

# Master's Thesis Defense

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## Serially Concatenated Coded Continuous Phase Modulation for Aeronautical Telemetry

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## Publications

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- K. Damodaran and E. Perrins, "Serially Concatenated High Rate Convolutional Codes with Continuous Phase Modulations," in *Proceedings of the International Telemetry Conference*, Las Vegas, NV, October, 2007.
- K. Damodaran and E. Perrins, "Turbo Product Codes with Continuous Phase Modulations," in *Proceedings of the International Telemetry Conference*, San Diego, CA, October, 2008.
- K. Damodaran and E. Perrins, "Spectrally Efficient Concatenated Convolutional Codes with Continuous Phase Modulations," in *Proceedings of the International Telemetry Conference*, San Diego, CA, October, 2008.
- K. Damodaran and E. Perrins, "Serially Concatenated Codes for Aeronautical Telemetry," in review for *Proceedings of the IEEE Military Communications Conference (MILCOM' 08)*, San Diego, CA, November, 2008.

## Outline

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- Introduction
- Error Control Coding
- Continuous Phase Modulation
- Serially Concatenated Codes
- Bit Error Rate Performance
- Conclusion and Future Work
- Appendix - Performance analysis

## Outline

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- **Introduction**
  - Motivation
  - Objective
- Error Control Coding
- Continuous Phase Modulation
- Serially Concatenated Codes
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## Introduction

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- **Motivation**
  - Effective transmission - efficient utilization of power, bandwidth, and complexity
  - Error control codes increases power efficiency; reduces bandwidth efficiency
  - Aeronautical telemetry: constant envelope waveforms
    - Pulse code modulation/frequency modulation (PCM/FM)
    - Shaped offset quadrature phase shift keying (known as SOQPSK-TG)
  - Forward error correction for aeronautical telemetry - preliminary attention to date

## Introduction

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- Objective
  - Develop bandwidth-efficient serially concatenated codes (SCCs) for aeronautical telemetry
  - Inner codes: SOQPSK-TG, PCM/FM
  - Outer codes: Convolutional codes (CCs), Turbo-Product codes (TPCs), Repeat-Accumulate codes (RACs)
  - Analyze coded coherent and noncoherent modulations
  - Compare coding gain performances of coded CPMs

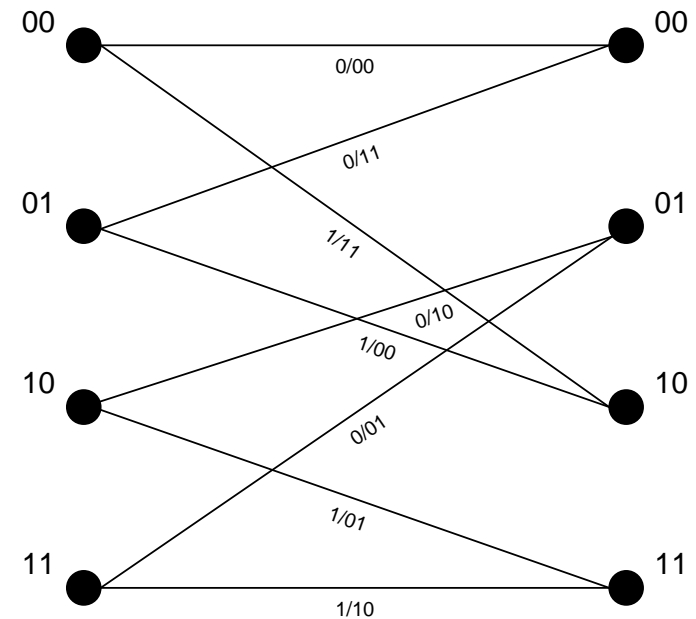
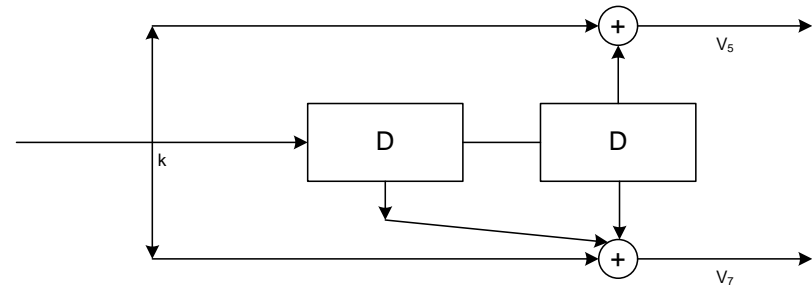
## Outline

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- Introduction
- Error Control Coding
  - Convolutional Codes
    - Encoding
    - Puncturing of Convolutional Codes
    - Soft-Input Soft-Output Decoding
  - Turbo-Product Codes
    - Encoding
    - Chase Decoding
  - Repeat-Accumulate Codes
    - Encoding
    - Sum-Product Decoding
- Continuous Phase Modulation
- Serially Concatenated Codes
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## Convolutional Codes

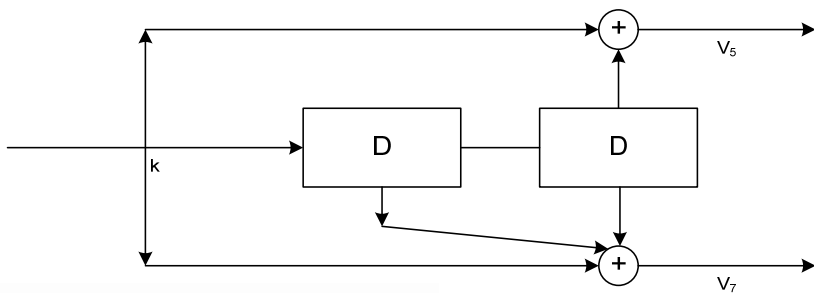
- Reasons
  - Satisfies the properties stated in [Benedetto1998]
  - Strong coding gains
  - Rate flexibility
  - Simple structure
- Encoding
  - Rate  $(R) = u/n$
  - Constraint length of convolutional code 1 (CC1) and convolutional code 2 (CC2) is 2 and 4
  - Encoder: 4-state, time-invariant trellis





## Convolutional Codes

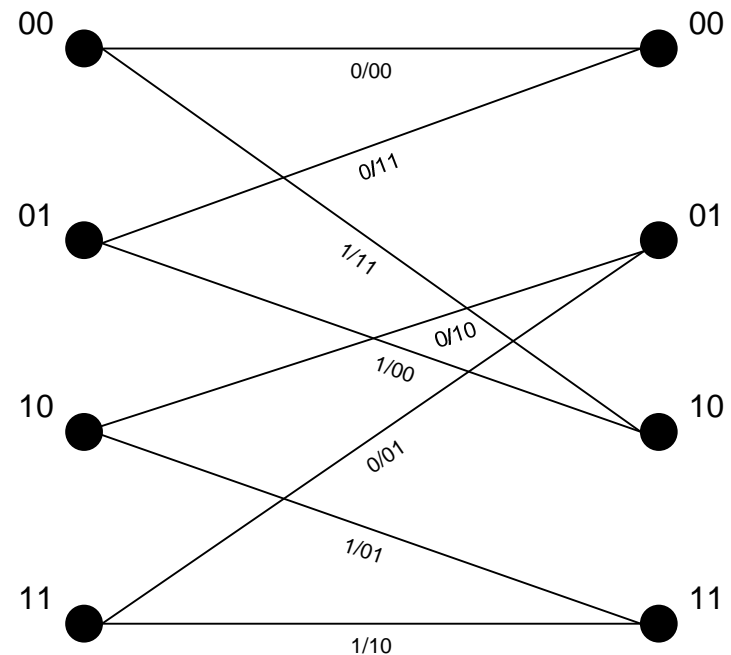
- Puncturing
  - Improves spectral efficiency
  - Rate selectable encoder/decoder
  - Viterbi decoding of punctured CCs: simple
  - High rate punctured CCs: from basic rate 1/2 CCs
  - A map: 0's indicate deleted bits



Code Rate	K = 2	N	S
1/2	1(5) 1(7)	2048	32
2/3	10 11	1536	27
3/4	101 110	1364	26
4/5	1011 1100	1280	25
5/6	10111 11000	1230	24
6/7	101111 110000	1197	24
7/8	1011111 1100000	1168	24
8/9	10111111 11000000	1152	24
9/10	101111111 110000000	1140	23

## Convolutional Codes

- Soft-input soft-output (SISO) decoding algorithm
  - SISO: 4 port device; 2 inputs, 2 outputs
  - Input: probability distribution of information bit and codeword symbols ( $P(u;I)$  and  $P(c;I)$ , respectively)
  - Output: update on input probability distributions based on code constraints ( $P(u;O)$  and  $P(c;O)$ )
  - Calculate *extrinsic* information
  - Modified "max-log" SISO is used



## Convolutional Codes

- Soft-input soft-output decoding algorithm (contd.)
  - Similar to branch metrics in Viterbi algorithm: forward and backward recursion branch metrics ( $A_k(\cdot)$  and  $B_k(\cdot)$ )

$$A_k(s) = \sum_{e:s^E(e)=s} A_{k-1}[s^S(e)]P_k[u(e); I]P_k[c(e); I]$$

$$B_k(s) = \sum_{e:s^S(e)=s} B_{k+1}[s^E(e)]P_{k+1}[u(e); I]P_{k+1}[c(e); I]$$

- Output probability distributions: calculated based upon forward, backward recursive branch metrics and input *a priori* probability distribution

$$\tilde{H}_c^j = \sum_{e:c_k^1(e)=c^j} A_{k-1}[s^S(e)]P_k[u(e); I]P_k[c(e); I]B_k[s^E(e)]$$

$$\tilde{H}_u^j = \sum_{e:u_k^1(e)=u^j} A_{k-1}[s^S(e)]P_k[u(e); I]P_k[c(e); I]B_k[s^E(e)]$$

## Turbo-Product Codes

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- **Reasons**
  - Large coding gain
  - Rate flexibility
  - Simple structure
  - Modest synchronization requirements
  - Availability of commercial encoder and decoder integrated circuits
- **Encoding**
  - $(n, k, \delta)$  TPC: product of  $(n_1, k_1, \delta_1)$  and  $(n_2, k_2, \delta_2)$  linear block code
  - $n = n_1 * n_2$ ;  $k = k_1 * k_2$ ;  $\delta = \delta_1 * \delta_2$
  - $R = k/n$

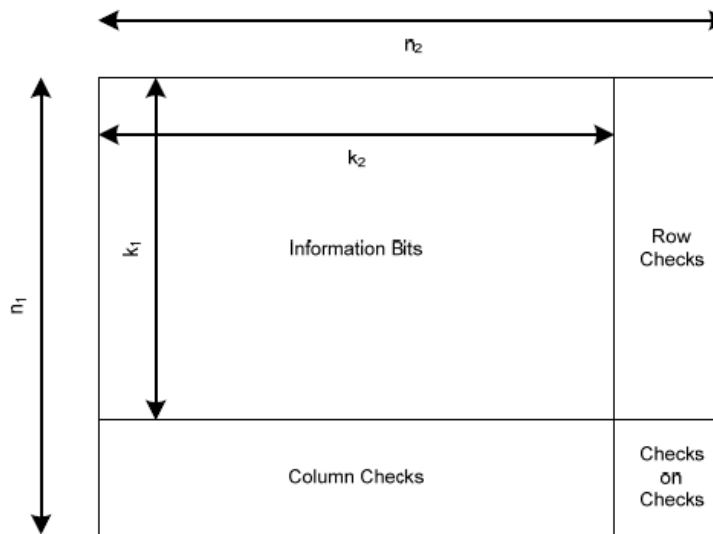
## Turbo-Product Codes

- Example TPCs:  $(64, 57, 4)$ ,  $(32, 26, 4)$ ,  $(128, 120, 4)$  TPCs

- $n = 64 * 64$

- $k = 57 * 57$

- $\delta = 4 * 4$



- Iterative Chase decoding algorithm
  - Key idea: reduce the number of reviewed codewords

## Turbo-Product Codes

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- Iterative Chase decoding algorithm (contd.)
  - Compute an optimum codeword  $D$  and a competing codeword  $C$
  - With  $D$  and  $C$  known, calculate extrinsic information

$$w_j = \left( \frac{|R - C|^2 - |R - D|^2}{4} \right) d_j - r_j.$$

- With no competing codeword  $C$ , extrinsic information is calculated as

$$w_j = \beta \times d_j \text{ with } \beta \geq 0.$$

- Soft input for the next decoding step is

$$[R(m)] = [R] + \alpha(m)[W(m)] \text{ with } R(0) = R.$$

- Updated  $R$ : refined by further iterations

## Repeat-Accumulate Codes

- **Reasons**
  - Simple code, small encoding complexity
  - Exceptional iterative decoding performance
- **Encoding**
  - $N$ -bit input block is repeated  $q$  times and scrambled by a  $qN \times qN$  interleaver
  - Output from interleaver: encoded by a rate 1 accumulator.



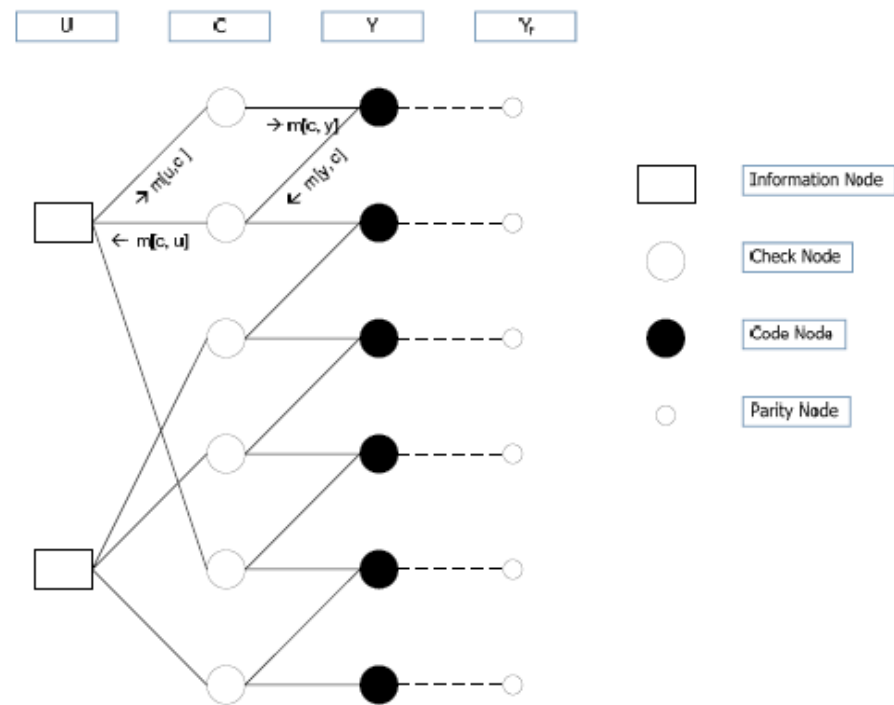
- **Sum-product decoding algorithm**
  - RACs perform well with maximum-likelihood (ML) decoding; complexity - prohibitively large.
  - Sum-product decoding: approximates the performance of RACs with ML decoding.

## Repeat-Accumulate Codes

- Sum-product decoding algorithm (contd.)
  - Tanner graph - bipartite with variable nodes and check nodes
  - Initialization: set branch messages to zero and update them in each iteration.
  - At the end of K iterations, calculate

$$s(u) = \sum_c m[u, c]$$

- If  $s(u) > 0$ , decoded bit is 1, else 0
- Example: repetition 3 RAC





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- Introduction
- Error Control Coding
- **Continuous Phase Modulation**
  - **Pulse Code Modulation/Frequency Modulation (PCM/FM)**
  - **Shaped-Offset Quadrature Phase Shift Keying (SOQPSK-TG)**
- Serially Concatenated Codes
- Bit Error Rate Performance
- Conclusion and Future Work
- Appendix - Performance analysis

## Continuous Phase Modulation

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- **Continuous Phase Modulation (CPM)**
  - CPMs: natural choice for inner codes of a SCC
  - Demodulators are SISO: designed and implemented in [Kumaraswamy2008]
- **PCM/FM**
  - $M = 2, h = 7/10$ .
  - raised cosine frequency pulse shape; duration  $L = 2$  symbol times (2RC)
  - High detection efficiency, low spectrum efficiency, moderate decoding complexity
- **SOQPSK-TG**
  - Precoder: converts binary information symbols to ternary channel symbols
  - $h = 1/2$ ; uses a custom frequency pulse shape
  - low decoding complexity, compared to PCM/FM: twice the spectral efficiency, low detection efficiency.

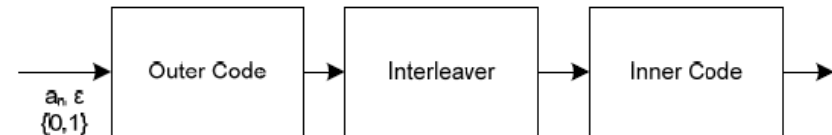
## Outline

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- Introduction
- Error Control Coding
- Continuous Phase Modulation
- **Serially Concatenated Codes**
  - *Serially Concatenated Convolutionally Coded CPM*
  - *Turbo-Product Coded CPM*
  - *Repeat-Accumulate Coded CPM*
- Bit Error Rate Performance
- Conclusion and Future Work
- Appendix - Performance analysis

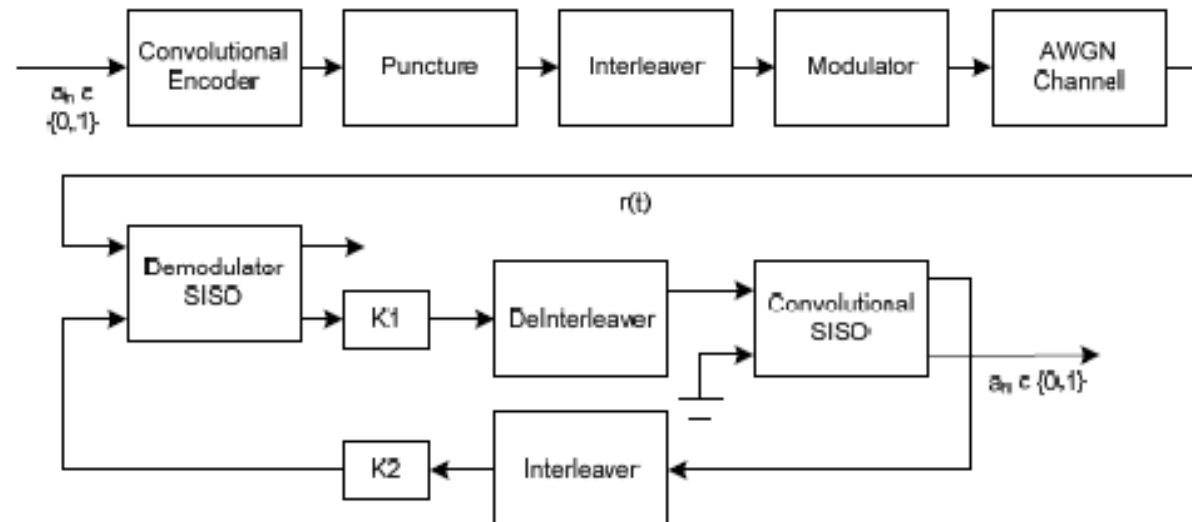
## Serially Concatenated Codes

- Why serially concatenated codes (SCC)
  - standard for applications where high coding gains are needed
  - Cascade of simpler codes; effective than a single complex code
  - SCCs: believed to be superior alternatives to PCCs [Benedetto1998]
- Good Outer Code
  - Properties of outer code adopted from [Benedetto1998]
  - Outer encoder should be a non-recursive encoder
  - Length of the input block should be large
  - Constraint length should be less than 4
  - Outer code should have maximum odd free distance



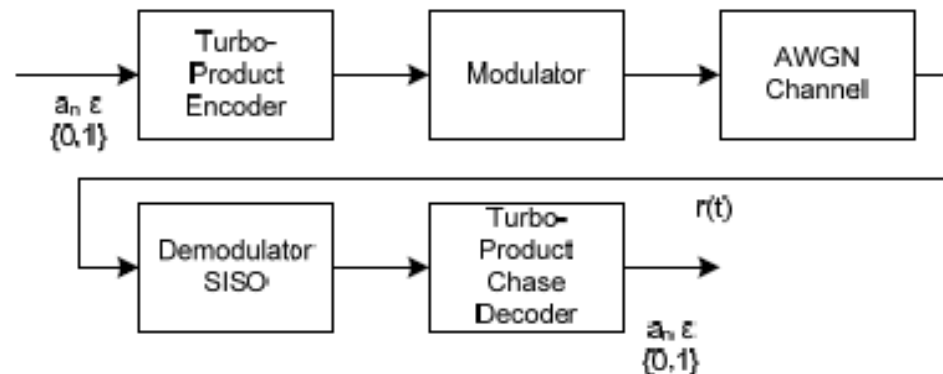
## Serially Concatenated Codes

- **Serially Concatenated Convolutionally Coded CPM (SCCC-CPM)**
  - Outer codes:  $CC1, CC2$ ; Inner modulation: SOQPSK-TG, PCM/FM
  - Inner demodulator and outer decoder: based on SISO decoding algorithm.
  - SOQPSK-TG:  $K1 = K2 = 0.75$
  - PCM/FM:  $K1 = K2 = 0.65$



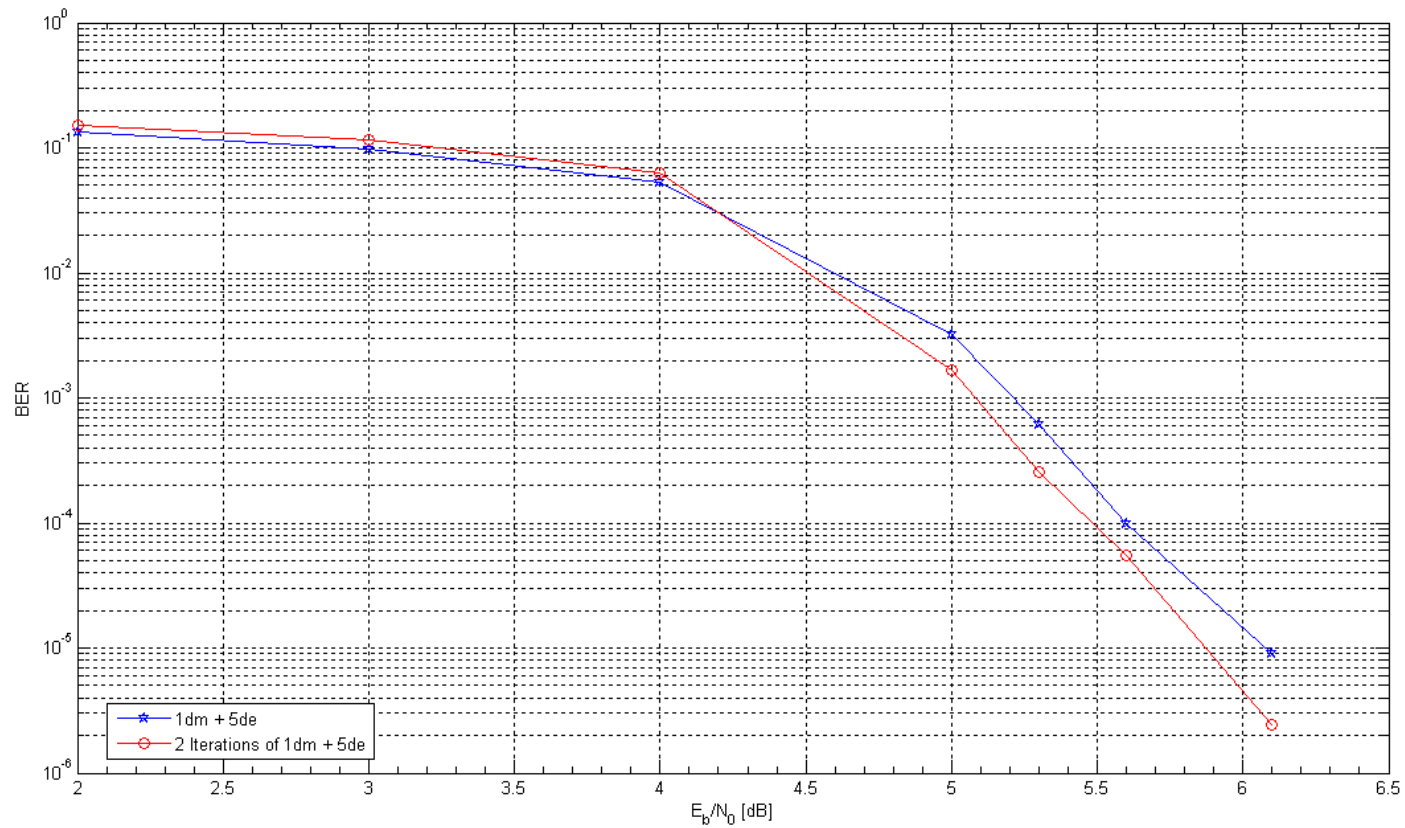
## Serially Concatenated Codes

- Turbo-Product Coded CPM (TPC-CPM)
  - Initial study: [Goeghegan2003]
  - Encoder:  $(64,57) \times (64,57)$ ,  $(32,26) \times (32,26)$ ,  $(128,120) \times (128,120)$  TPC
  - Modulation: SOQPSK-TG, PCM/FM
  - Best performance: realized with 2 receiver iterations; each receiver iteration involves a CPM SISO demodulation followed by 5 decode iterations
  - Trade off: with 1 receiver iteration the performance is only 0.2 dB less.



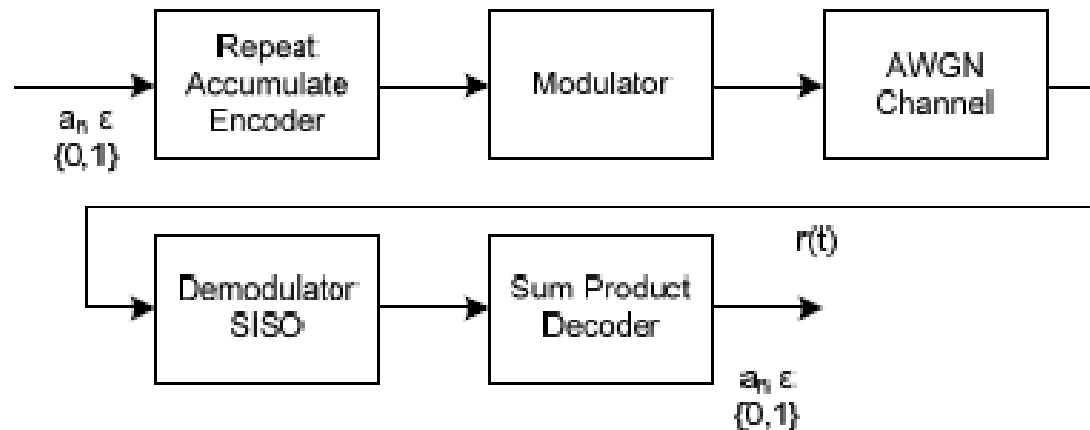
## Serially Concatenated Codes

- Why a single receiver iteration?



## Serially Concatenated Codes

- Repeat-Accumulate Coded CPM (RAC-CPM)
  - Encoder: RAC with a repetition factor  $q = 3$  or  $q = 4$
  - Modulation: SOQPSK-TG or PCM/FM
  - Decoder: Iterative sum-product algorithm





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- Serially Concatenated Codes
- **Bit Error Rate Performance**
  - **SOQPSK-TG vs. PCM/FM**
  - **Coherent vs. Noncoherent Demodulation**
  - **Convolutional Code 1 vs. Convolutional Code 2**
  - **Performance of Turbo-Product Coded CPM**
- Conclusion and Future Work
- Appendix - Performance analysis

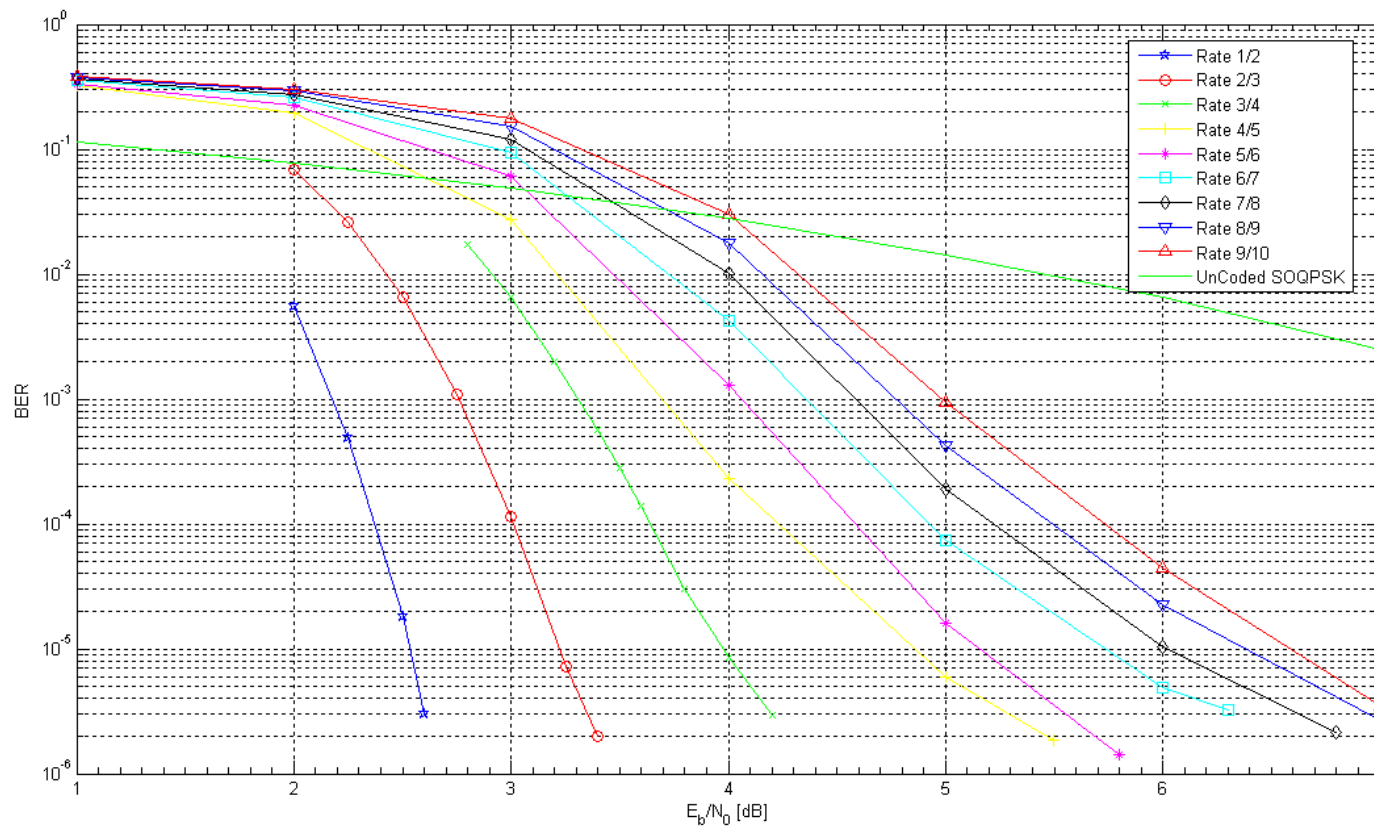
## Bit Error Rate Performance

- Coded SOQPSK-TG vs. PCM/FM
  - Coding gains measured at a bit error rate (BER) =  $10^{-5}$
  - Uncoded SOQPSK-TG: BER =  $10^{-5}$  at  $E_b/N_0 = 10.56$  dB
  - Uncoded PCM/FM: BER =  $10^{-5}$  at  $E_b/N_0 = 8.44$  dB
  - SOQPSK-TG has twice the bandwidth efficiency of PCM/FM: better choice for coded aeronautical telemetry.

Code	Modulation	Code Rate	BER = $10^{-5}$	Gain dB
CC1	SOQPSK-TG	1/2	2.6	8.0
CC1	SOQPSK-TG	7/8	6.0	4.6
CC1	PCM/FM	1/2	1.8	6.6
CC1	PCM/FM	7/8	3.8	4.6

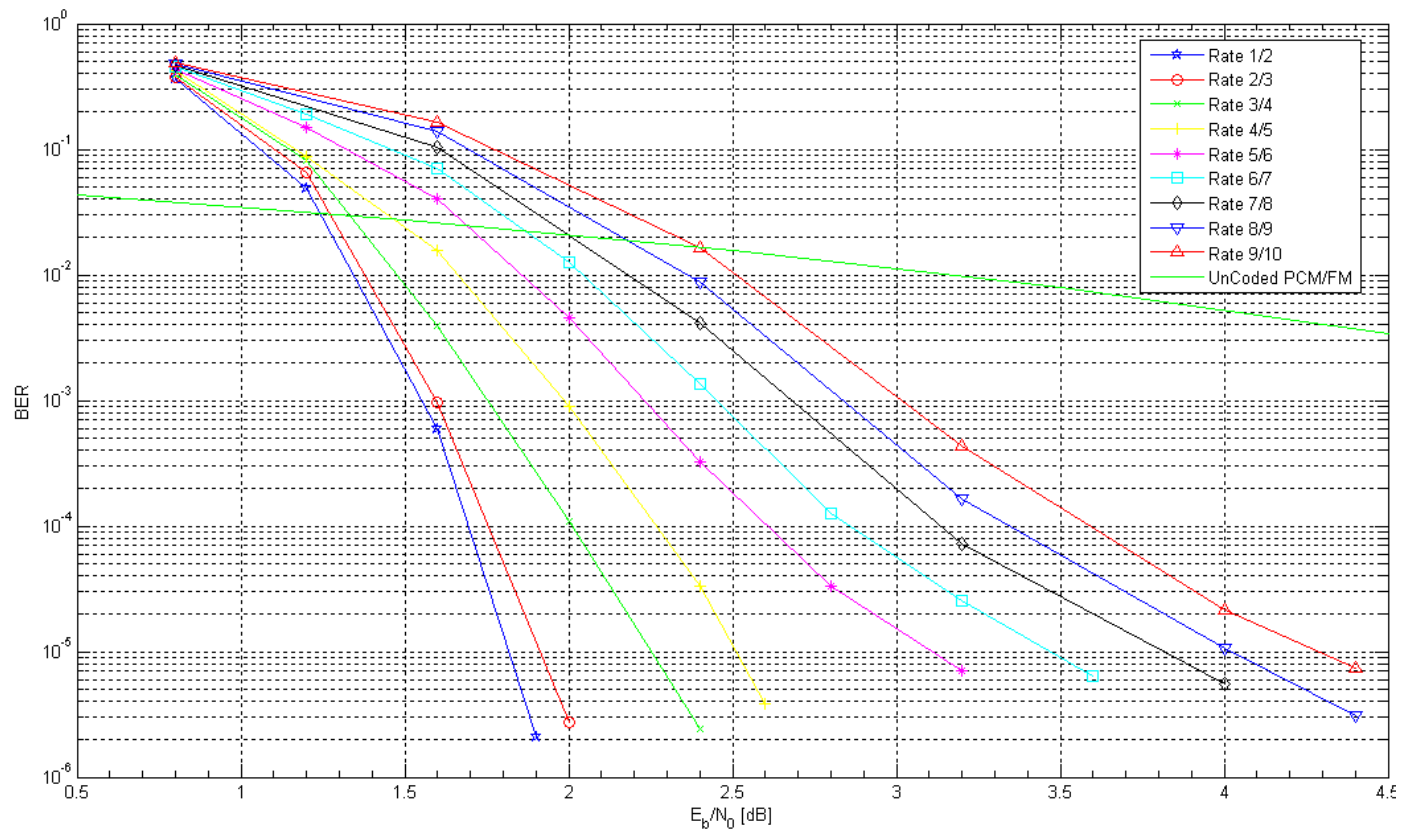
## Bit Error Rate Performance

- CC1 with SOQPSK-TG



## Bit Error Rate Performance

- CC1 with PCM/FM



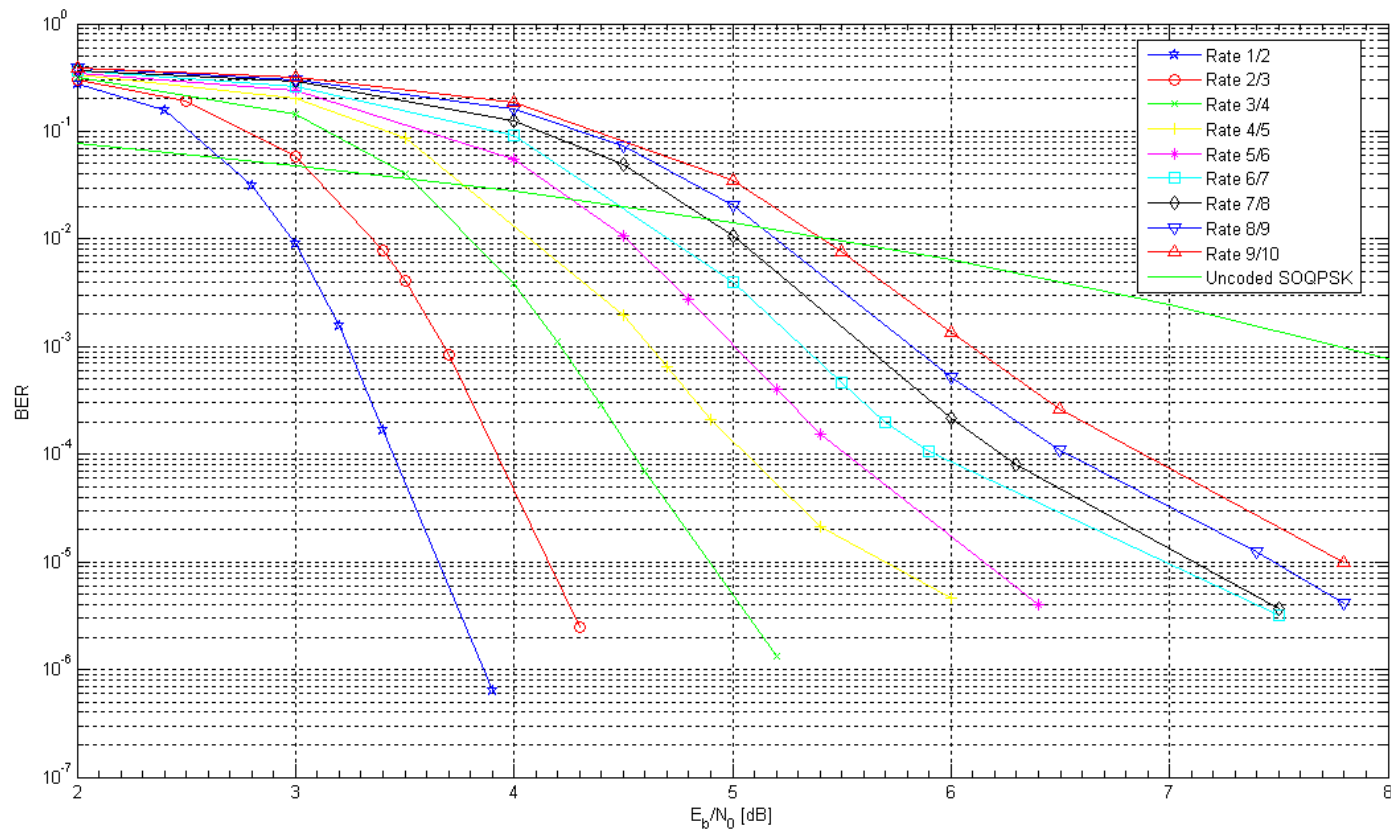
## Bit Error Rate Performance

- Coded coherent vs. noncoherent demodulation
  - Noncoherent demodulation: performance about 1 dB less than coherent demodulation
  - Noncoherent demodulators: perfect tradeoff between complexity and performance

Code	Noncoherent Modulation	Code Rate	BER = $10^{-5}$	Gain dB	Gain dB (Coherent)	Difference dB
CC1	SOQPSK-TG	1/2	3.6	7.0	8.0	1.0
CC1	SOQPSK-TG	7/8	7.1	3.5	4.6	1.1
CC1	PCM/FM	1/2	2.5	5.9	6.6	0.7
CC1	PCM/FM	7/8	4.8	3.6	4.6	1.0

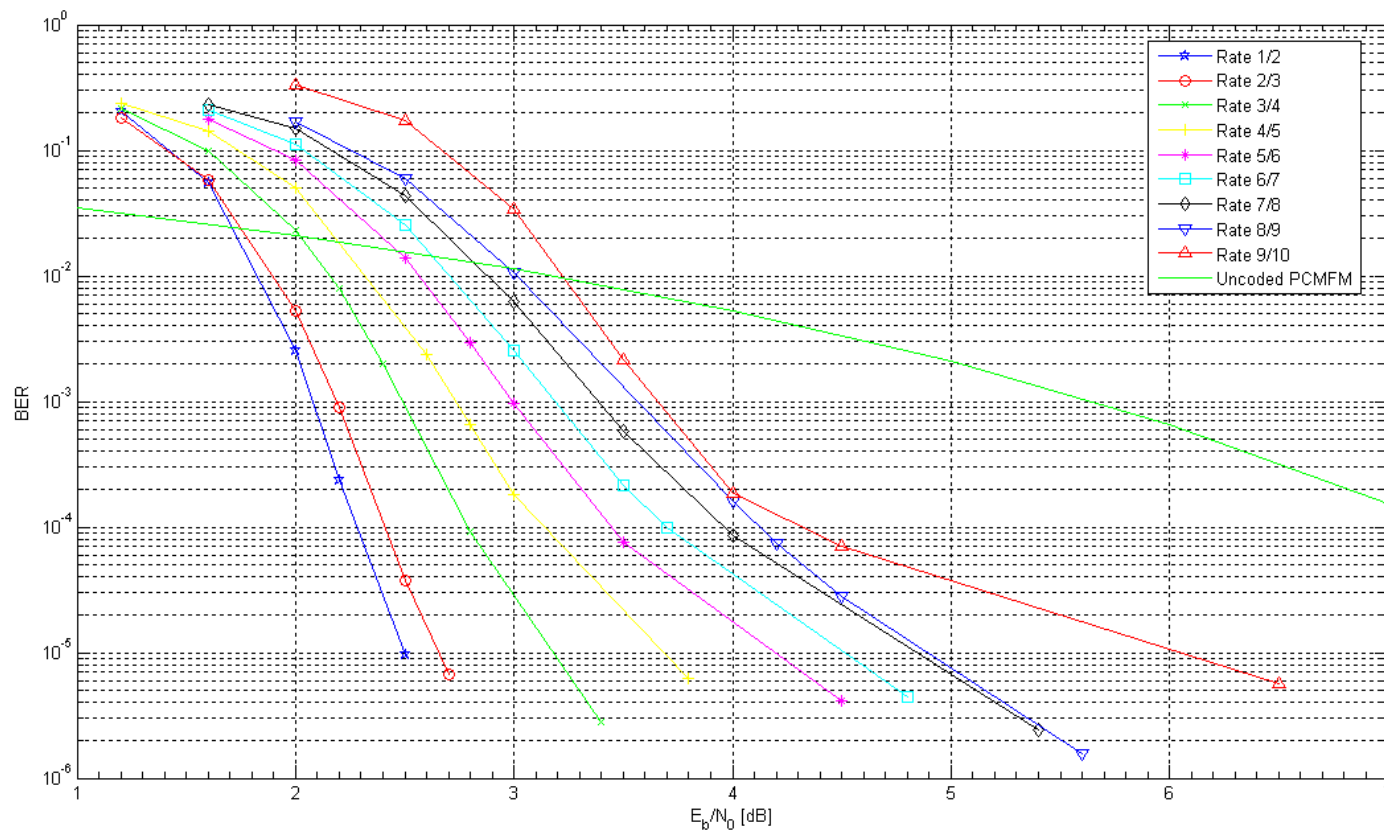
## Bit Error Rate Performance

- Noncoherent demodulation - CC1 with SOQPSK-TG



## Bit Error Rate Performance

- Noncoherent demodulation - CC1 with PCM/FM



## Bit Error Rate Performance

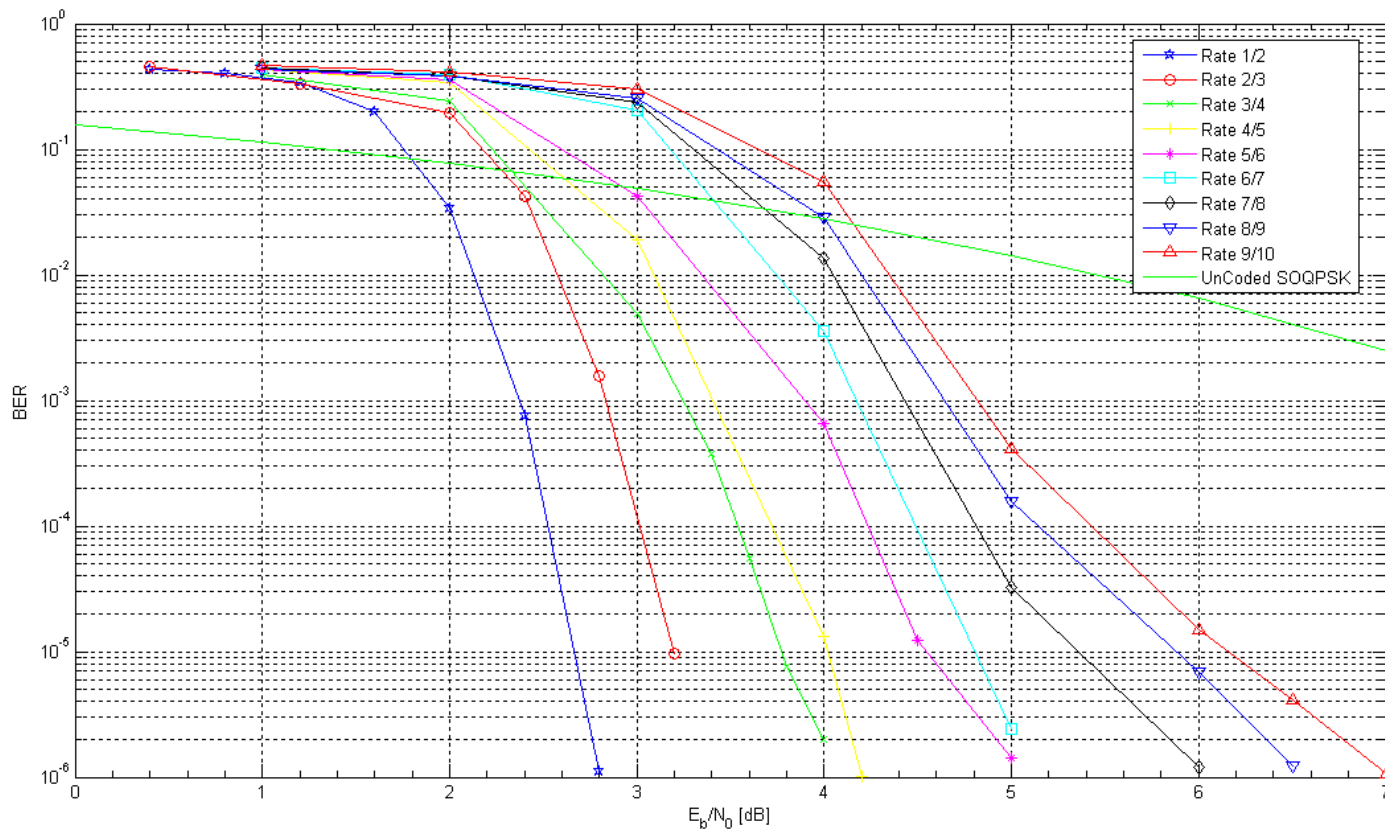
- *CC1 vs. CC2*
  - SOQPSK-TG: *CC2* outperforms *CC1* at higher code rates; lower code rates similar performance
  - PCM/FM: *CC1* outperforms *CC2* at lower code rates; higher code rates similar performance

Code	Modulation	Code Rate	BER = $10^{-5}$	Gain dB
<i>CC1</i>	SOQPSK-TG	1/2	2.6	8.0
<i>CC2</i>	SOQPSK-TG	1/2	2.7	7.9
<i>CC1</i>	SOQPSK-TG	7/8	6.0	4.6
<i>CC2</i>	SOQPSK-TG	7/8	5.4	5.2
<i>CC1</i>	PCM/FM	1/2	1.8	6.6
<i>CC2</i>	PCM/FM	1/2	2.1	6.3
<i>CC1</i>	PCM/FM	7/8	3.8	4.6
<i>CC2</i>	PCM/FM	7/8	3.8	4.6



## Bit Error Rate Performance

- CC2 with SOQPSK-TG



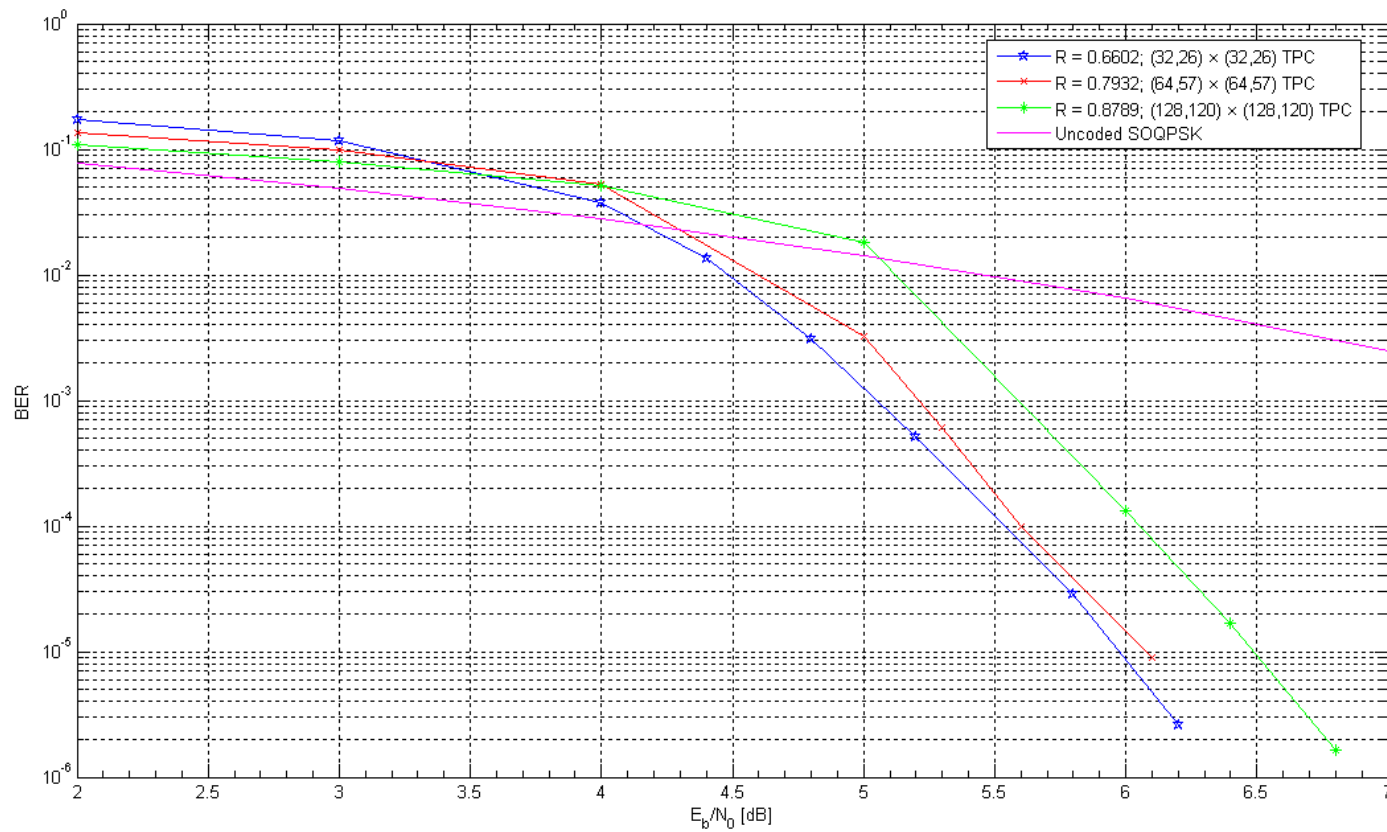
## Bit Error Rate Performance

- Performance of TPC-CPM
  - Noncoherent demodulation performs closely to coherent demodulation
  - TPC-CPMs built here performs 0.8 dB better than similar system built in [Geoghegan2003]
  - Performance improvement: use of near-optimal SISO algorithm for CPM demodulation instead of *ad hoc* soft demodulation techniques used in [Geoghegan2003].

Code	Modulation	BER = $10^{-5}$	Gain dB	Geoghegan BER = $10^{-5}$
TPC	SOQPSK-TG	6.1	4.5	6.9
TPC	PCM/FM	4.4	4.0	5.0

## Bit Error Rate Performance

- TPC with SOQPSK-TG



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## Conclusion and Future Work

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- **Conclusion**
  - SCC-CPMs with iterative SISO demodulation and decoding were built
  - Outer Codes: CCs, TPCs, RACs
  - inner modulation: SOQPSK-TG, PCM/FM
  - Coded SOQPSK-TG is better suited to aeronautical telemetry than coded PCM/FM
  - Noncoherent demodulation performs within 1 dB of coherent demodulation
  - With SOQPSK-TG, CC2 outperforms CC1 at higher code rates
  - With PCM/FM, CC1 outperforms CC2 at lower code rates
  - TPC-CPM built here outperforms a similar system developed in [Geoghegan2003]

## Conclusion and Future Work

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- **Conclusion (contd.)**
  - CCs with CPM provide better coding gain performance than TPCs and RACs with CPM
  - Coding gain performance of SCCC-CPMs increases with an increase in input block size and number of decoding iterations
- **Future Work**
  - For a bandwidth efficiency and decoding complexity: find a optimum combination of coding and CPM
  - Low-density parity-check (LDPC) codes can be optimally combined with CPM to develop LDPC-CPM
  - SCC-CPMs can be built with advanced range telemetry modulation (ARTM)-CPM as an inner modulation.

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- **Appendix - Performance analysis**
  - Performance of Repeat-Accumulate Coded CPM
  - Convolutional Codes vs. Turbo-Product Codes and Repeat-Accumulate Codes
  - Performance Variations due to Increase in Input Block Size and Number of Decoding Iterations.

## Appendix - Performance Analysis

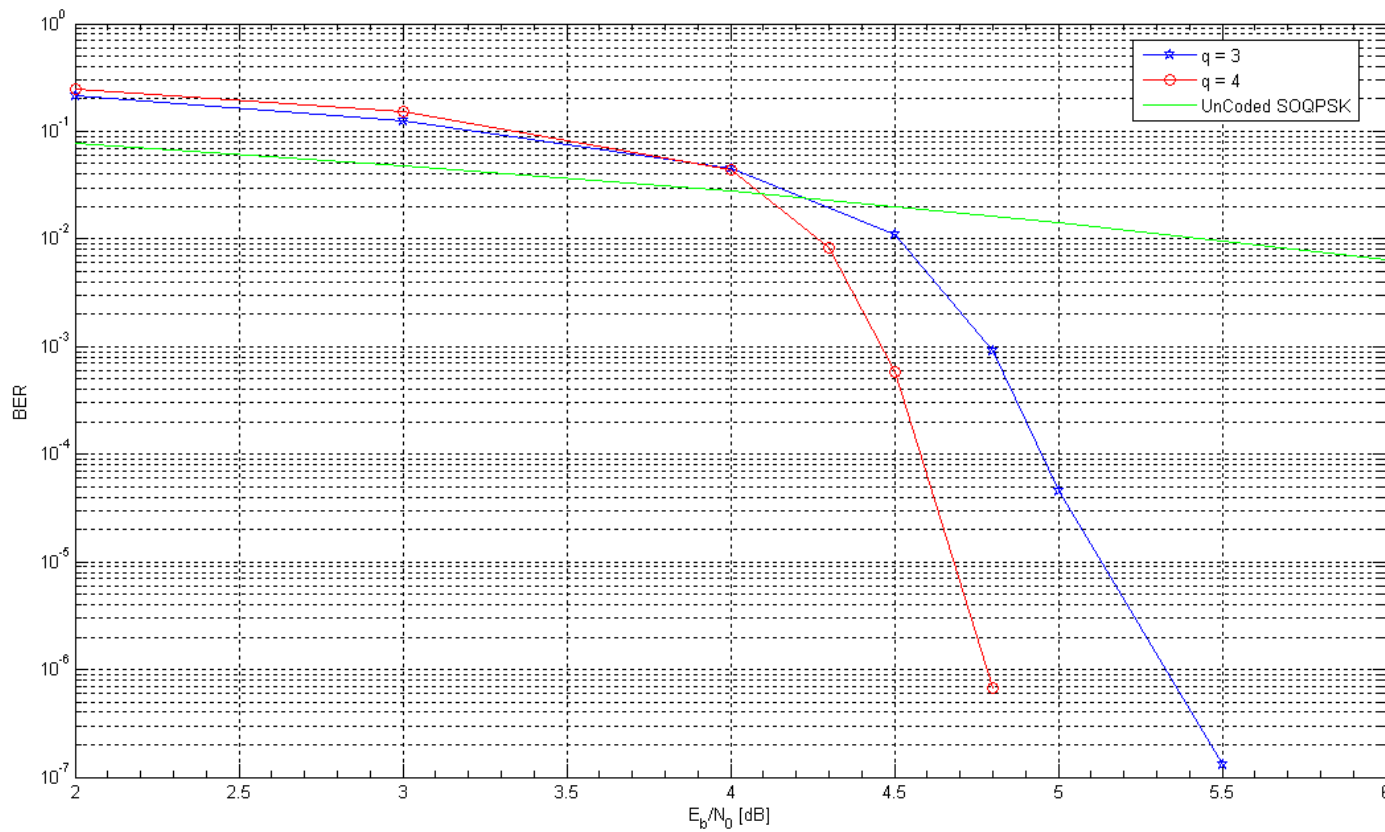
- Performance of RAC-CPM
  - RAC-CPMs lose their significance because of their lower code rates.

Code	Repetition factor	Modulation	BER = $10^{-5}$	Gain dB
RAC	$q = 3$	SOQPSK-TG	5.1	5.5
RAC	$q = 4$	SOQPSK-TG	4.7	5.9
RAC	$q = 3$	PCM/FM	5.2	3.2
RAC	$q = 4$	PCM/FM	5.0	3.4



## Appendix - Performance Analysis

- RAC with SOQPSK-TG



## Appendix - Performance Analysis

- *CCs vs. TPCs and RACs*
  - From table: *CCs with CPM* provide better coding gain performance.

Code	Modulation	Code rate	BER = $10^{-5}$	Gain dB
CC1	SOQPSK-TG	4/5	4.9	5.7
CC2	SOQPSK-TG	4/5	4.0	6.6
TPC	SOQPSK-TG	0.7932	6.1	4.5
RAC	SOQPSK-TG	1/3	5.1	5.5
CC1	PCM/FM	4/5	2.5	5.9
CC2	PCM/FM	4/5	2.7	5.7
TPC	PCM/FM	0.7932	4.4	4.0
RAC	PCM/FM	1/3	5.2	3.2

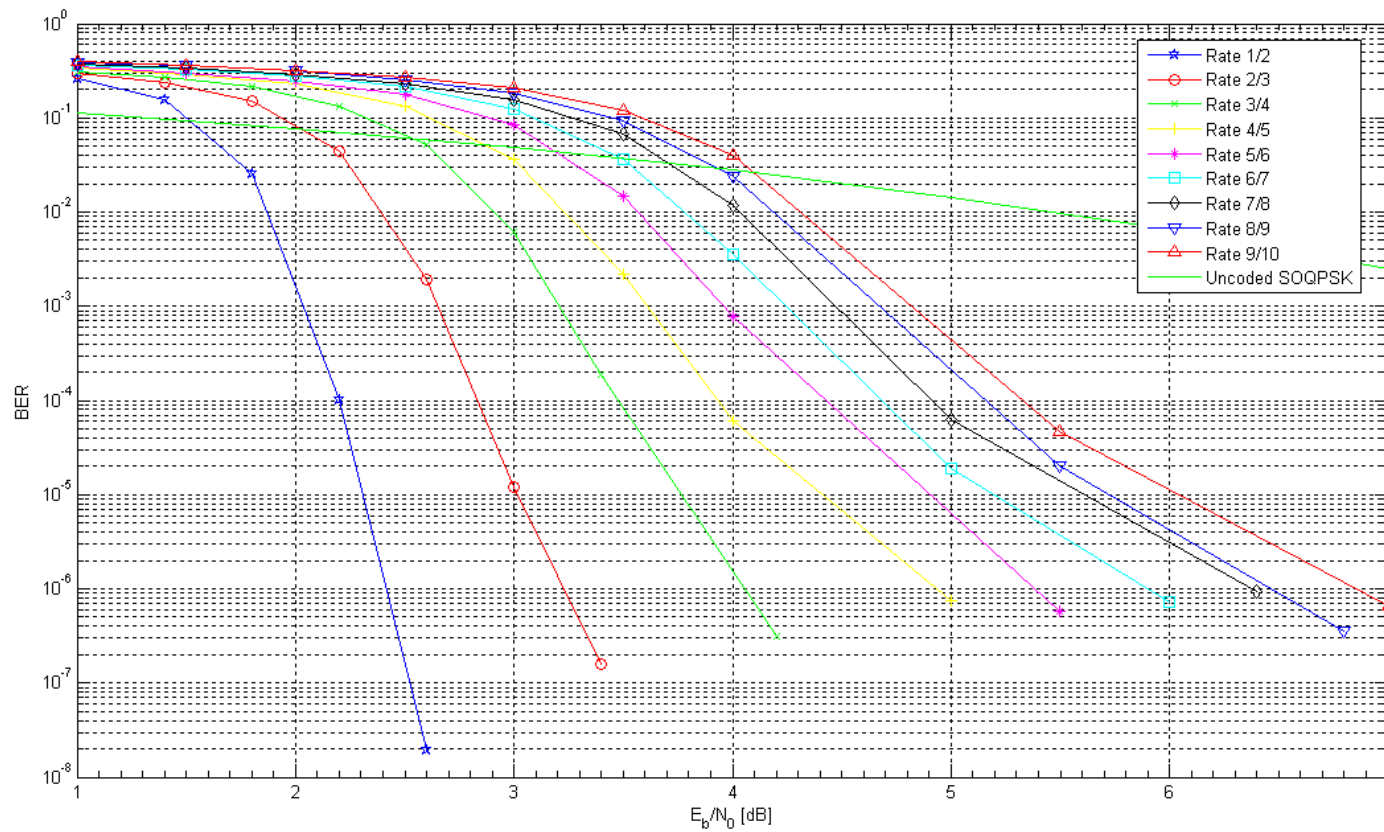
## Appendix - Performance Analysis

- Increase in input block size
  - Input block: 4096 bits.
  - Expected performance: With a large input block - decoding performance increases; complexity increases.

Code	Modulation	Code Rate	BER = $10^{-5}$	Gain dB	Gain dB - 1024 bit block	Difference dB
CC1	SOQPSK-TG	1/2	2.3	8.3	8.0	0.3
CC1	SOQPSK-TG	7/8	5.6	5.0	4.6	0.4
CC2	PCM/FM	1/2	1.6	6.8	6.3	0.5
CC2	PCM/FM	7/8	3.2	5.2	4.6	0.6

## Appendix - Performance Analysis

- CC1 with SOQPSK-TG



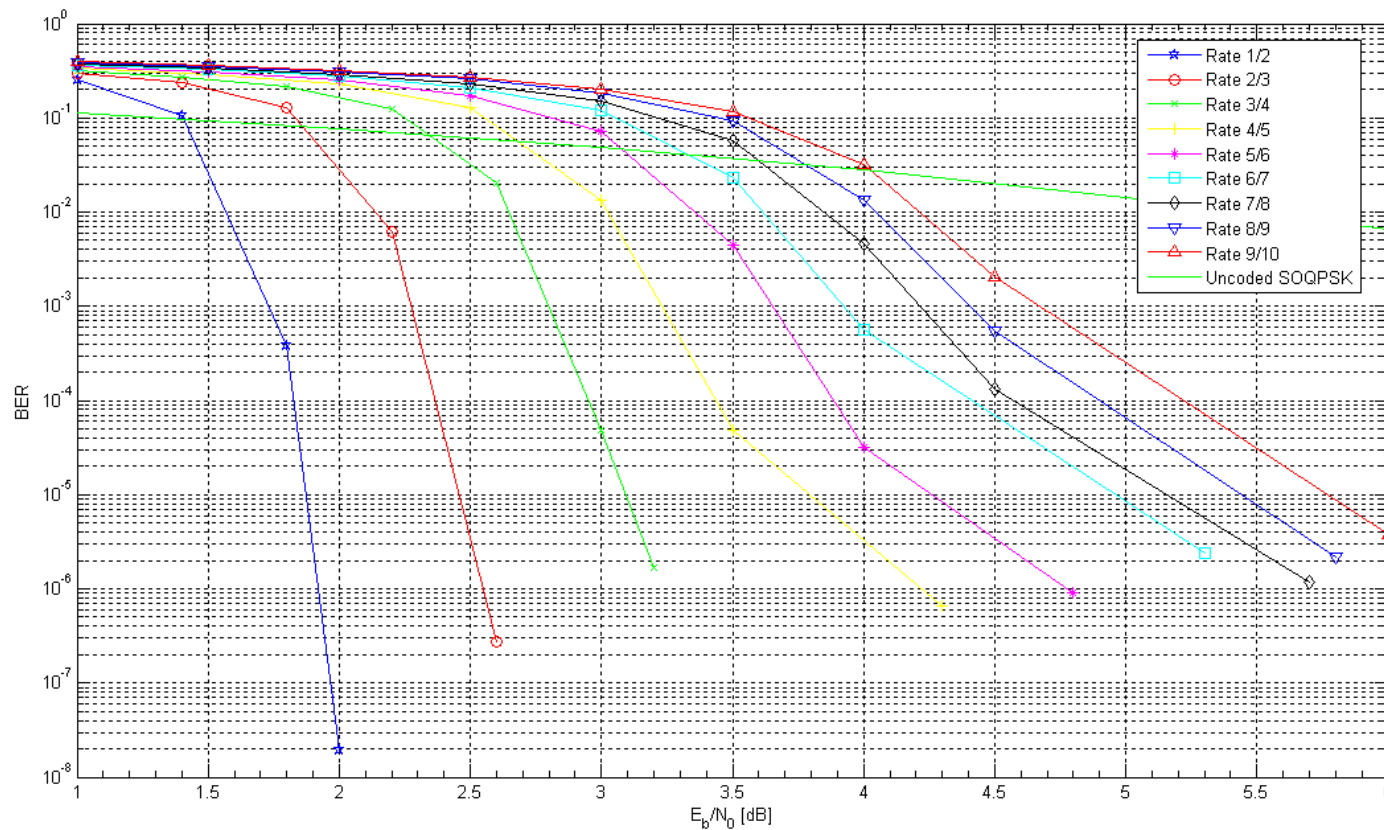
## Appendix - Performance Analysis

- Increase in number of decoding iterations
  - Input Block: 4096 bits; Decoding iterations: 10
  - Expected performance: increase in the number of decoding iterations - better performance; increased complexity

Code	Modulation	Code Rate	BER = $10^{-5}$	Gain dB	Gain dB(4096 bits, 5 iterations)	Difference dB
CC1	SOQPSK-TG	1/2	1.9	8.7	8.3	0.4
CC1	SOQPSK-TG	7/8	5.2	5.4	5.0	0.4
CC2	PCM/FM	1/2	1.4	7.0	6.8	0.2
CC2	PCM/FM	7/8	2.55	5.85	5.2	0.65

## Appendix - Performance Analysis

- CC1 with SOQPSK-TG



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Questions?

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Thank You

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