2.7 – Lossy Transmission Lines

Reading Assignment: *pp. 79-82*

Recall that we have been **approximating** low-loss transmission lines as lossless ($R = G = 0$):

$$\alpha = 0 \qquad\qquad \beta = \omega\sqrt{LC}$$

But, **long** low-loss lines require a **better** approximation:

$$\alpha = \frac{1}{2}\left(\frac{R}{Z_0} + GZ_0\right) \qquad\qquad \beta = \omega\sqrt{LC}$$

Now, if we have **really long** transmission lines (e.g., long distance communications), we can apply **no** approximations at all:

$$\alpha = Re\{\gamma\} \qquad\qquad \beta = Im\{\gamma\}$$

For these **very** long transmission lines, we find that $\beta = Im\{\gamma\}$ is a **function** of signal **frequency** ω. This results in an extremely serious problem—signal **dispersion**.

HO: The Distortionless Line

The Distortionless Line

Recall that the **phase velocity** v_p (i.e., propagation velocity) of a wave in a transmission line is:

$$v_p = \frac{\omega}{\beta}$$

where:

$$\beta = Im\{\gamma\}$$
$$= Im\left\{\sqrt{(R + j\omega L)(G + j\omega C)}\right\}$$

Thus, for a lossy line, the phase velocity v_p is a function of frequency ω (i.e., $v_p(\omega)$)—this is **bad**!

Any signal that carries significant **information** must has some non-zero **bandwidth**. In other words, the signal energy (as well as the information it carries) is **spread** across many frequencies.

If the different frequencies that comprise a signal travel at different velocities, that signal will arrive at the end of a transmission line **distorted**. We call this phenomenon signal **dispersion**.

Recall for **lossless** lines, however, the phase velocity is **independent** of frequency—**no** dispersion will occur!

$$v_p = \frac{1}{\sqrt{LC}} \qquad [R = 0, G = 0]$$

Of course, a perfectly lossless line is impossible, but we find phase velocity is **approximately** constant if the line is low-loss.

Therefore, dispersion distortion on low-loss lines is **most often** not a problem.

Q: You say *"**most** often"* not a problem—that phrase seems to imply that dispersion sometimes **is** a problem!

A: Even for low-loss transmission lines, dispersion can be a problem **if** the lines are **very** long—just a small difference in phase velocity can result in significant differences in propagation delay **if** the line is very long!

Modern examples of long transmission lines include phone lines and cable TV. However, the **original** long transmission line problem occurred with the **telegraph**, a device invented and implemented in the 19th century.

Telegraphy was the essentially the **first** electrical engineering technology ever implemented, and as a result, led to the first ever **electrical engineers**!
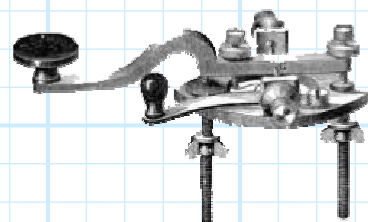
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Early telegraph "engineers" discovered that if they made their telegraph lines **too long**, the dots and dashes characterizing Morse code turned into a muddled, indecipherable **mess**. Although they did not realize it, they had fallen victim to the heinous effects of **dispersion**!

Thus, to send messages over long distances, they were forced to implement a series of intermediate "**repeater**" stations, wherein a human operator received and then **retransmitted** a message on to the next station. This **really** slowed things down!
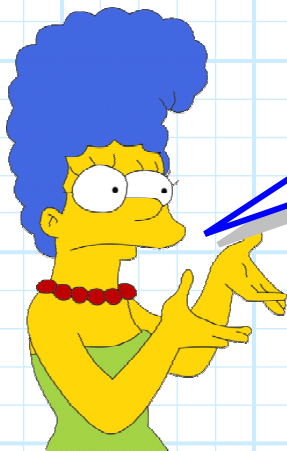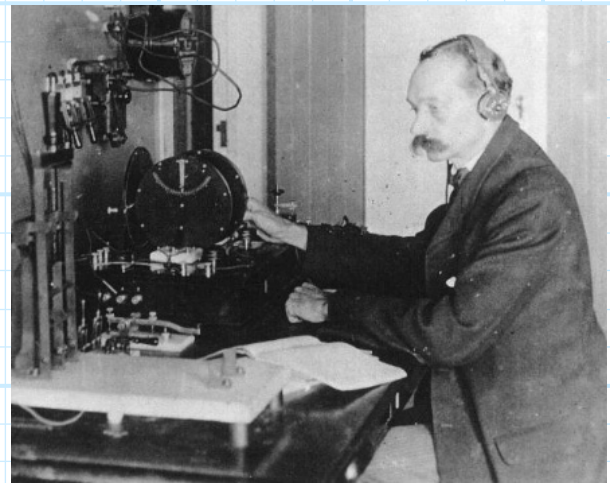
Q: *Is there any way to* ***prevent*** *dispersion from occurring?*

A: You bet! **Oliver Heaviside** figured out how in the **19th** Century!

Heaviside found that a transmission line would be distortionless (i.e., no dispersion) **if** the line parameters exhibited the following **ratio**:

$$\frac{R}{L} = \frac{G}{C}$$

Let's see **why** this works. Note the complex propagation constant γ can be expressed as:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$
$$= \sqrt{LC\,(R/L + j\omega)(G/C + j\omega)}$$

Then **IF**:

$$\frac{R}{L} = \frac{G}{C}$$

we find:

$$\gamma = \sqrt{LC\,(R/L + j\omega)(G/C + j\omega)}$$
$$= \sqrt{LC\,(R/L + j\omega)(R/L + j\omega)}$$
$$= (R/L + j\omega)\sqrt{LC}$$
$$= R\sqrt{\frac{C}{L}} + j\,\omega\sqrt{LC}$$

Thus:
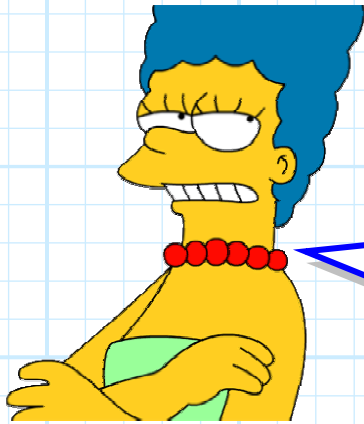
$$\alpha = Re\{\gamma\} = R\sqrt{\frac{C}{L}}$$

$$\beta = Im\{\gamma\} = \omega\sqrt{LC}$$

The propagation **velocity** of the wave is thus:

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

The propagation velocity is **independent** of frequency! This lossy transmission line is **not** dispersive!

> **Q:** *Right. All the transmission lines I use have the property that $R/L > G/C$. I've **never** found a transmission line with this **ideal** property $R/L = G/C$!*

A: It is true that typically $R/L > G/C$. But, we can reduce the ratio R/L (until it is equal to G/C) by adding series **inductors** periodically along the transmission line.

This was **Heaviside's** solution—and it worked! **Long** distance transmission lines were made possible.

Q: *Why don't we increase G instead?*

A:

"[Oliver Heaviside](#) was born in the same London slums as Dickens. Scarlet fever left him partly deaf. He compensated with shyness and sarcasm. Heaviside finished his only schooling in 1865. He was 16 and a top student, but he'd failed geometry. He loathed all that business of deducing one fact from another. He meant to invent knowledge -- not to compute it.

Heaviside went to work as a telegrapher. That drew him into the study of electricity. Then he read Maxwell's new _Treatise on Electricity and Magnetism_, and it seemed to have mystical beauty. It changed his life. He quit work and sealed himself in a room in his family's house. There he reduced Maxwell's whole field theory into two equations. He gave electric theory its modern shape and form. Hertz got the credit for that. But in the fine print Hertz admits his ideas came from Heaviside.

Next Heaviside picked up the radical new idea of vector analysis. His most important ally was the reclusive American genius J. Willard Gibbs. Vector analysis won out, but only after Heaviside -- this shy man with his acid pen -- had started a war. He brought that war to full pitch a few years later with something called operational calculus.

He invented this strange new math by leaping over logic. It was a powerful tool, but it wasn't rigorous. Only people like Kelvin, Rayleigh, and Hertz saw the brilliance that was driving Heaviside faster than method could follow. He knew what he was doing. He growled at his detractors, "Shall I refuse my dinner because I do not fully understand ... digestion?"

Like vector analysis, Heaviside's calculus stood the test of time. So did the rest of his work. He gave us the **theory for long distance telephones**. His math has served and shaped engineering. Yet his biographer, Paul Nahin, writes a sad ending. Heaviside grew sick of fighting and faded off to Torquay in Southwest England. There he lived out his last 25 years in a bitter retreat. You don't see much of Heaviside's name today. But his magnificent works have been woven into the fabric of our textbooks. He deserved a better end. Yet his huge accomplishments force a happy ending on a sad life." **(Taken from http://www.uh.edu/engines/epi426.htm)**