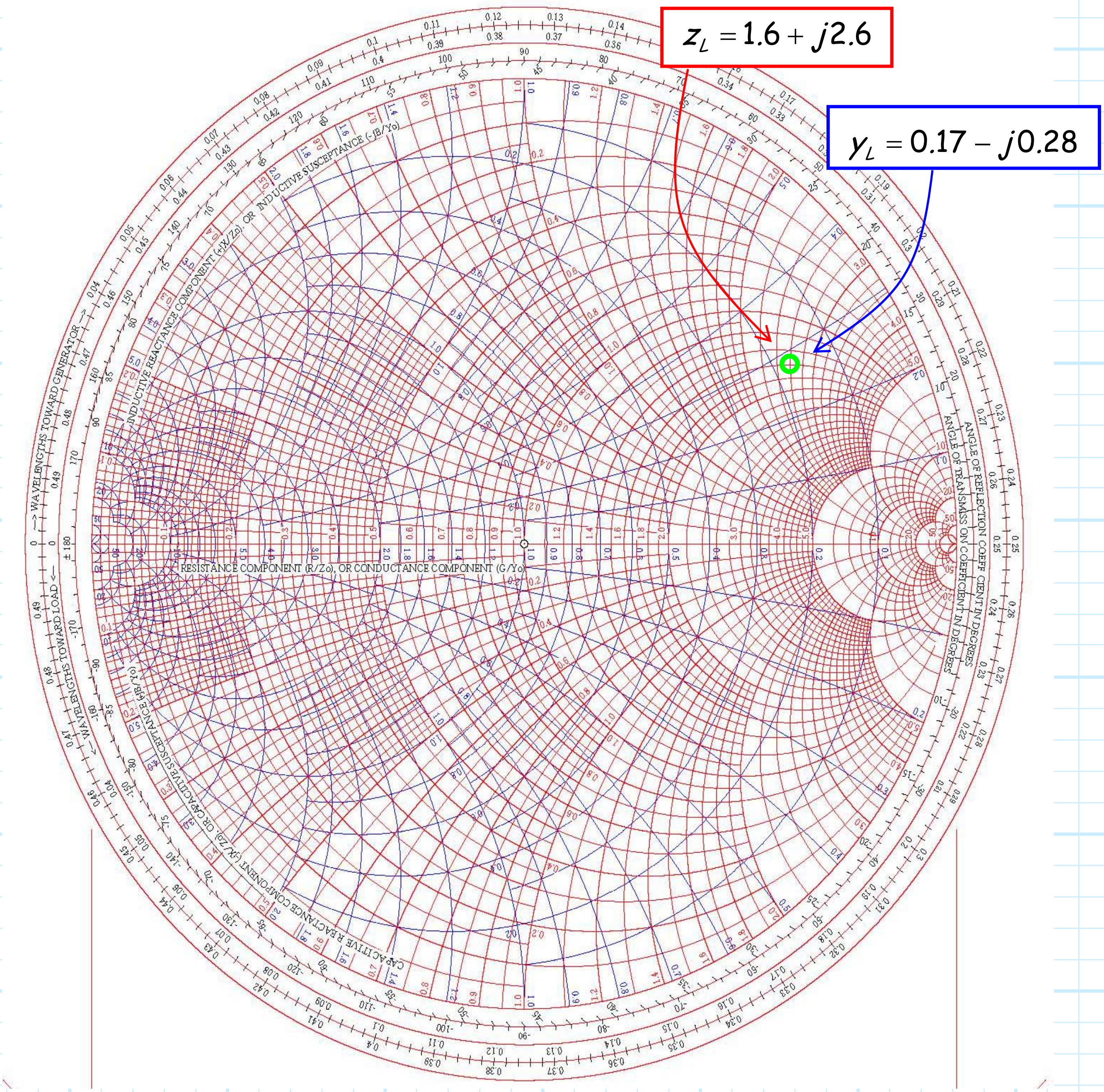
First, we need to determine the normalized **input** admittance of the transmission line:



There are **two ways** to determine this value!

Method 1

First, we express the load $z_L = 1.6 + j2.6$ in terms of its **admittance** $y'_L = 1/z_L$. We can calculate this complex value—or we can use a **Smith Chart**!

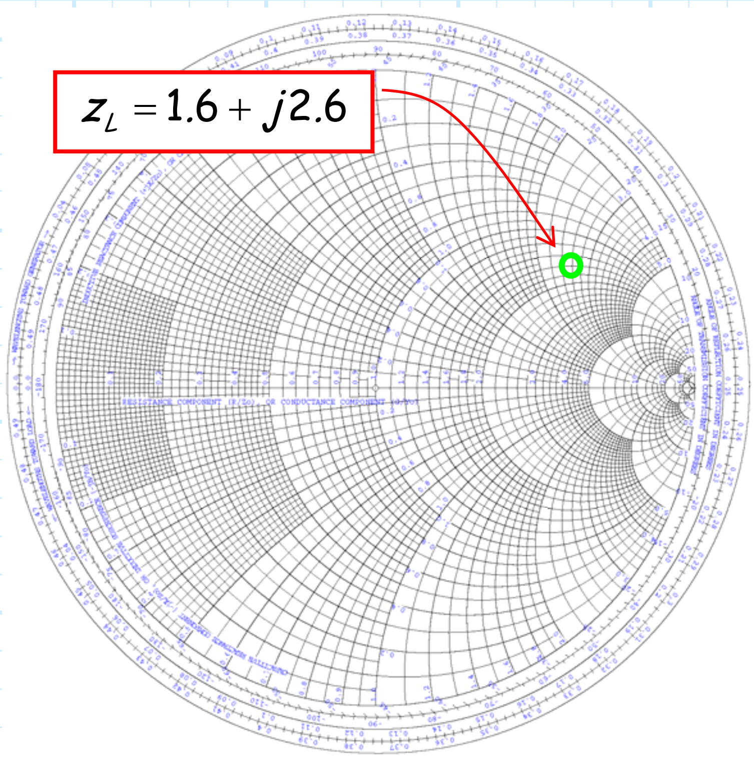


The Smith Chart above shows **both** the **impedance** mapping (red) and **admittance** mapping (blue). Thus, we can locate the impedance $z_L = 1.6 + j2.6$ on the impedance (red) mapping, and then determine the value of that **same** Γ_L point using the admittance (blue) mapping.

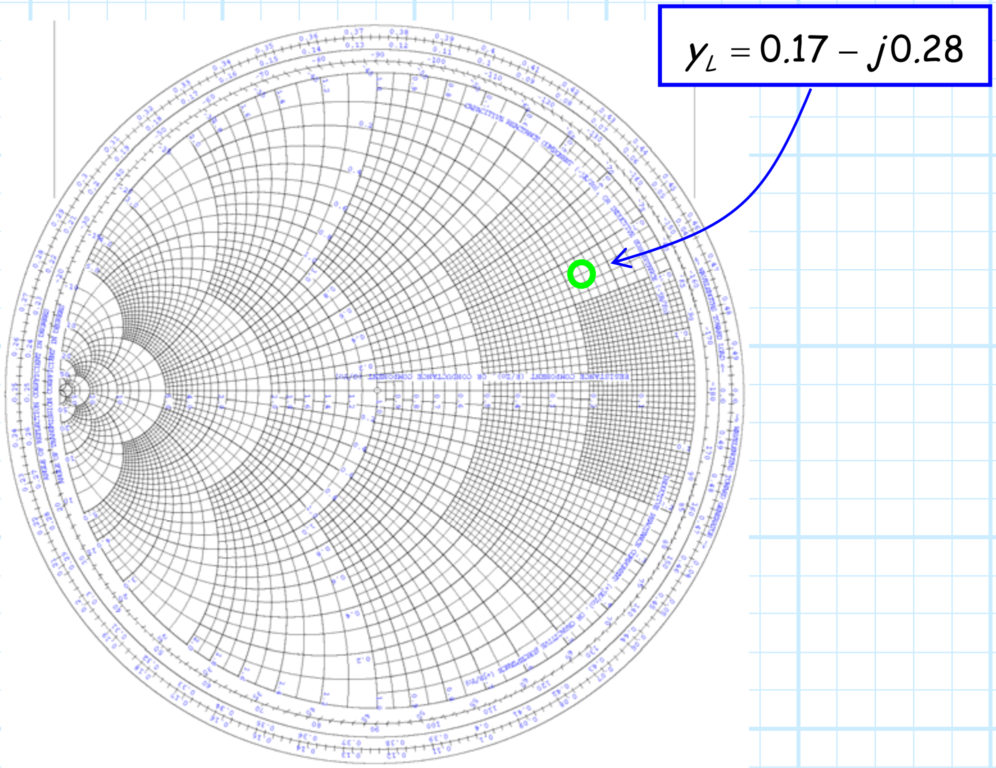
From the chart above, we find this admittance value is **approximately** $y_L = 0.17 - j0.28$.

Now, you may have noticed that the Smith Chart above, with both impedance and admittance mappings, is very **busy** and **complicated.** Unless the two mappings are printed in different colors, this Smith Chart can be very **confusing** to use!

But remember, the two mappings are precisely identical—they're just **rotated** 180° with respect to each other. Thus, we can **alternatively** determine y_L by again first locating $z_L = 1.6 + j2.6$ on the impedance mapping :



Then, we can rotate the **entire** Smith Chart 180°--while keeping the point Γ_L location on the complex Γ plane **fixed**.

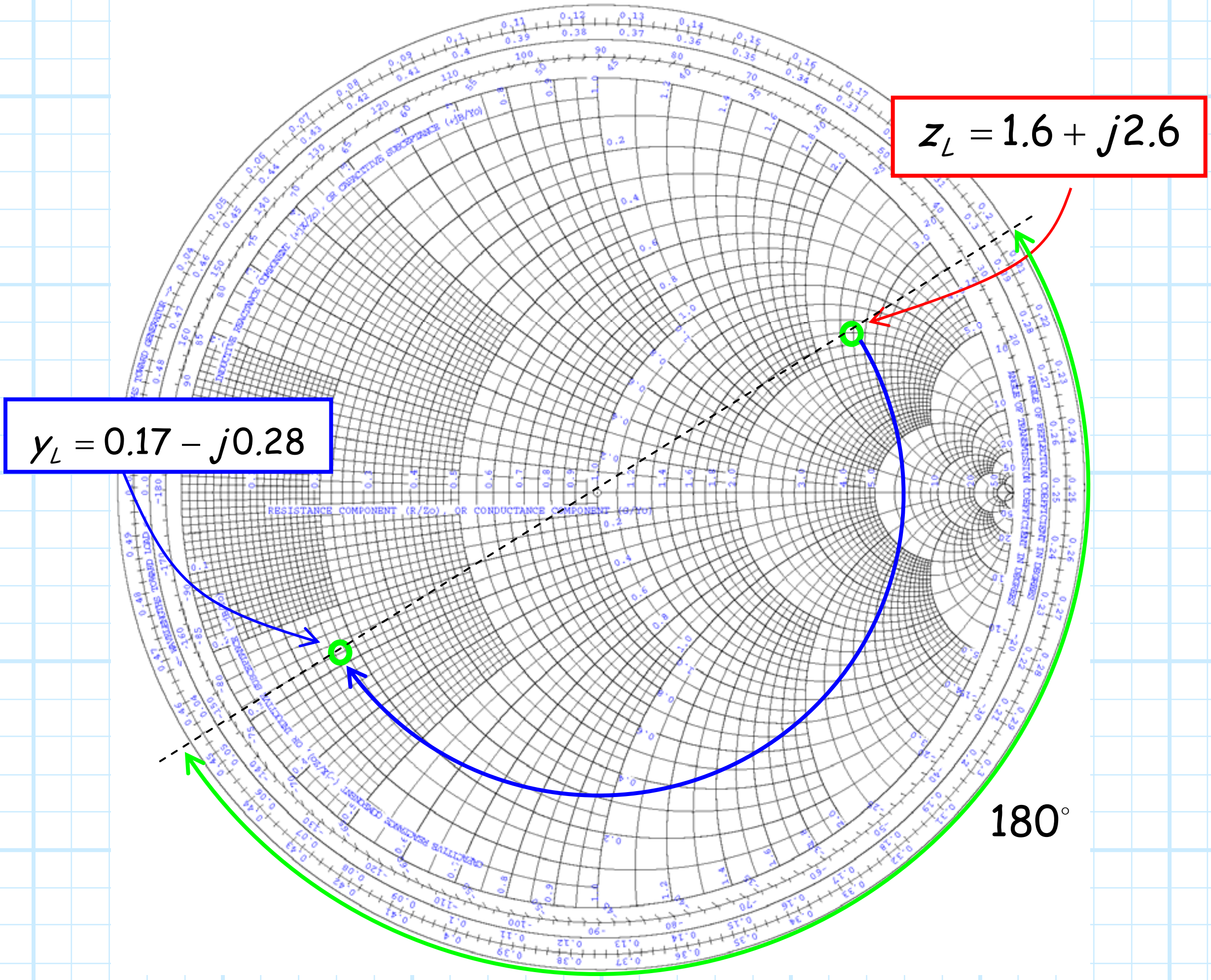


Thus, use the **admittance** mapping at that point to determine the admittance value of Γ_L.

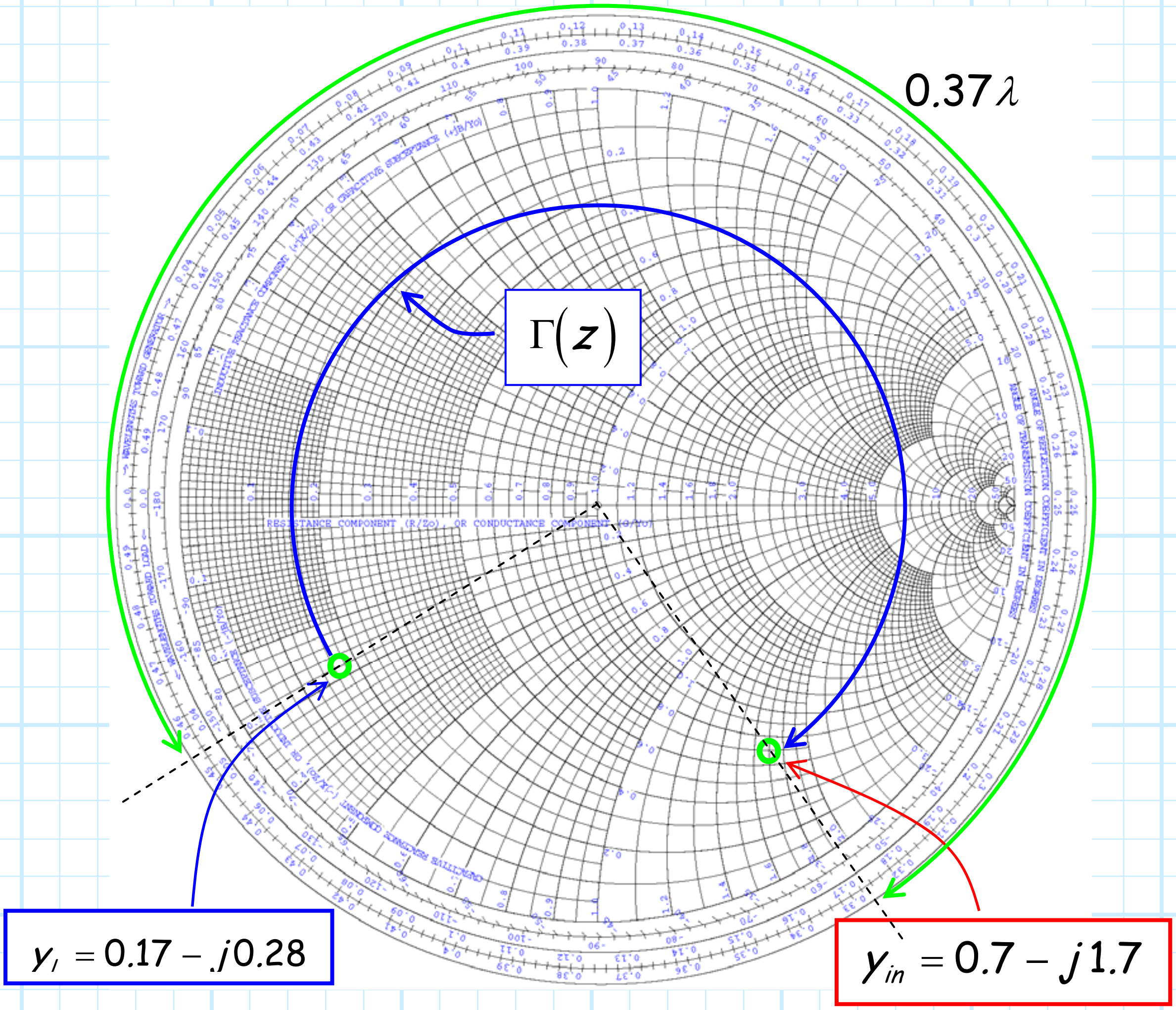
Note that rotating the **entire** Smith Chart, while keeping the point Γ_L fixed on the complex Γ plane, is a **difficult** maneuver to successfully—as well as accurately—execute.

But, realize that rotating the entire Smith Chart 180° with respect to point Γ_L is **equivalent** to rotating 180° the **point** Γ_L with respect to the entire Smith Chart!

This maneuver (rotating the **point** Γ_L) is **much** simpler, and the **typical** method for determining admittance.



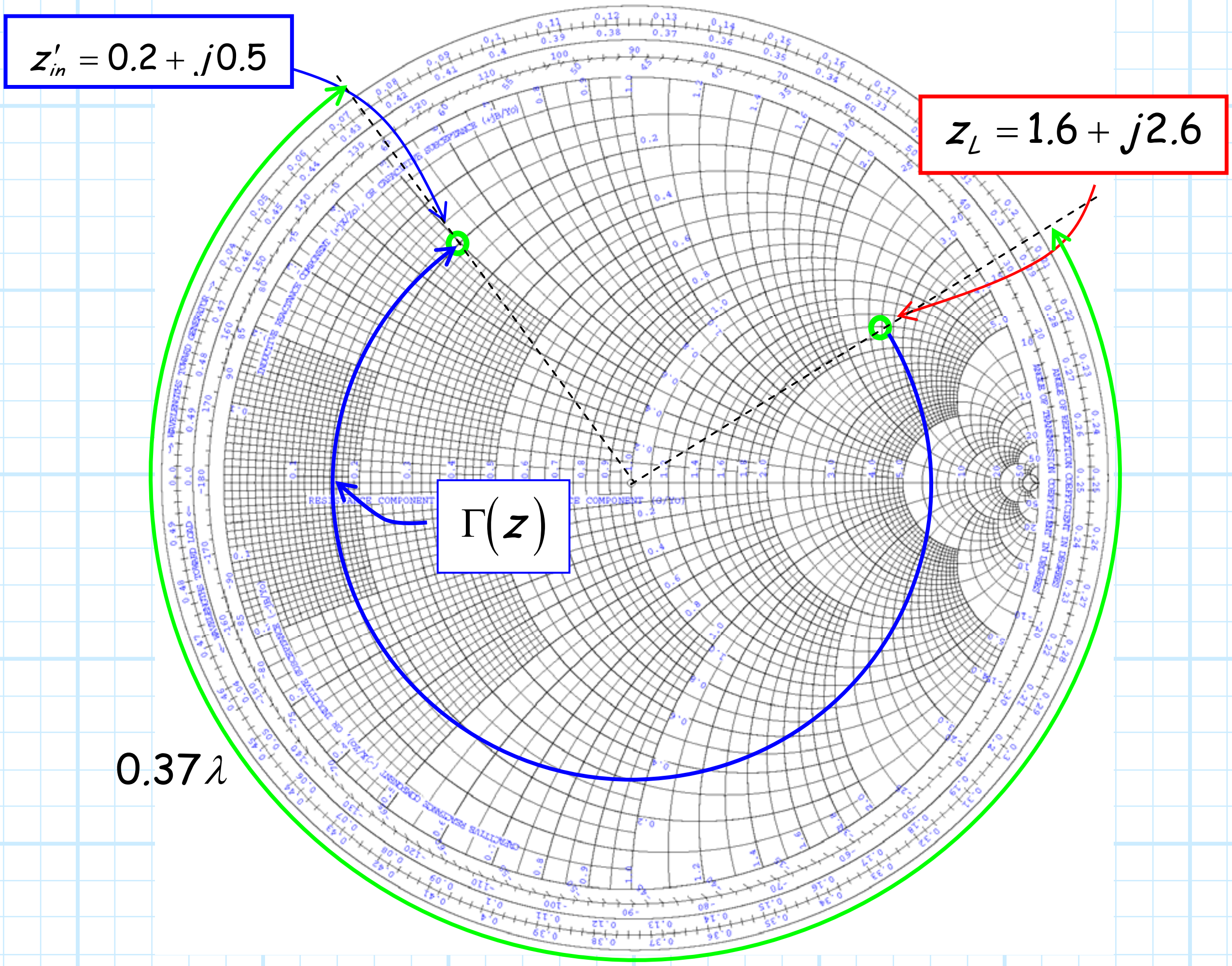
Now, we can determine the value of y'_{in} by simply **rotating** clockwise $2\beta\ell$ from y'_L, where $\ell = 0.37\lambda$:



Transforming the load admittance to the beginning of the transmission line, we have determined that $y'_{in} = 0.7 - j1.7$.

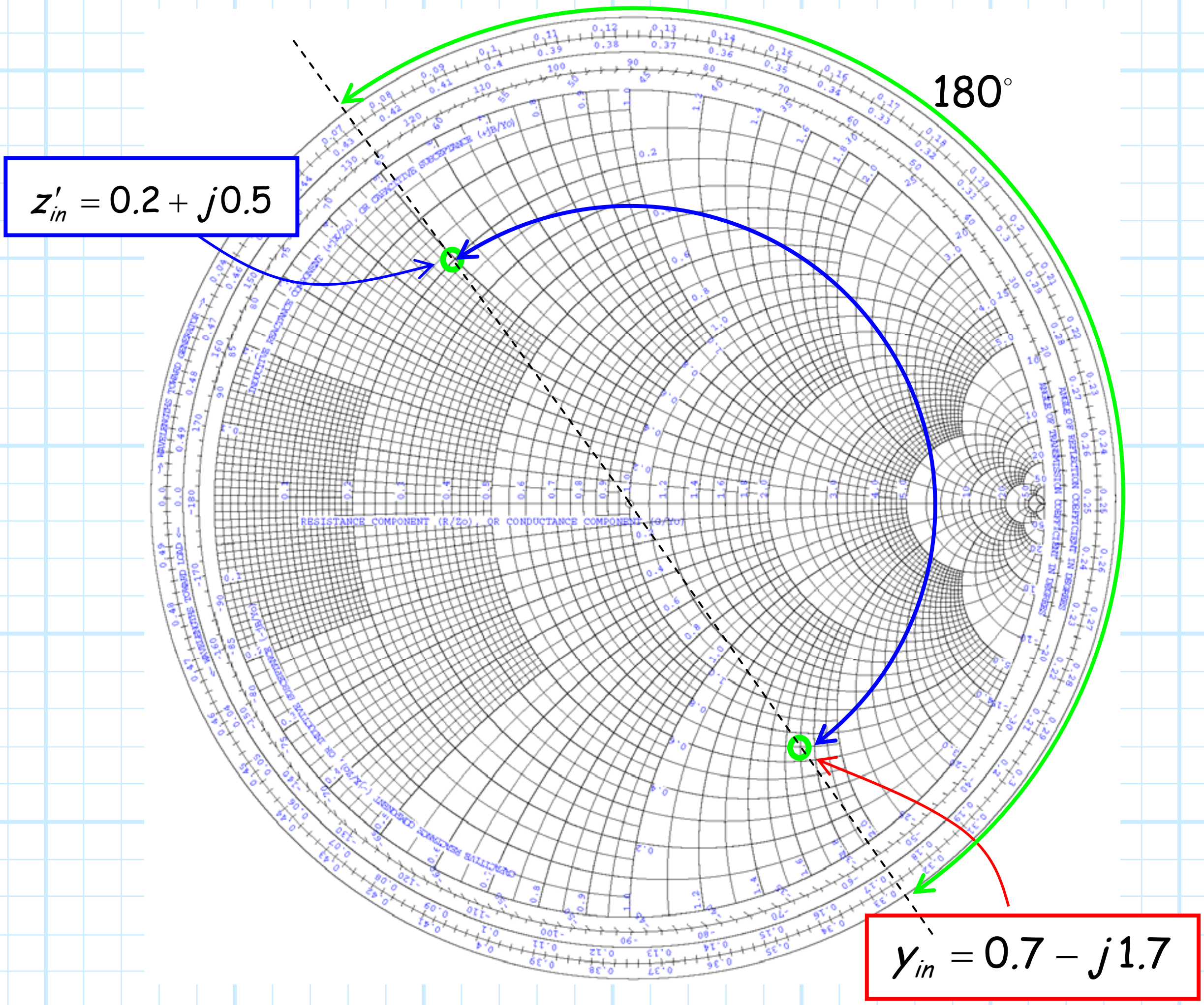
Method 2

Alternatively, we could have **first** transformed impedance z'_L to the **end** of the line (finding z'_{in}), and then determined the value of y'_{in} from the **admittance** mapping (i.e., rotate 180° around the Smith Chart).



The **input impedance** is determined after rotating clockwise $2\beta\ell$, and is $z'_{in} = 0.2 + j0.5$.

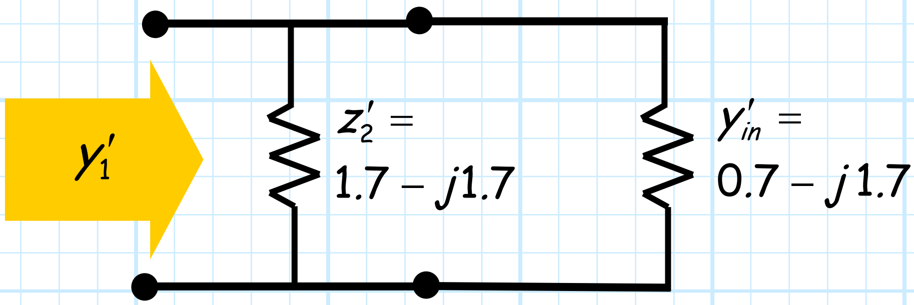
Now, we can rotate this point 180° to determine the **input admittance** value y'_{in}:



The result is the **same** as with the earlier method-- $y'_{in} = 0.7 - j1.7$.

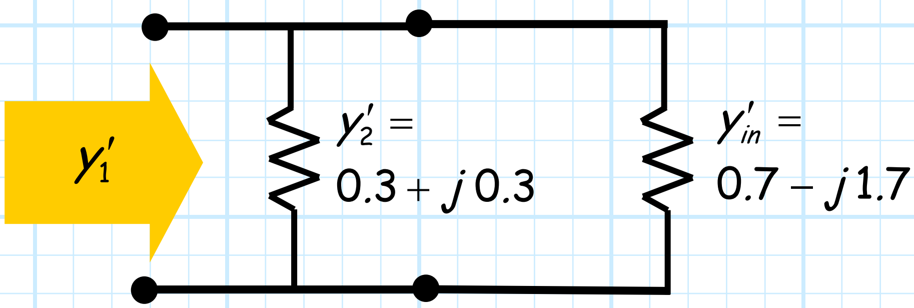
Hopefully it is **evident** that the two methods are equivalent. In method **1** we **first** rotate 180°, and **then** rotate $2\beta\ell$. In the second method we **first** rotate $2\beta\ell$, and **then** rotate 180°--the result is thus the **same**!

Now, the remaining **equivalent** circuit is:



Determining y_1' is just **basic circuit theory**. We first express z_2' in terms of its admittance $y_2' = 1/z_2'$.

Note that we could do this using a **calculator**, but could likewise use a **Smith Chart** (locate z_2' and then rotate 180°) to accomplish this calculation! Either way, we find that $y_2' = 0.3 + j\,0.3$.



Thus, y_1' is simply:

$$\begin{aligned} y_1' &= y_2' + y_{in}' \\ &= (0.3 + j\,0.3) + (0.7 - j\,1.7) \\ &= 1.0 - j\,1.4 \end{aligned}$$