TIME FREQUENCY ANALYSIS – An Application to FMCW Radars

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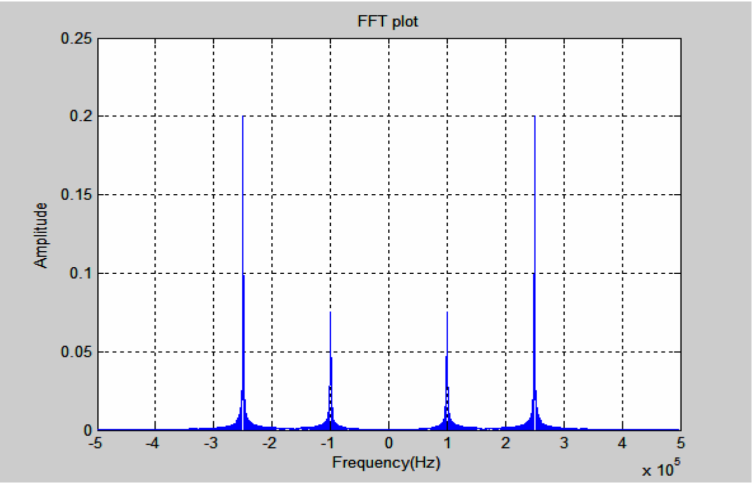
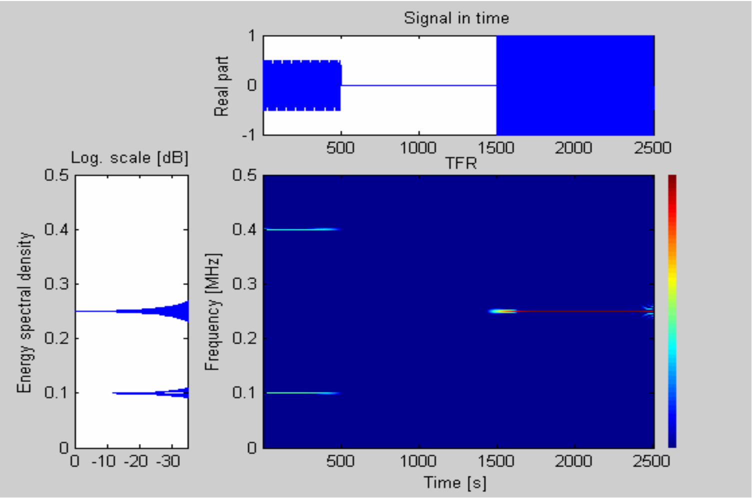
OUTLINE

- Introduction
  - What is Joint Time – Frequency analysis ?
  - Application of JTFA to radar signal processing
- Background
  - FMCW (sea-ice) radar system design & specifications
  - Need for Time – Frequency analysis of radar range profiles
- Time – Frequency Representation
  - Different techniques – classification & description
- Experiments and Results
  - Ideal simulations
  - Sea-ice radar testing
- Conclusions & Future Work

What is Joint Time – Frequency Analysis ?

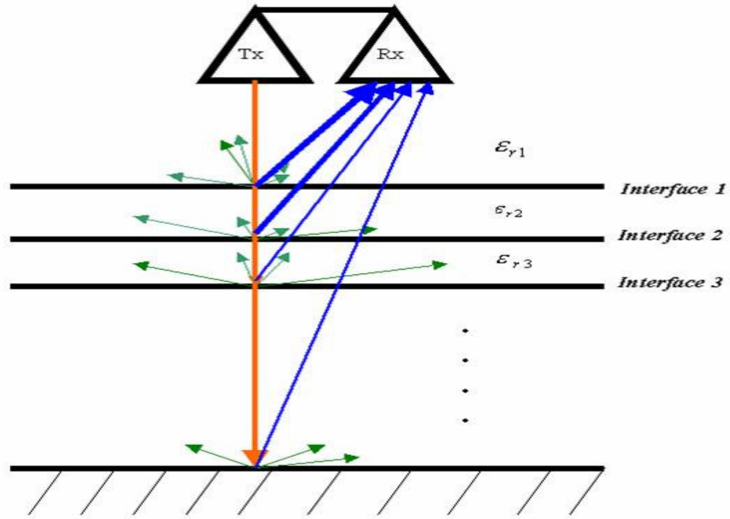
- Fourier Analysis
  - Signal – superposition of weighted sinusoidal functions
  - Frequency attributes are exactly described
- Drawbacks
  - Inability to express signals whose frequency contents change over time
  - Examples – speech & music
- Joint Time – Frequency transforms
  - Characterize behavior of time-varying frequency content of signal
  - Powerful tool for removing noise & interference





Applications of TFD to Radar Signal Processing

- Radar
  - Electromagnetic instrument used for detection & location of targets
  - High-resolution image
    - Transmits electromagnetic energy to target
    - Processes returned signal from target & clutter



- Use of TFD
  - Generate range profile
  - Target recognition features
  - Characterize *'top'* & *'bottom'* of range profiles

FMCW Radar Design

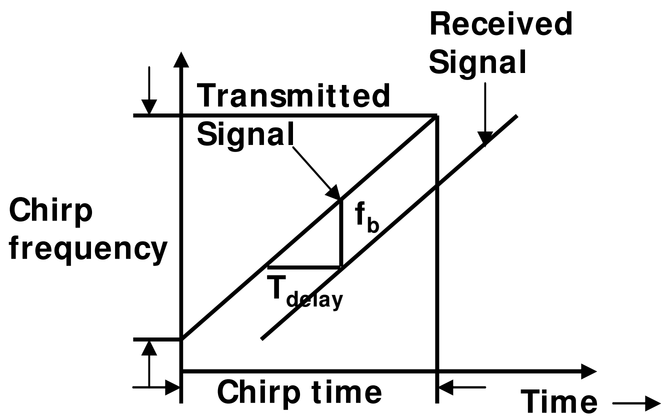
- Background
  - Measurement of sea-ice thickness
  - VHF pulse radars did not have sufficient range resolution

- Frequency Modulated Continuous Wave Radar
  - Developed by RSL at University of Kansas
  - Different types
    - 50 – 250 MHz radar ⇒ thick 1st year/multiyear sea-ice thickness in Arctic region
    - 300 – 1300 MHz radar ⇒ Antarctic region and thin sea-ice in the Arctic

- Design
  - Generates linear chirp signal of frequency 4.5 – 6Ghz & down-converted
  - DAC : 16-bit analog-to-digital converter, sampling beat frequencies at 5MHz

Sea – ice Radar Specifications

Calculation of beat frequency



$$\frac{f_b}{BW} = \frac{T_{delay}}{t_{chirp}}$$

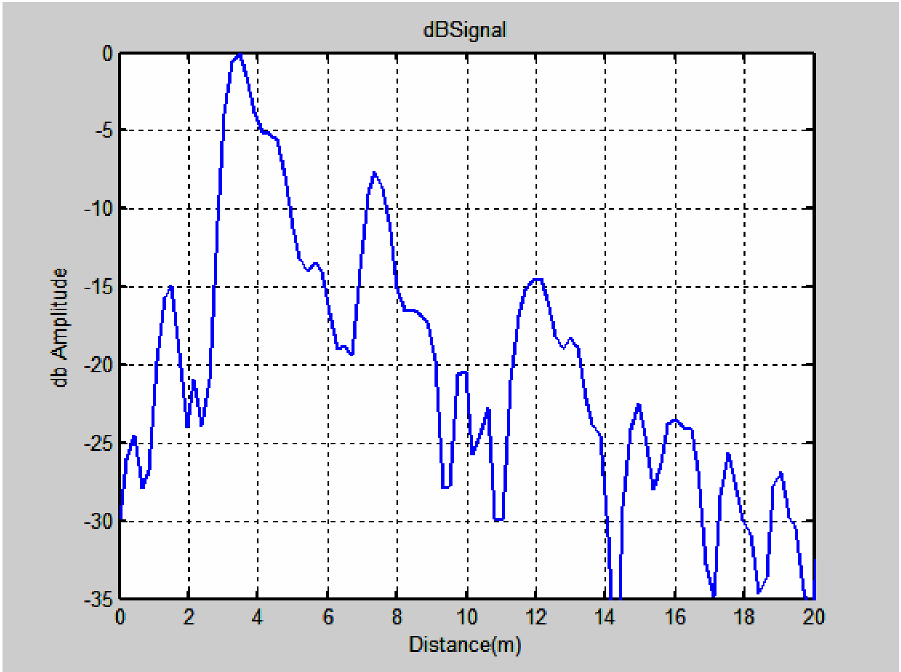
$$T_{delay} = 2R / c$$ where c=3e8 m/sec

$$\Rightarrow f_b = \frac{2R \times BW}{c \times t_{chirp}}$$

System Parameters	Value
Chirp Frequency Range	50 – 250 MHz
Unambiguous Range	3 – 30 meters
Transmit Power	20 dBm
Chirp Time	5 msec
Range Resolution	75 cms

Need for Time – frequency analysis of Radar range profiles

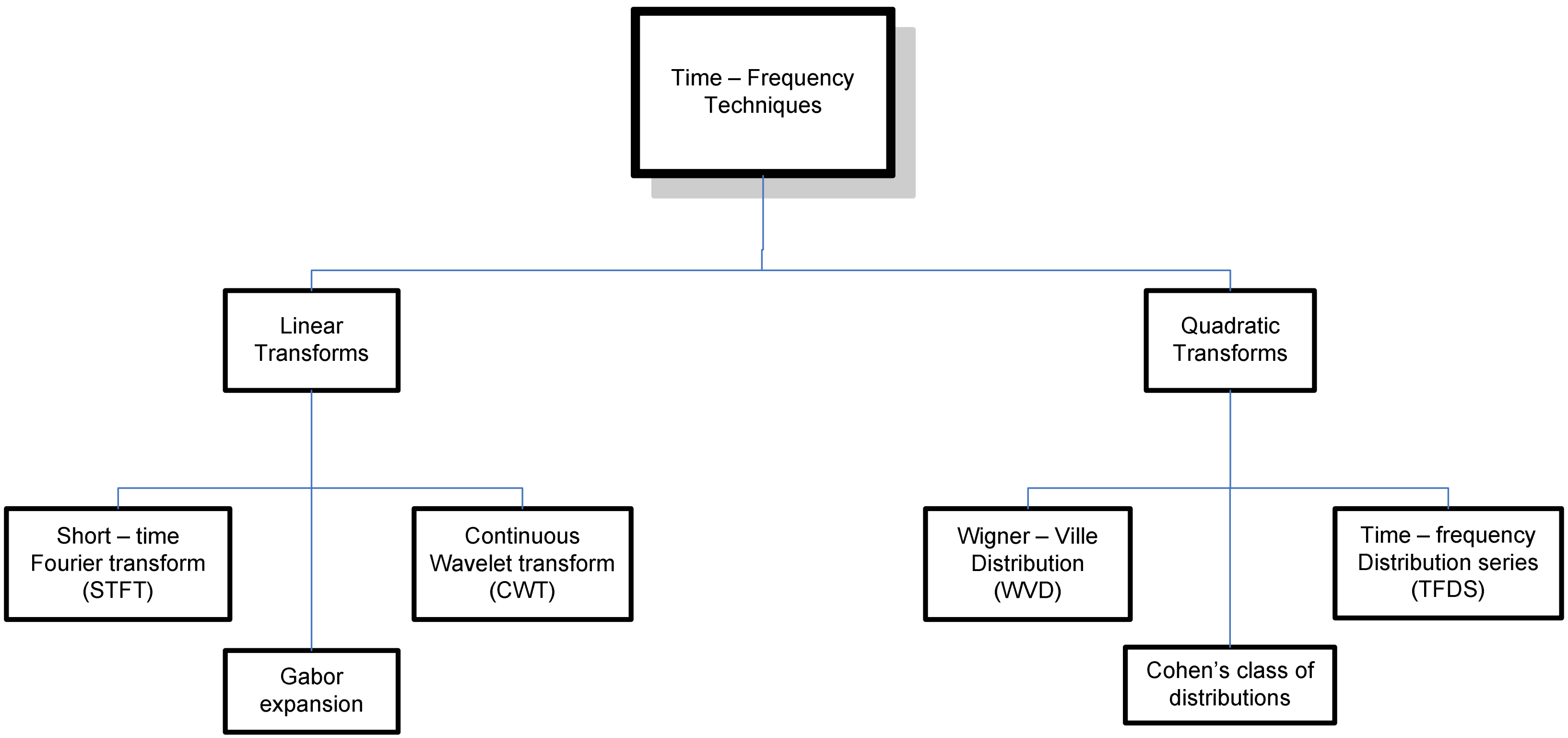
- Fourier Spectrum
  - Variation of signal amplitude in decibels over distance traveled by radar signal
  - Amplitude-scope of sea-ice radar range profile from '*traverse2.bin*'
- Features
  - Signals of varying amplitudes over different distances
  - Highest signal peak at 0dB indicating '*Top*' of range profile
- Drawbacks
  - Prediction of ice-bottom
  - Distinguish surface returns from noise signals and multiples



Need for Time – frequency analysis (contd…)

- Time – frequency spectrum
  - 2 – dimensional analysis
    - Determine range to a target – function of time
    - Measure the target speed – function of frequency
  - Indicates position of different layers
    - Layers are identified by peaks at specific frequencies for all time
    - Attempts to distinguish between *top* and *bottom* of range profiles from other noise signals
  - Time – varying filtering
    - Separating noise from data signal

Time – Frequency Representation

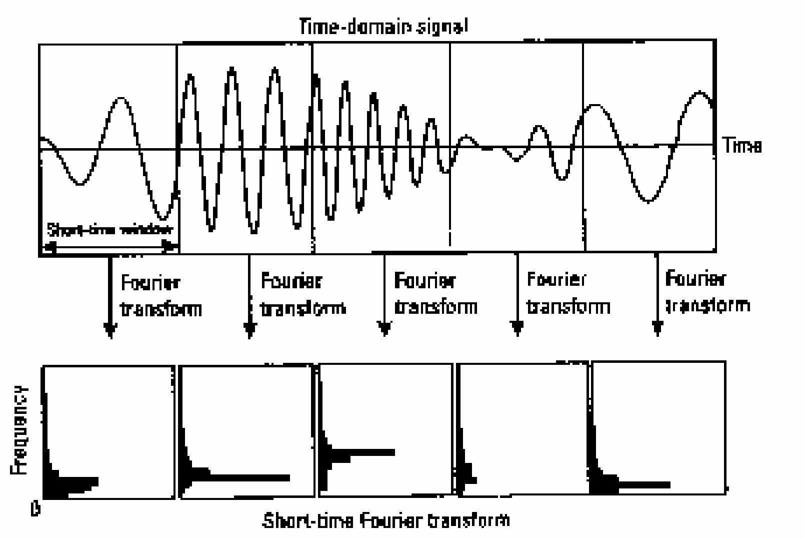


Short Time – Fourier Transform (STFT)

- STFT
  - Modified Fourier transform by comparing signals with elementary functions localized in time & frequency

$$STFT(t,\omega) = \int s(\tau)\gamma^{*}_{t,\omega}(\tau)d\tau = \int s(\tau)\gamma^{*}(\tau - t)e^{-j\omega\tau}d\tau$$

  - Computes the Fourier transform on a block-by-block basis
  - Analysis window function $\gamma(t)$ balances time & frequency resolutions
    - Smaller the time duration of $\gamma(t)$, the better the time resolution (poorer frequency resolution) and vice-versa



Short – time Fourier transform (contd…)

- STFT spectrogram
  - Squared magnitude of STFT
  - Simple & often used time-dependant spectrum
- Signal reconstruction
  - Sampled version of STFT $STFT(mT,n\Omega)=\int_{-\infty}^{+\infty}s(t)\gamma^{*}(t-mT)e^{-jn\Omega t}dt$
    - T, Ω - time & frequency sampling steps
    - Useful in determining relationship between STFT and Gabor expansion

Gabor expansion

- Definition
  - Use coefficients as description of signal's local property $s(t) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} C_{m,n} h_{m,n}(t)$
    - $C_{m,n}$ are the Gabor coefficients
  - Gaussian-type signal was chosen as elementary function

$$g(t) = \left(\frac{\alpha}{\pi}\right)^{\frac{1}{4}} \exp\left\{-\frac{\alpha}{2} t^2\right\}$$

    - Offered optimal joint time-frequency concentration
  - Necessary condition for existence : $T\Omega \le 2\pi$
    - $T\Omega = 2\pi$ - critical sampling (gives most compact representation)
- Relationship with STFT
  - $C_{m,n} = \int s(t)\gamma^*_{m,n}(t)dt = STFT(mT, n\Omega)$ i.e. STFT $\Leftrightarrow$ Gabor coefficient
  - Gabor expansion – inverse of STFT

Continuous Wavelet Transform (CWT)

- Alternative approach to STFT
  - Spectrogram is limited in resolution by extent of sliding window function
- Differences between STFT & CWT
  - Fourier transforms of windowed signals are not taken
  - Width of window changed as transform is computed
- Definition : $$CWT_x^{\psi}(\tau, s) = \Psi_x^{\psi}(\tau, s) = \frac{1}{\sqrt{|s|}} \int x(t)\psi^*\left(\frac{t-\tau}{s}\right)dt$$
  - ψ*(t)* denotes the *mother wavelet* , *s* represents scale index
- Wavelet Denoising
  - Basis is the principle of *'noise decorrelation'*
  - Types – soft thresholding & hard thresholding

Wigner – Ville Distribution (WVD)

- Introduction
  - WVD is defined as $WVD_s(t,\omega) = \int s\left(t+\frac{\tau}{2}\right)s^*\left(t-\frac{\tau}{2}\right)\exp\{-j\omega\tau\}d\tau$
    - where time – dependant autocorrelation function is $R(t,\tau) = s\left(t+\frac{\tau}{2}\right)s^*\left(t-\frac{\tau}{2}\right)$
- Properties
  - Satisfies time marginal & frequency marginal condition
    - $\int_{freq} TFR \rightarrow |s(t)|^2$ i.e. Instantaneous energy of signal at particular instance
    - $\int_{time} TFR \rightarrow |S(\omega)|^2$ i.e. Power spectrum of signal at a particular frequency
  - Mean frequency of WVD at time *t* is equal to signal's weighted average instantaneous frequency
  - Energy of WVD is same as the energy content in signal

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} WVD(t,\omega)dtd\omega = \int_{-\infty}^{\infty}|s(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}|S(\omega)|^2 d\omega$$

Wigner – Ville Distribution (contd…)

- Advantages
  - No window effect
  - Better time & frequency resolutions compared to STFT spectrogram
- Drawbacks
  - Cross – term interference
    - 2 points of TFR interfere to create a contribution on 3rd point located at their geometrical midpoint
    - Oscillate perpendicularly to line joining two points interfering, with a frequency proportional to distance between two points
- Alternatives
  - Cohen's class of distributions
  - Gabor spectrogram

Cohen's class of distributions

- Smoothed Pseudo – WVD
  - Pseudo WVD
    - Windowed version of WVD because of difficulty in determining $R(t,\tau)$

$$PWVD_s(t,\upsilon) = \int_{-\infty}^{\infty} h(\tau)s(t+\tau/2)s^*(t-\tau/2)e^{-j2\pi\upsilon\tau}d\tau$$

    - Equivalent to frequency smoothing of WVD where *h(t)* is a regular window
    - Oscillating nature attenuates interferences
    - Drawback : controlled only by short – time window *h(t)*
  - SPWVD
    - Separable smoothing kernel $\Psi_T(t,f) = g(t)H(f)$ where *g* and *h* are two even windows with $h(0) = G(0) = 1$
    - Progressive and independent control, in both time & frequency

Choi – Williams Distribution

- Kernel design
  - Theory of interference distributions - developed by Choi & Williams
  - Exponential kernel: $\Phi(\vartheta,\tau)=\exp\{-\frac{(\pi\vartheta\tau)^2}{2\sigma^2}\}$ where σ is scaling parameter
- Properties
  - Suppresses the cross-terms created by two functions having different time & frequency centers
  - σ controls the decay speed
    - as σ decreases the interference is reduced
    - When $\sigma \to \infty$ we obtain the WVD.
  - Essentially a low – pass filter in (ν,τ) plane which preserves properties of WVD while reducing cross-term interference

Time – Variant Filter

- Application of TFR
  - Detection & estimation of noise-corrupted signals
  - SNR is substantially improved in joint time-frequency domain
- Filtering mechanism
  - Based on both linear & bilinear time-frequency representations
  - Gabor expansion-based filter is most widely used
- Techniques
  - Least Square Error (LSE) filter
  - Iterative Time – Variant Filter

Experiments & Results – Outline

- Ideal Simulations
  - Sum of frequency tones
  - Linear chirp signal
- Sea – ice radar data
  - Measured depth from field tests
  - How does TFD distinguish surface return from noise ?
- Time – frequency techniques
  - Linear transforms – STFT
  - Quadratic transforms – WVD, SPWVD, CWD
- Time – variant filtering
  - Drawbacks of aforementioned techniques
  - Wavelet denoising

Ideal Simulations

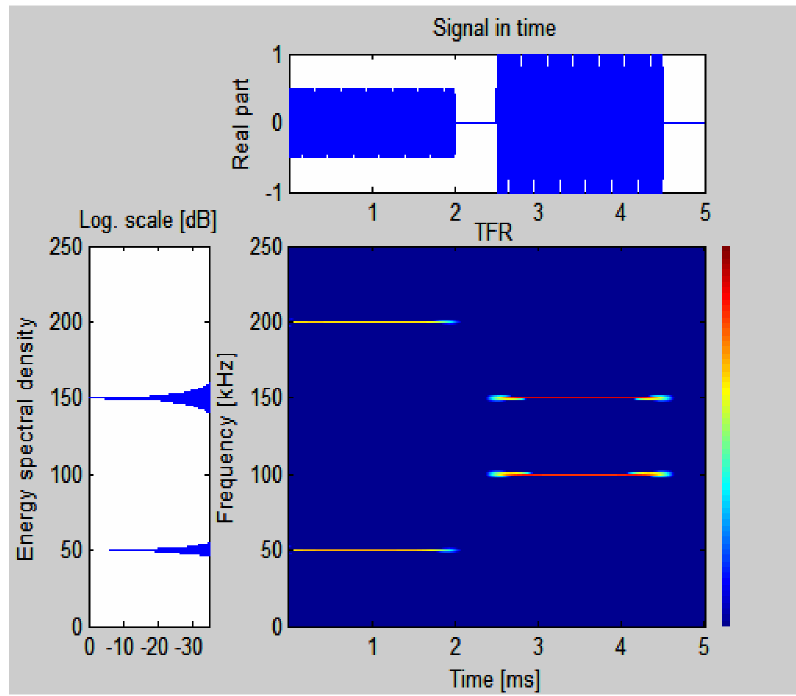
- Test of TFR with *cosine* signal
  - Input frequency tones :

$$x_1[n] = a\cos(2*\pi*f_1*n_1Ts), a = 0.5; f_1 = 50KHz$$
$$x_2[n] = b\cos(2*\pi*f_2*n_2Ts), b = 1; f_2 = 150KHz$$
$$x[n] = x_1[n] + x_2[n]$$

where $n_1 = 0:999, n_2 = 1250:2249$ and

$$f_s = 1/Ts = 500Khz$$

  - Power spectrum does not indicate when frequency tones occur



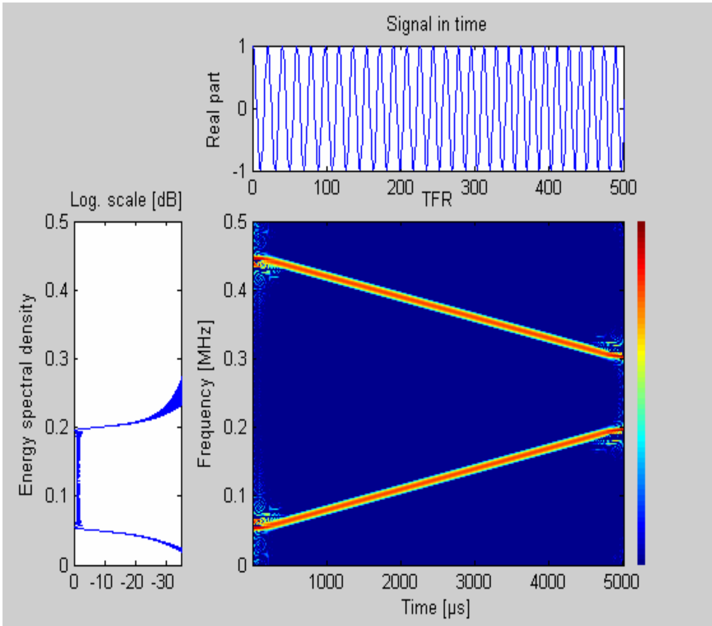
- TFR results
  - Frequency tones at 50KHz & 150KHz varying from (0-2ms), (2.5-4.5ms)
  - Image frequencies at 200KHz and 100KHz respectively
  - Differences in amplitudes indicated by respective colormap scales of frequency tones

Ideal Simulations (contd…)

- Test of TFR with *chirp* signal
  - Input chirp signal:

$$s(t) = \cos(2\pi(f_0 t + \frac{1}{2}\alpha . t^2)), \alpha = \frac{f_1 - f_0}{T}$$

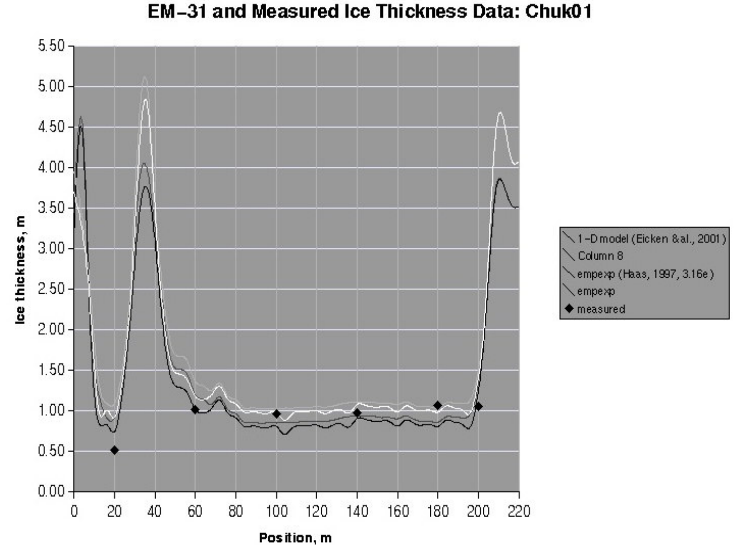
where $f_0 = 50Khz, f_1 = 200Khz, T = 5m\sec$



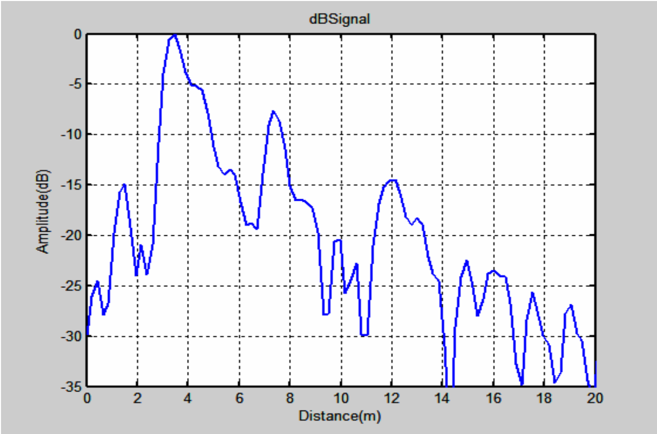
- TFR results
  - SPWVD applied to linear swept-frequency signal
  - Signal with linearly varying frequency for full duration of time of 5msec
  - Image frequency shown as another chirp from 450-300KHz

Sea – ice radar experimental data

- Sea-ice (FMCW) radar
  - Data set from field experiments in Barrow, Alaska
  - Measured sea-ice depth compared with depth calculated from signal processing experiments



- Ice thickness data
  - Field experiments show the measured ice thickness at various depths
  - Ascope-60 of file *traverse2.bin* at distance of 0-20m from 1st point
  - Calculations suggest
    - Antenna feedthrough – 3.45m
    - Ice bottom – 7.35m

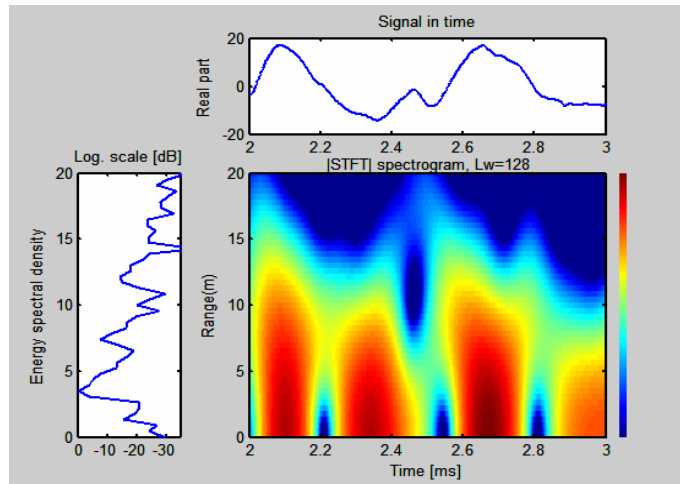


How does TFD distinguish surface return from noise ?

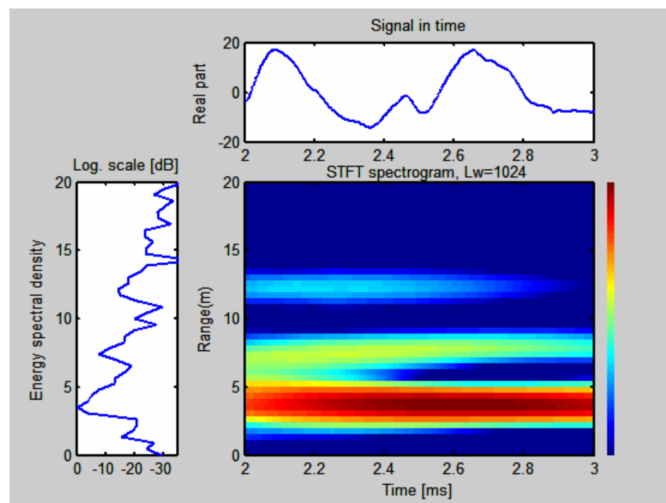
- Frequency is expressed as function of distance or range
- Time – dependant spectrum expresses variation of beat signal at different instances of time for a given frequency
- Presence of surface return
  - Signal exists for entire duration of time interval at given frequency
  - Otherwise, signal is assumed to be noise or multiple return

STFT – based Spectrogram

- Narrow Window
  - Good time resolution & poor frequency resolution
  - Peaks are well separated from each other in time
  - In frequency domain, every peak covers a range of frequencies instead of a single frequency

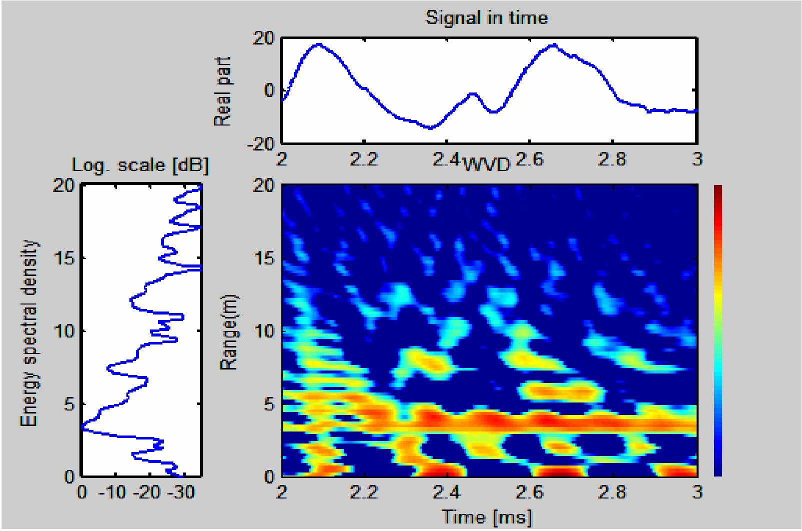


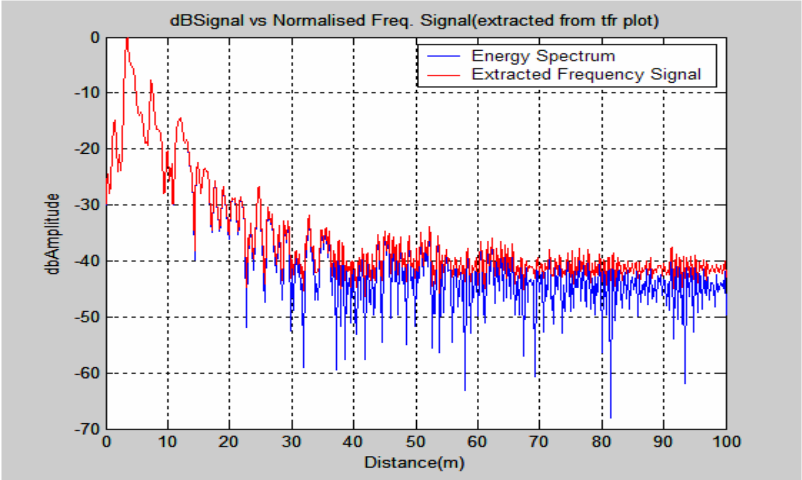
- Wide Window
  - Good frequency resolution & poor time resolution
  - Frequency resolution is much better with continuous variation in time
  - In time domain, peaks are not observed



Wigner – Ville Distribution

- *Top* of the range profile
  - Observed at distance of around 3.5m
  - Varying over all instances of time (high colormap scale)
- *Ice - bottom*
  - Observed at distance of around 7.5m
  - Yellow colormap scale which is 6dB lower than highest scale
- Drawbacks
  - Suffers from cross-term interference effects
- Best performance
  - Energy distribution being optimally concentrated in the joint time-frequency domain



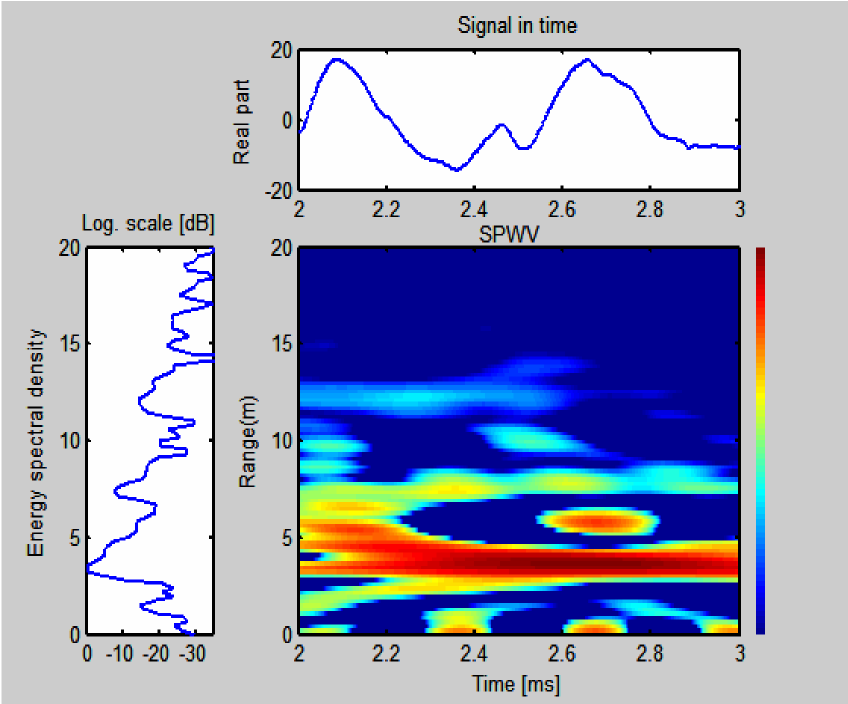


Smoothed Pseudo WVD

- Defined by *smoothing kernel*

  $$\psi_T(t,f) = g(t)H(f)$$

  - *g & h are time and frequency smoothing windows respectively*
- Trade – off
  - *Improves* the cross-term interference at the cost of lower resolution
  - More the smoothing in time and/or frequency, the poorer the resolution in time and/or frequency
- Surface returns
  - clearly visible

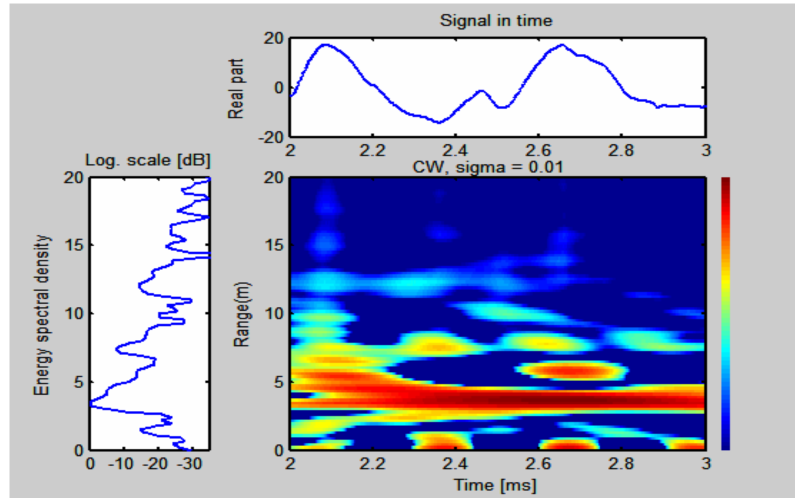


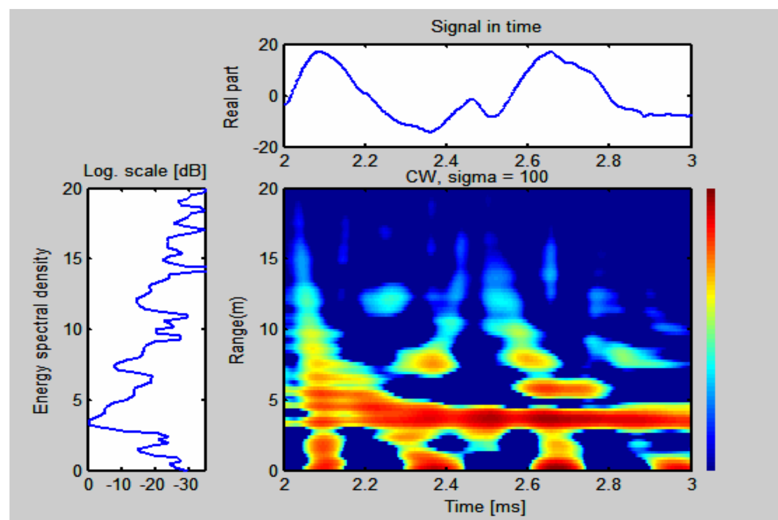
Choi – Williams Distribution

- Employs the exponential kernel

  $$\psi(v,\tau) = \exp\{-v^2\tau^2 / \sigma\}$$

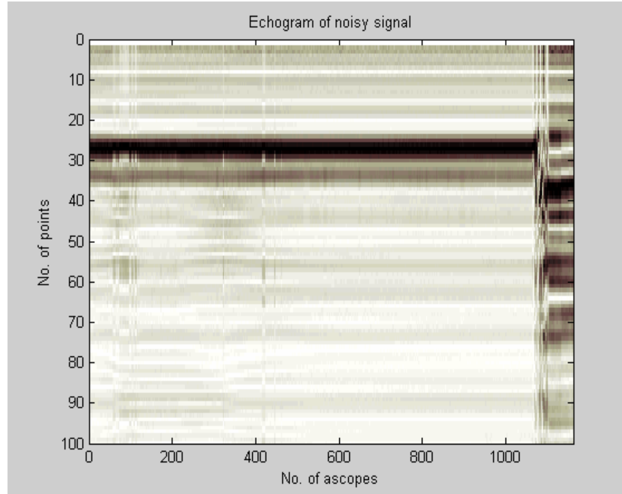
  where σ is a scaling factor
- Effect of σ :
  - $\sigma = 0.01$
    - cross-terms diminish in size
    - width of the signal component spreads
    - surface returns distinguished easily
    - mild loss in resolution
  - $\sigma \rightarrow \infty$
    - approaches the Wigner transform, since the kernel is nearly constant
    - interference terms become more prominent
    - Frequency & time resolution are comparable to that of WVD

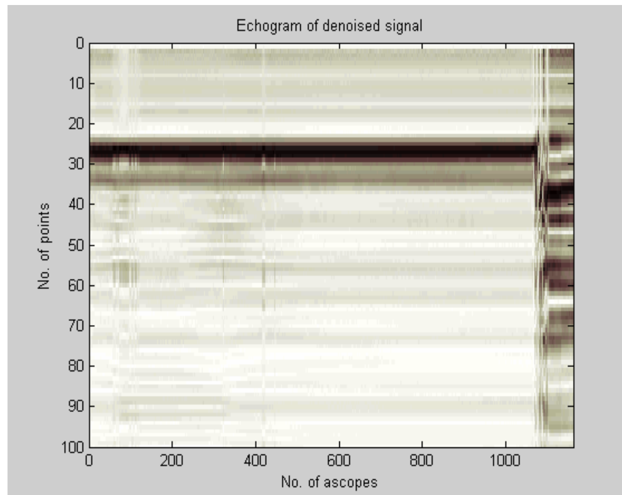




Time – Variant Filtering

- Time – variant denoising
  - Investigated for FMCW radar signals
  - Discrete Gabor transform is used
  - Not suitable for radar chirp signals
- Alternative
  - Wavelet transforms can be used
  - Currently used for '*depth sounder radar'* in RSL
- Wavelet denoising
  - Radar echogram showing the noisy signal
  - SNR of denoised signal : 1.4 dB (clean signal)





CONCLUSIONS

- Comparison between Fourier analysis & Joint time – frequency analysis
- Time – frequency analysis
  - Classification
  - Need for TFA of radar range profiles
- Signal processing experiments
  - STFT spectrogram – worst resolutions
  - WVD – best performance / optimal concentration in joint time-frequency domain
    - surface returns clearly visible
    - Depth from radar matched that of measured depth
  - Cohen's class of distributions – compromise between interference reduction & loss in resolution
- Time – variant filtering
  - Discrete Gabor transform cannot be used

FUTURE WORK

- Wavelet denoising can be investigated for FMCW radars
- Time – variant filtering can be attempted for other radar signals
  - Particularly for moving targets
- Applications of Time – frequency analysis
  - Speech & music signal processing

THANK YOU

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