# 11.2 Stability

**Reading Assignment:** ***pp. 542-548***

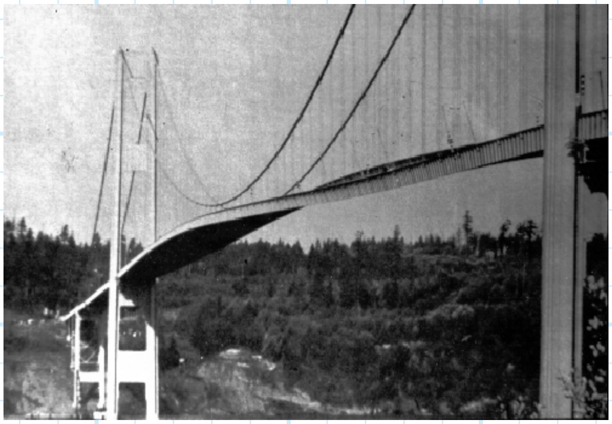
A gain element is an active device. One potential problem with every active circuit is its stability

HO: STABILITY

# Stability

**Q:** *So all there is to making a good microwave amplifier is the design of proper **matching networks**?*

**A:** There is one other problem that confronts the microwave amplifier designer. That problem is **stability** (of the amplifier, not the designer).



An unstable amplifier is also known as an **oscillator**—a source of microwave energy!

**Q:** *Under what **conditions** will an amplifier oscillate?*

**A:** An amplifier will go **unstable** if **either** of these two conditions are true:

$$|\Gamma_{out}| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} \right| > 1.0$$

$$|\Gamma_{in}| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| > 1.0$$

In other words, the amplifier will oscillate if either the input or output reflection coefficient of the gain element has a **magnitude greater than one**.

**Q:** *Hey wait! I thought we learned that the **maximum** value of any reflection coefficient magnitude was 1 (i.e.,* $|\Gamma| \leq 1.0$ *)—this defined the **validity region** of our Smith Chart!*

**A:** Remember, the inequality $|\Gamma| \leq 1.0$ is true for any **passive** load or device. Our gain element is an **active** device—it must have a DC source of power.

As a result, we find that $|\Gamma| > 1.0$ is quite **possible**!

**Q:** *But, we learned that the region outside the* $|\Gamma| = 1.0$ *circle on the Smith Chart corresponded to loads with negative **values of resistance**. Does this mean that* $Z_{in}$ *or* $Z_{out}$ *could have real (i.e. resistive) components that are **negative**?*

**A:** That's **exactly** what it means!

**Q:** *What **is** a negative resistor exactly?*

**A:** Ohm's law **still** applies—negative resistance or not. As a result, we would find for a $-10\ \Omega$ resistor that:

$$\frac{V}{I} = -10 \qquad \Rightarrow \qquad V = -10\, I$$

**Q:** ???

**A:** The result above simply means that the current through a negative resistor is $180^\circ$ **out-of-phase** with the voltage across it.

→ The resistor current is at its minimum value when the voltage across it is at it maximum—and **vice versa**!

This behavior drives our amplifier circuit a little **wacky**, and it begins to oscillate!

**Q:** *So how do we **avoid** this unfortunate occurrence?*

**A:** Recall that amplifier instability occurs when:

$$\left|\Gamma_{out}\right| = \left| S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s} \right| > 1.0$$

$$\left|\Gamma_{in}\right| = \left| S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L} \right| > 1.0$$

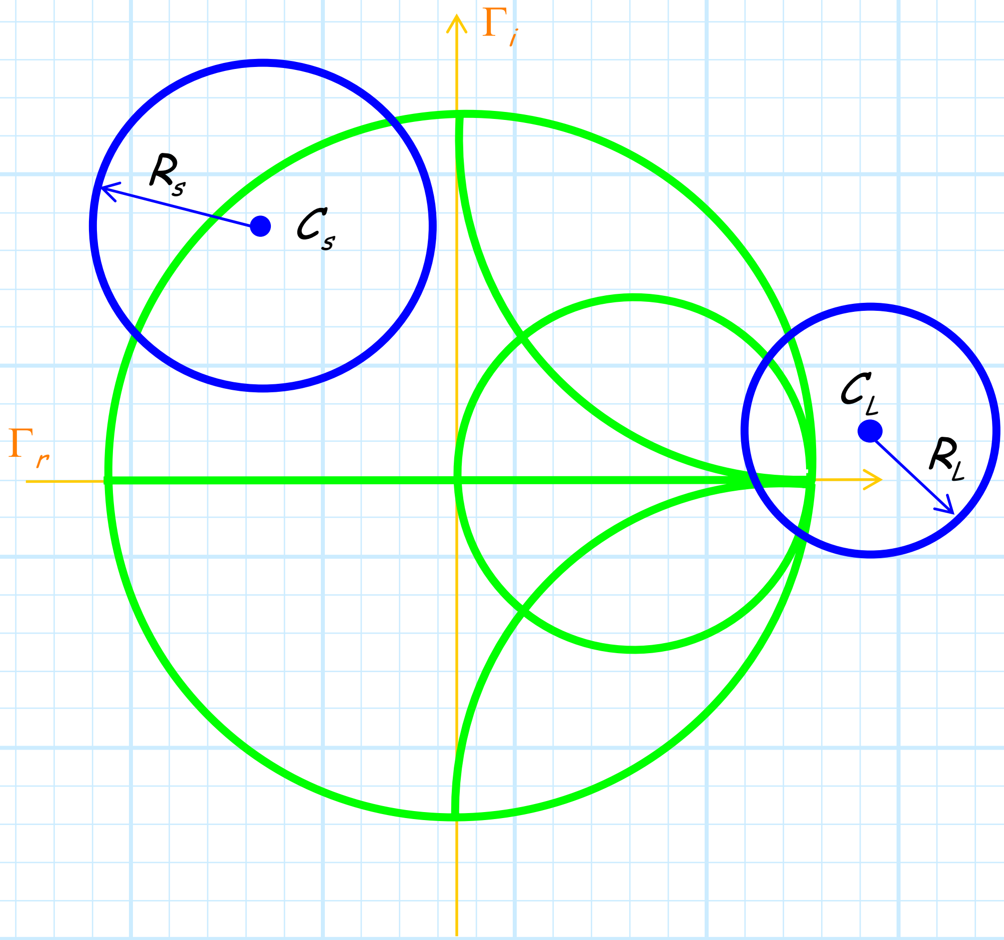
Thus, for a **given** gain element (i.e., $S_{11}, S_{21}, S_{22}, S_{12}$), amplifier stability is determined by the value of $\Gamma_L$ and $\Gamma_s$.

We can **solve** the above equations to determine the specific range of values of $\Gamma_L$ and $\Gamma_s$ that will **induce oscillation**. The results are provided in page 543 and 544 of your text.

We find that these unstable values—when plotted on the **complex $\Gamma$ plane**—form a circle. These circles are know as a **stability circles**.

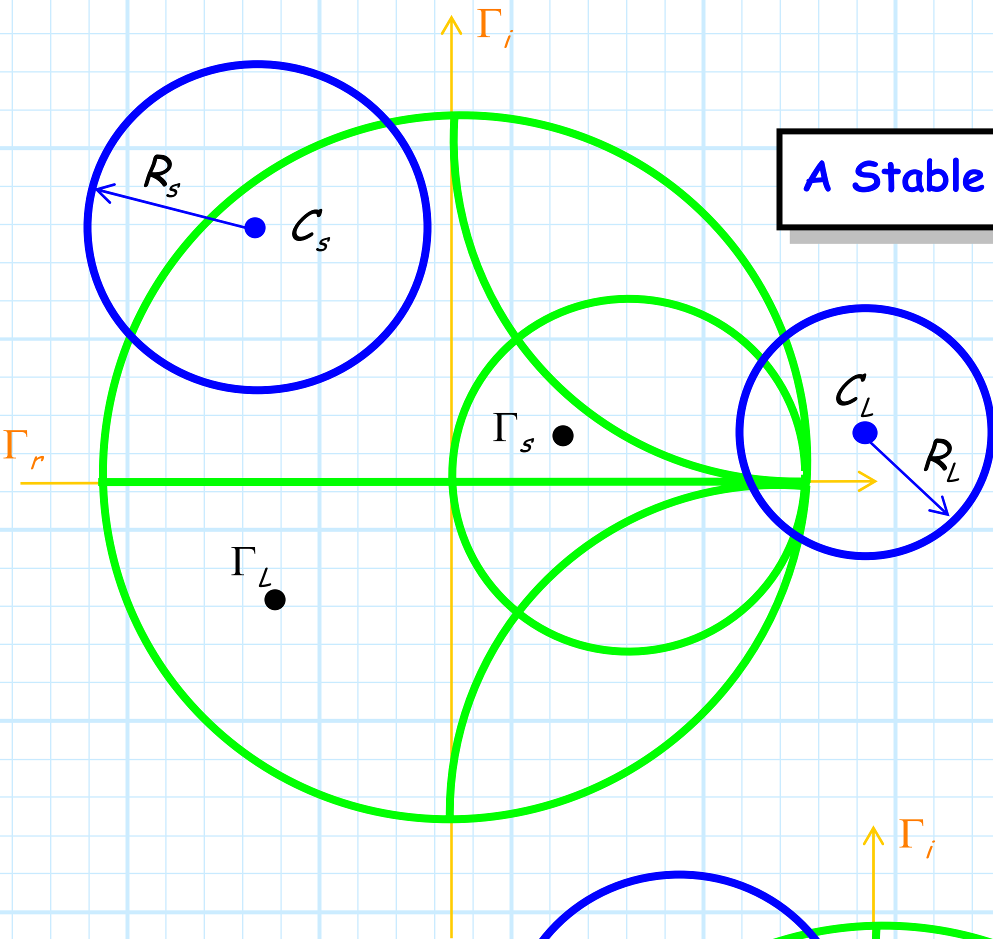
These gain circles are defined as a complex value $C$, which specifies the location of the stability circle **center** on the complex $\Gamma$ plane, and a real value $R$, which specifies the **radius** of the stability circle.

There is **one** stability circle for $\Gamma_L$ (i.e., $C_L$ and $R_L$) and **another** for $\Gamma_s$ (i.e., $C_s$ and $R_s$). Typically, the $\Gamma$ values that lie **inside** the circle will create amplifier oscillation.

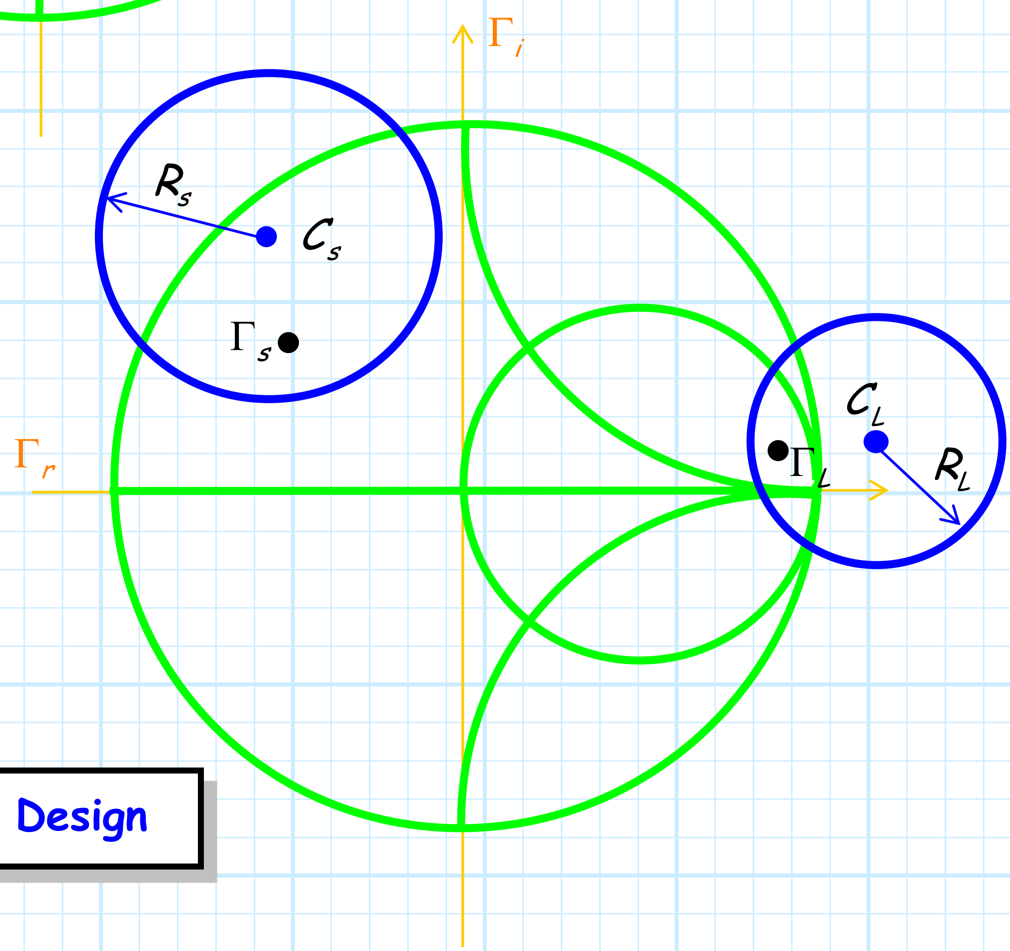



**Q:** *So what do we **use** these stability circles for?*

**A:** As an amplifier designer, we must make sure that our **design values** $\Gamma_L$ and $\Gamma_s$ lie **outside** these circles—otherwise, our well-designed amplifiers will **oscillate**!




A Stable Design



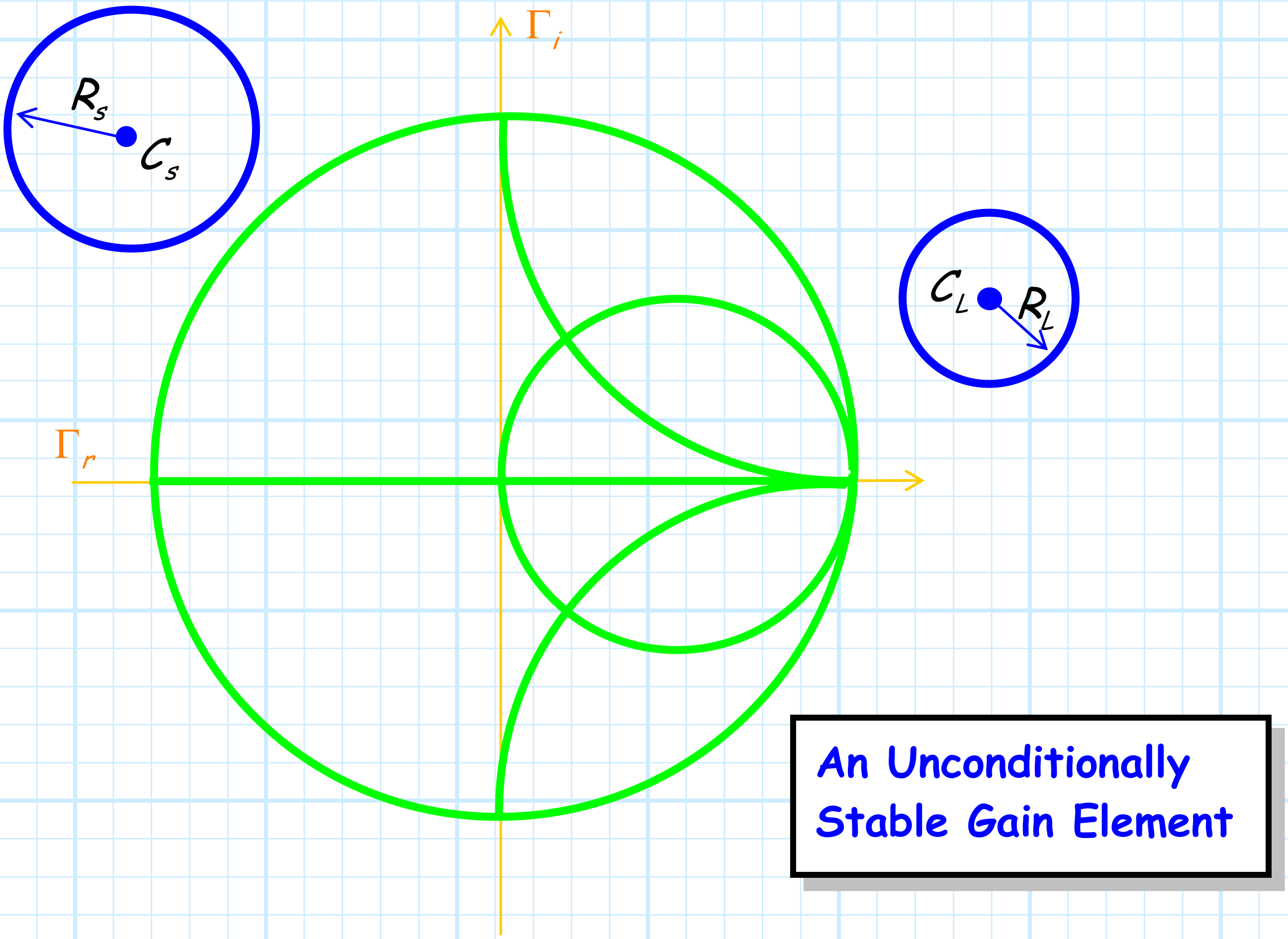

An Unstable Design

**Q:** *Yikes! Must we **always** determine these circles and check our design for instability?*

**A:** Not necessarily! Some gain elements are **unconditionally stable**. As the name suggests, these gain elements result in stable amplifiers for any and all **realizable** values of $\Gamma_L$ and $\Gamma_s$.

**Q:** *So an unconditionally stable gain element has stability circles with **zero radius** (i.e., $R = 0$)?*

**A:** **Could** be, but all that is required for a gain element to be unconditionally stable is for its stability circles to lie completely **outside** the $|\Gamma| = 1$ circle.



For this condition, we find that the values of $\Gamma_L$ and $\Gamma_s$ that result in an unstable amplifier must have a magnitude **greater than 1** (i.e., $|\Gamma_L| > 1$ or $|\Gamma_s| > 1$).

But, we can presume that our the loads and sources attached to our amplifier will **always** have **positive** resistances, such that $|\Gamma_L| < 1$ and $|\Gamma_s| < 1$.

Thus, an **amplifier** constructed with an unconditionally stable gain element will likewise be **unconditionally stable**!

**Q:** *How will I **recognize** an unconditionally stable gain element if I see one? Must I determine and plot the stability circles?*

**A:** There are **tests** that we can apply—using the scattering parameters $S_{11}, S_{21}, S_{22}, S_{12}$—to more **directly** determine if a gain element is unconditionally stable.

First, we find some **necessary conditions** for a gain element to be unconditionally stable are:

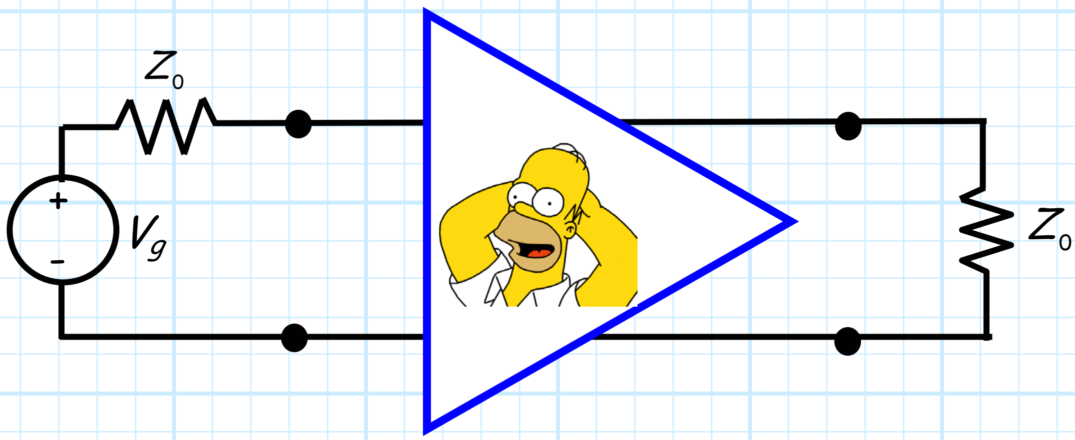
$$|S_{11}|^2 < 1 \qquad \text{and} \qquad |S_{22}|^2 < 1$$

If our gain element is **unilateral** (i.e., $S_{12} = 0$ or approx. $|S_{12}| \ll |S_{21}|$) then the above conditions are likewise **sufficient** for unconditional stability.

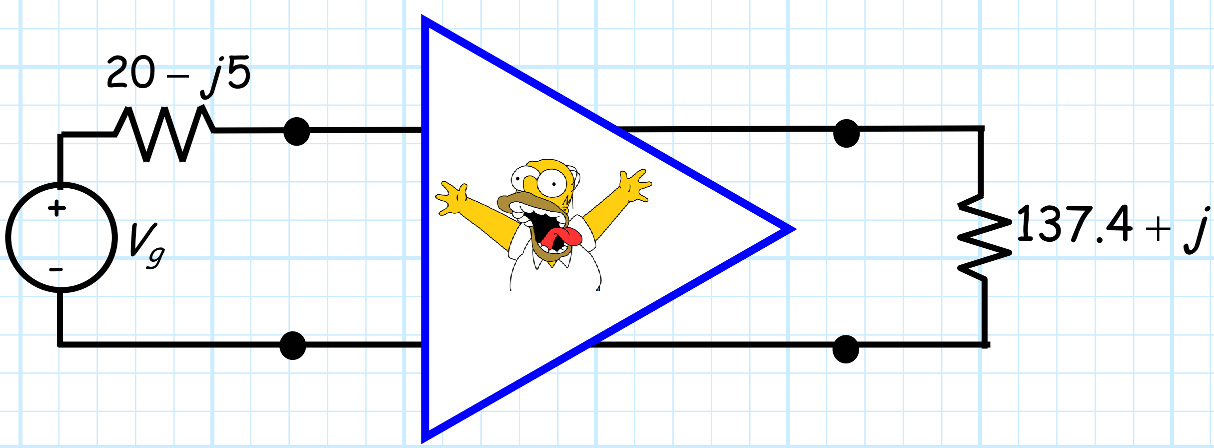
Otherwise, (for $S_{12} \neq 0$ ) we find that our gain element must pass **two more tests**—tests that are provided on page 545 of your text.

**Q:** *Do we really **care** if our design is unconditionally stable? Aren't we really **just** concerned with whether our design values $\Gamma_L$ and $\Gamma_s$ lie inside the stability circle?*

**A:** Remember, the values $\Gamma_L$ and $\Gamma_s$ are determined for the **specific** values of source and load impedances connected to the amplifier (**presumably** $Z_0$).




But what if the resulting amplifier is **not** connected to these ideal sources? The ideal source or load impedance $Z_0$ is **never** achieved with **perfection**, and often achieved **not at all** (consider all the **narrow-band** devices we have studied!).



Thus, since we **do not specifically** know what source and load impedances our amplifier might encounter, we had generally design an amplifier that is stable for them **all**—one that's **unconditionally stable**!

**Q:** *Anything **else** we need to know about amplifier stability?*

**A:** One last **very** important thing.

Recall that amplifiers, like all microwave devices, are **dependent on frequency**. Thus, all of the important values involved in our design (e.g., $\Gamma_s, S_{11}, S_{21}, S_{22}, S_{12}, \Gamma_o$) will **change** as a function of frequency!

**Q:** *I see, amplifier performance, most notably **gain**, will change as a function of frequency, and so maximum power transfer will occur at just our **design frequency**.*

*We've seen this kind of thing **before**!*

**A:** **True**, but for amplifiers there is also a **new** twist.

The amplifier stability conditions (i.e., stability tests) must be satisfied at **any and all frequencies**!

If for **even one frequency** we find that either:

$$\left|\Gamma_{out}\right| = \left|S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s}\right| > 1.0 \quad \text{or} \quad \left|\Gamma_{in}\right| = \left|S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}\right| > 1.0$$

then our amplifier will **oscillate—even** if that frequency is **not** our "design frequency"!

This makes amplifier stability a much more **significant** and **difficult** problem than you might otherwise think.

→ An unconditionally stable amplifier must be unconditionally stable at **all frequencies**!