Decision Weighted Adaptive Algorithms with Applications to Wireless Channel Estimation

Shane Martin Haas

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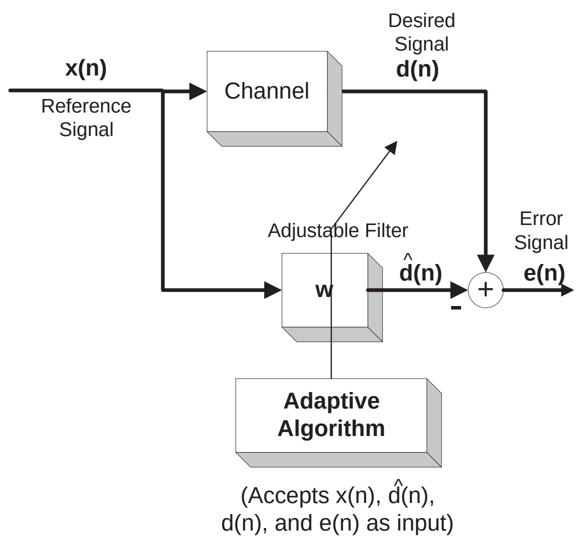
Department of Electrical Engineering and Computer Science
University of Kansas

Introduction

⇒ What is an Adaptive Algorithm?

⇒ What is System Identification?



⇒ Benefits of Channel Estimation

⇒ Training Sequence Versus Blind Estimation

Presentation Overview

⇒ Theoretical Development

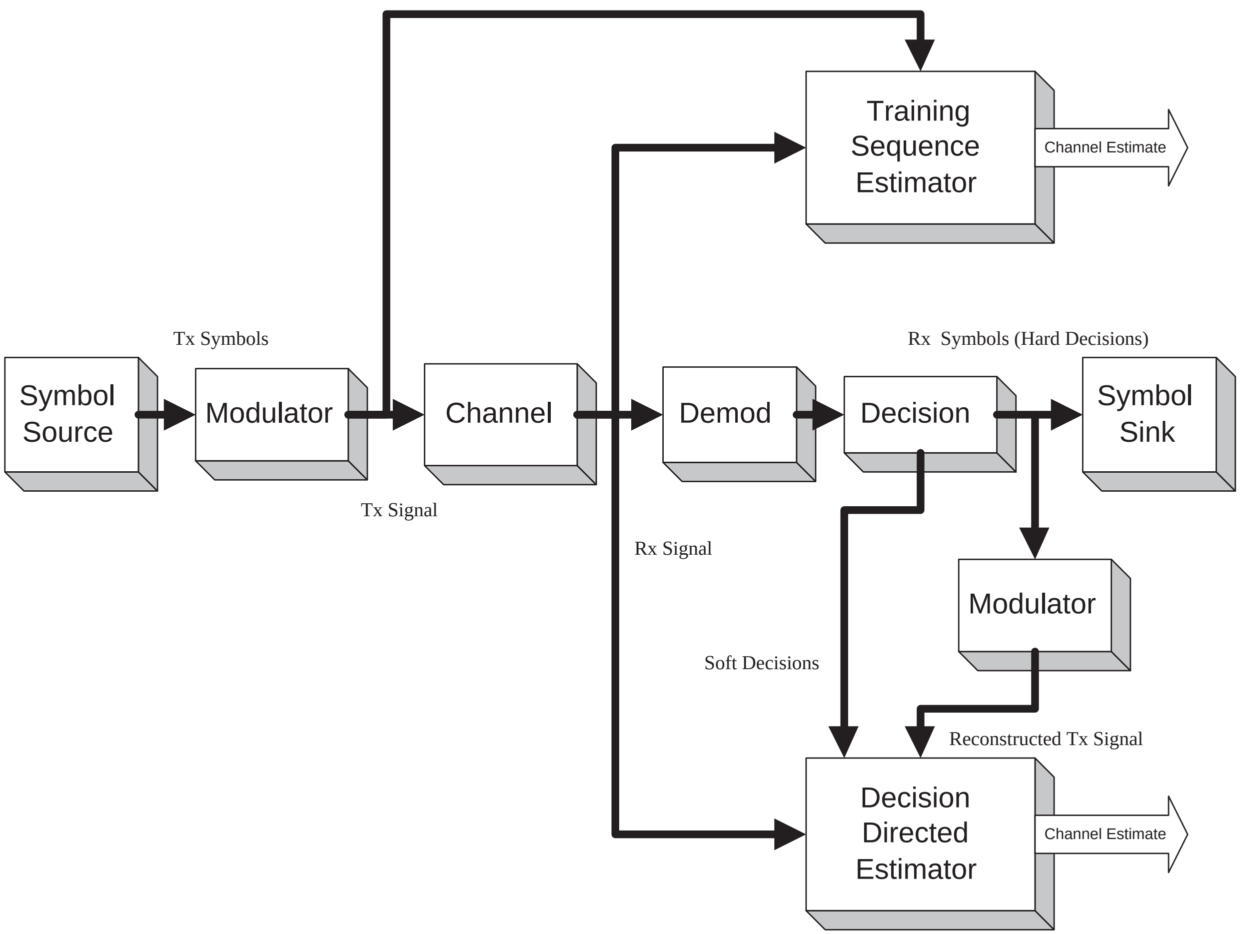
- Problem Formulation
- Multiple Phase Shift Keying
- Characterizing Wireless Communication Channels
- Bandpass to Low-Pass Conversion of Signals and Systems
- Adaptive Algorithms
  - Linear and LMS Estimation Algorithms
  - Properties of Decision Weighted Algorithms

⇒ Simulation Methodology

⇒ Simulation Results

⇒ Conclusions

Problem Formulation



Multiple Phase Shift Keying (MPSK)

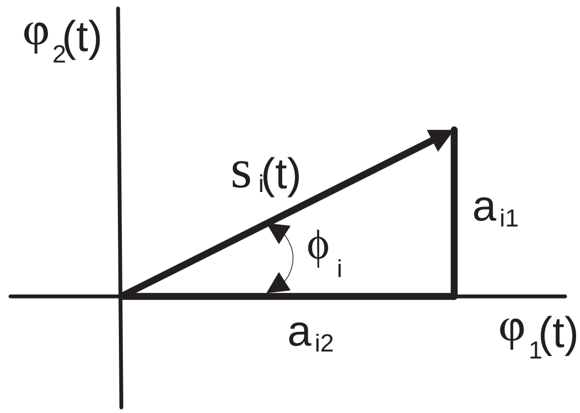
⇒ Modulation

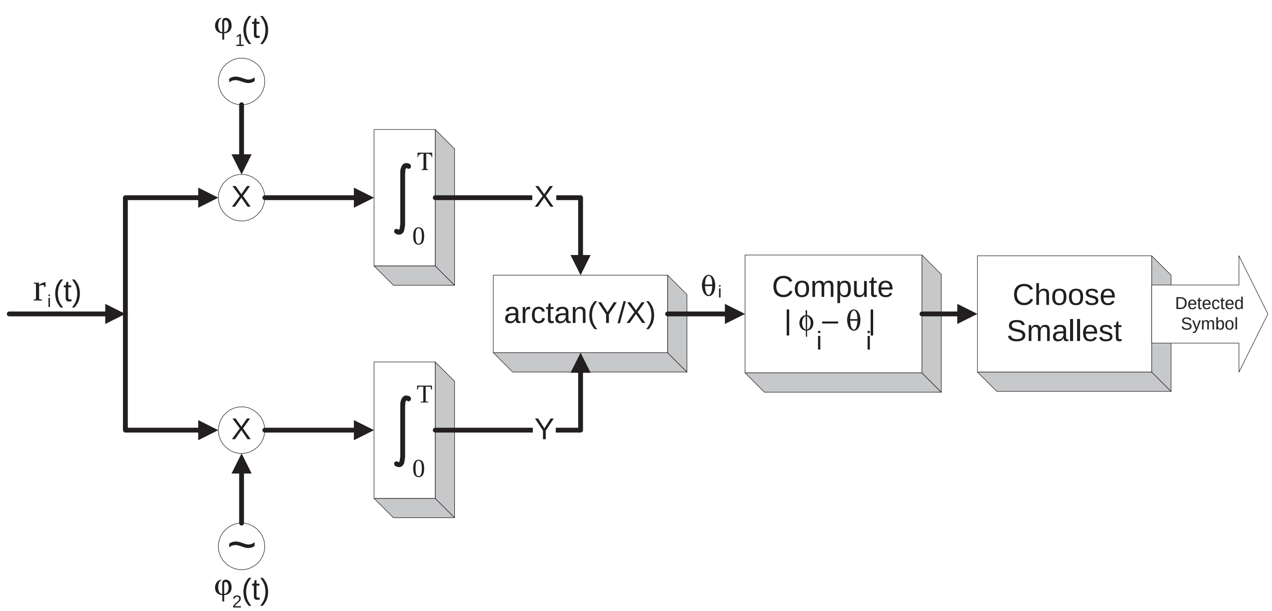
$$
\begin{aligned}
s_i(t) &= \sqrt{\frac{2E}{T}}\cos\left(2\pi f_c t + \frac{2\pi i}{M}\right) \qquad \text{for } 0 \le t < T \\
&= a_{i1}\psi_1(t) + a_{i2}\psi_2(t)
\end{aligned}
$$

with

$$
\begin{aligned}
\psi_1(t) &= \sqrt{\frac{2}{T}}\cos(2\pi f_c t) \\
\psi_2(t) &= -\sqrt{\frac{2}{T}}\sin(2\pi f_c t) \\
a_{i1} &= \sqrt{E}\cos(\phi_i) \\
a_{i2} &= \sqrt{E}\sin(\phi_i) \\
\phi_i &= \frac{2\pi i}{M}
\end{aligned}
$$

⇒ Demodulation

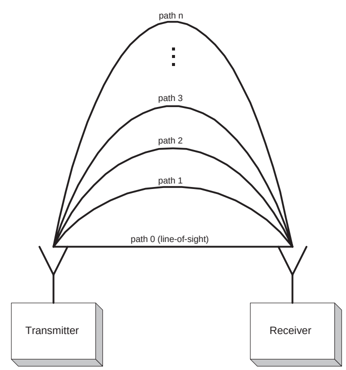




Characterizing Wireless Communication Channels

⇒ Multipath

$$y(t) = \sum_n \alpha_n(t) s(t - \tau_n(t))$$



⇒ Channel Models

* Radio Relay Three-Path (Rummler) Model
* Mobile Radio Channel Model

Bandpass to Low-Pass Conversion

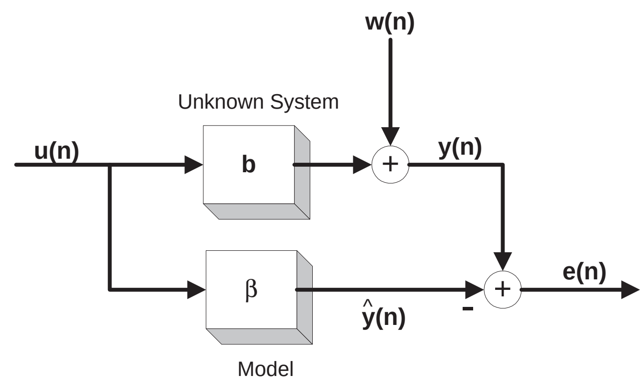
⇒ The Complex Envelope of the MPSK Signalling Waveform

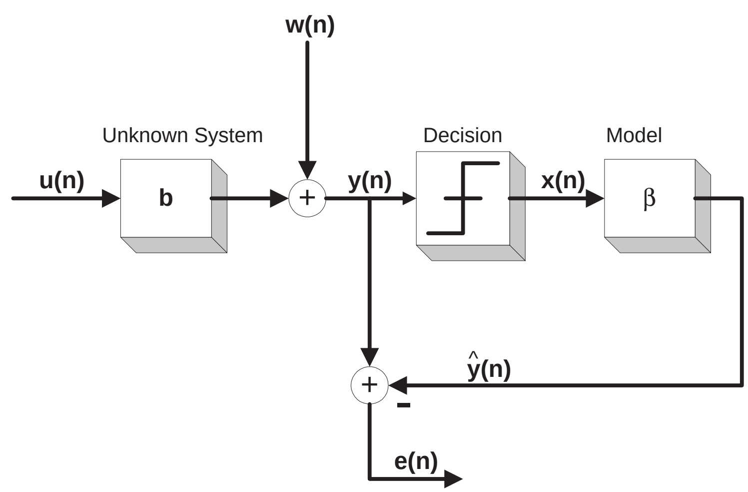
$$\widetilde{s}_i(t) = \sqrt{\frac{2E}{T}} \cos\left(\frac{2\pi i}{M}\right) + j\sqrt{\frac{2E}{T}} \sin\left(\frac{2\pi i}{M}\right)$$

⇒ Main Idea: Convolution of real bandpass signals is the same as the convolution of their complex envelope low-pass equivalents

Adaptive Algorithms

⇒ System Identification Problems





$\Rightarrow$ Linear Estimators

Define for the training sequence estimation problem:

$$\begin{aligned} y(n) &= b_1 u_1(n) + \cdots + b_M u_M(n) + w(n) \\ \widehat{y}(n) &= \beta_1 u_1(n) + \cdots + \beta_M u_M(n) \\ e(n) &= y(n) - \widehat{y}(n) = y(n) - (\beta_1 u_1(n) + \cdots + \beta_M u_M(n)) \end{aligned}$$

Define for the decision directed estimation problem:

$$\begin{aligned} y(n) &= b_1 u_1(n) + \cdots + b_M u_M(n) + w(n) \\ \widehat{y}(n) &= \beta_1 x_1(n) + \cdots + \beta_M x_M(n) \\ e(n) &= y(n) - \widehat{y}(n) = y(n) - (\beta_1 x_1(n) + \cdots + \beta_M x_M(n)) \end{aligned}$$

Observe the system for N sample periods and write

$$
\begin{aligned}
\mathbf{y} &= \begin{bmatrix} y(1) & y(2) & \cdots & y(N) \end{bmatrix}^T \\
\mathbf{w} &= \begin{bmatrix} w(1) & w(2) & \cdots & w(N) \end{bmatrix}^T \\
\mathbf{e} &= \begin{bmatrix} e(1) & e(2) & \cdots & e(N) \end{bmatrix}^T \\
\mathbf{b} &= \begin{bmatrix} b_1 & b_2 & \cdots & b_M \end{bmatrix}^T \\
\beta &= \begin{bmatrix} \beta_1 & \beta_2 & \cdots & \beta_M \end{bmatrix}^T \\
\mathbf{U} &= \begin{bmatrix} u_1(1) & \cdots & u_M(1) \\ \vdots & \ddots & \vdots \\ u_1(N) & \cdots & u_M(N) \end{bmatrix} \\
\mathbf{X} &= \begin{bmatrix} x_1(1) & \cdots & x_M(1) \\ \vdots & \ddots & \vdots \\ x_1(N) & \cdots & x_M(N) \end{bmatrix}
\end{aligned}
$$

The channel output is

$$\mathbf{y} = \mathbf{U}\mathbf{b} + \mathbf{w}$$

The error for the training sequence estimation problem is

$$\mathbf{e} = \mathbf{y} - \mathbf{U}\beta$$

While that for the decision directed estimation is

$$\mathbf{e} = \mathbf{y} - \mathbf{X}\beta$$

The error or loss function is

$$\mathbf{J}(\beta) = \mathbf{e}^{\mathbf{T}}\mathbf{R}\mathbf{e}$$

where $\mathbf{R}$ is a N x N matrix of weighting coefficients.

⇒ Linear Estimator

$$\widehat{\beta} = \left(\mathbf{U^T R U}\right)^{-1} \mathbf{U^T R y}$$

⇒ Recursive Weighted Least Squares Estimator

$$\mathbf{u}(n) = \begin{bmatrix} u_1(n) & u_2(n) & \cdots & u_M(n) \end{bmatrix}^T$$

$$\mathbf{R} = diag(\lambda^{n-1}a_1, \ldots, \lambda a_{n-1}, a_N) \qquad 0 < \lambda \leq 1$$

$$\begin{aligned} \widehat{\beta}_n &= \widehat{\beta}_{n-1} + a_n \mathbf{H}_n^{-1} \mathbf{u}(n) e(n) \\ \mathbf{H}_n &= \lambda \mathbf{H}_{n-1} + a_n \mathbf{u}(n) \mathbf{u}^T(n) \\ e(n) &= y(n) - \mathbf{u}(n)^T \widehat{\beta}_{n-1} \end{aligned}$$

⇒ Least Mean Squares (LMS) Estimator

$$\begin{aligned} \widehat{\beta}_n &= \widehat{\beta}_{n-1} + \mu\rho_n \mathbf{u}(n)e(n) \\ e(n) &= y(n) - \mathbf{u}(n)^T \widehat{\beta}_{n-1} \end{aligned}$$

⇒ Decision weighted estimators are decision directed estimators whose weights depend on the quality of the decisions.

* Ideal decision weighted estimators use knowledge of decision errors to calculate their weights. Specifically,

$$\mathbf{X^T R X} = \mathbf{X^T R U}$$

* Soft decision weighted estimators use receiver soft decisions to calculate their weights.

⇒ More on Ideal Decision Weighted Linear Estimators

* **Q**: How does one choose $\mathbf{R}$ such that $\mathbf{X^T R X} = \mathbf{X^T R U}$?
* **A**: If $\mathbf{X}$ and $\mathbf{U}$ differ in the jth *row*, choose the jth *column* of $\mathbf{R}$ orthogonal to each column in $\mathbf{X}$.
* **Q**: Are $\mathbf{X^T R X} = \mathbf{X^T R U}$ and $\mathbf{X^T R X}$ non-singular conflicting conditions?
* **A**: No, let $\mathbf{R}$ be an identity matrix with its jth column set to zero if $\mathbf{X}$ and $\mathbf{U}$ differ in the jth row. Under slightly more restrictive assumptions placed on $\mathbf{X}$ than in ordinary training sequence estimators, $\mathbf{X^T R X}$ is non-singular.

⇒ More on Soft Decision Weighted Estimators

For MPSK modulation define a soft decision as

$$p_i = 1 - \frac{|\phi_i - \theta_i|}{\pi/S}$$

A possible choice for the soft decision weight is

$$a_n = p_n p_{n-1} \cdots p_{n-M+1}$$

⇒ Biasness of Decision Directed Linear Estimators

If $\mathcal{E}\{\mathbf{w}|\mathbf{X},\mathbf{U}\} = \mathbf{0}$ then

$$\mathcal{E}\left\{\widehat{\beta}\right\} = \mathcal{E}\left\{\left(\mathbf{X}^{\mathbf{T}}\mathbf{R}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathbf{T}}\mathbf{R}\mathbf{U}\right\}\mathbf{b}$$

For ideal decision weighted estimators $\mathbf{X}^{\mathbf{T}}\mathbf{R}\mathbf{X} = \mathbf{X}^{\mathbf{T}}\mathbf{R}\mathbf{U}$, and therefore the estimator is unbiased.

⇒ Covariance of Decision Directed Linear Estimators

If $\mathcal{E}\{\mathbf{w}|\mathbf{X},\mathbf{U}\} = \mathbf{0}$ then

$$\mathbf{cov}\left\{\widehat{\beta}\right\} = \mathbf{cov}\{\mathbf{S}\mathbf{U}\mathbf{b}\} + \mathcal{E}\left\{\mathbf{S}\mathbf{V}\mathbf{S}^{\mathbf{T}}\right\}$$

where $\mathbf{S} = \left(\mathbf{X}^{\mathbf{T}}\mathbf{R}\mathbf{X}\right)^{-1}\mathbf{X}^{\mathbf{T}}\mathbf{R}$ and $\mathbf{V} = \mathcal{E}\left\{\mathbf{w}\mathbf{w}^{\mathbf{T}} \mid \mathbf{X},\mathbf{U}\right\}$. Notice for ideal decision weighted estimators $\mathbf{S}\mathbf{U} = \mathbf{I}$.

Simulation Methodology

⇒ Algorithm Summary

* **Training Sequence LMS (TLMS):** Uses training sequence LMS with $\rho_n = 1$
* **Blind LMS (BLMS):** Uses decision directed LMS with $\rho_n = 1$
* **Soft Decision Weighted LMS (SDWLMS):** Uses decision directed LMS with soft decision weights
* **Ideal Decision Weighted LMS (IDWLMS):** Uses decision directed LMS with $\rho_n = 1$ if $\mathbf{x}(\mathbf{n}) = \mathbf{u}(\mathbf{n})$ and zero otherwise
* **Training Sequence RLS (TRLS):** Uses training sequence WRLS with $a_n = 1$
* **Blind RLS (BRLS):** Uses decision directed WRLS with $a_n = 1$
* **Soft Decision Weighted RLS (SDWRLS):** Uses decision directed WRLS with soft decision weights

* **Ideal Decision Weighted RLS (IDWRLS):** Uses decision directed WRLS with $a_n = 1$ if $\mathbf{x}(\mathbf{n}) = \mathbf{u}(\mathbf{n})$ and zero otherwise
* **Modified Soft Decision Weighted RLS (MSDWRLS):** Uses decision directed WRLS with soft decision weights; however, we modify the matrix update $\mathbf{H}_n$ by removing the weight a_n, resulting in $\mathbf{H}_n = \lambda \mathbf{H}_{n-1} + \mathbf{x}(n)\mathbf{x}^T(n)$.
* **Modified Ideal Decision Weighted RLS (MSDWRLS):** Uses decision directed WRLS with $a_n = 1$ if $\mathbf{x}(\mathbf{n}) = \mathbf{u}(\mathbf{n})$ and zero otherwise; however, we modify the matrix update $\mathbf{H}_n$ by removing the weight a_n, resulting in $\mathbf{H}_n = \lambda \mathbf{H}_{n-1} + \mathbf{x}(n)\mathbf{x}^T(n)$.

⇒ General Methods for Delay Spread, SNR, and Doppler Frequency Tests

* **LMS Gain:** $\mu = 0.3$
* **RLS Forgetting Factor:** $\lambda = 0.99$
* **Sampling Rate:** 1 sample per second
* **Symbol Interval:** 4 samples per symbol
* **Modulation:** QPSK
* **Number of Symbols per Individual Simulation:** 300 symbols
* **Number of Individual Simulations to Perform per Test Point Iteration:** 20 simulations
* **Maximum Symbol Error Rate (SER):** 0.2
* **Number of Symbols to Skip Before Calculating Estimation Error (N_0):** 100 symbols
* **Initial Estimate:** the true response
* **Performance Criteria:** median average estimation error

Simulation Results

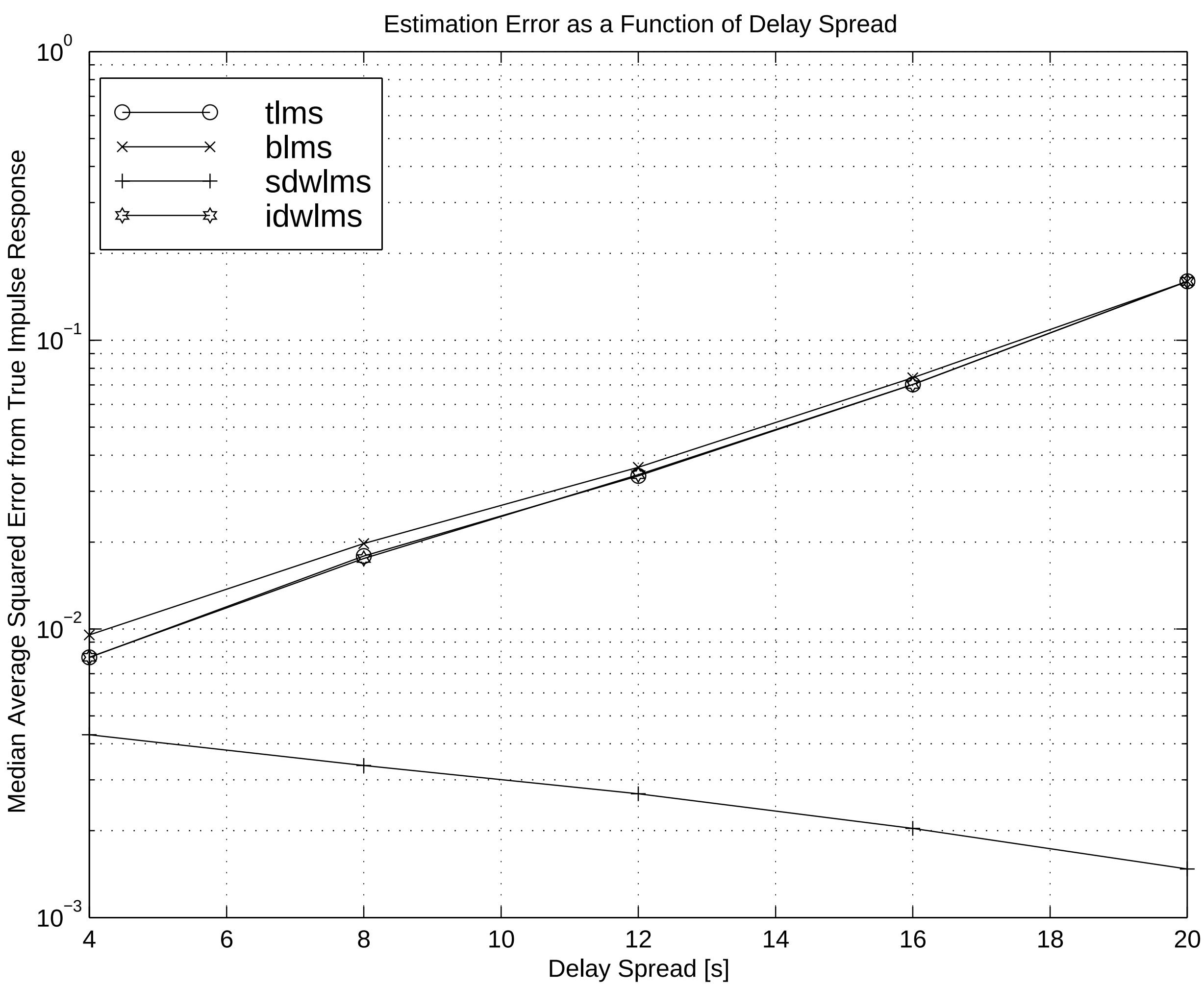


Figure 1: Median of the average squared error of LMS algorithms as a function of delay spread (SNR = 10 dB)

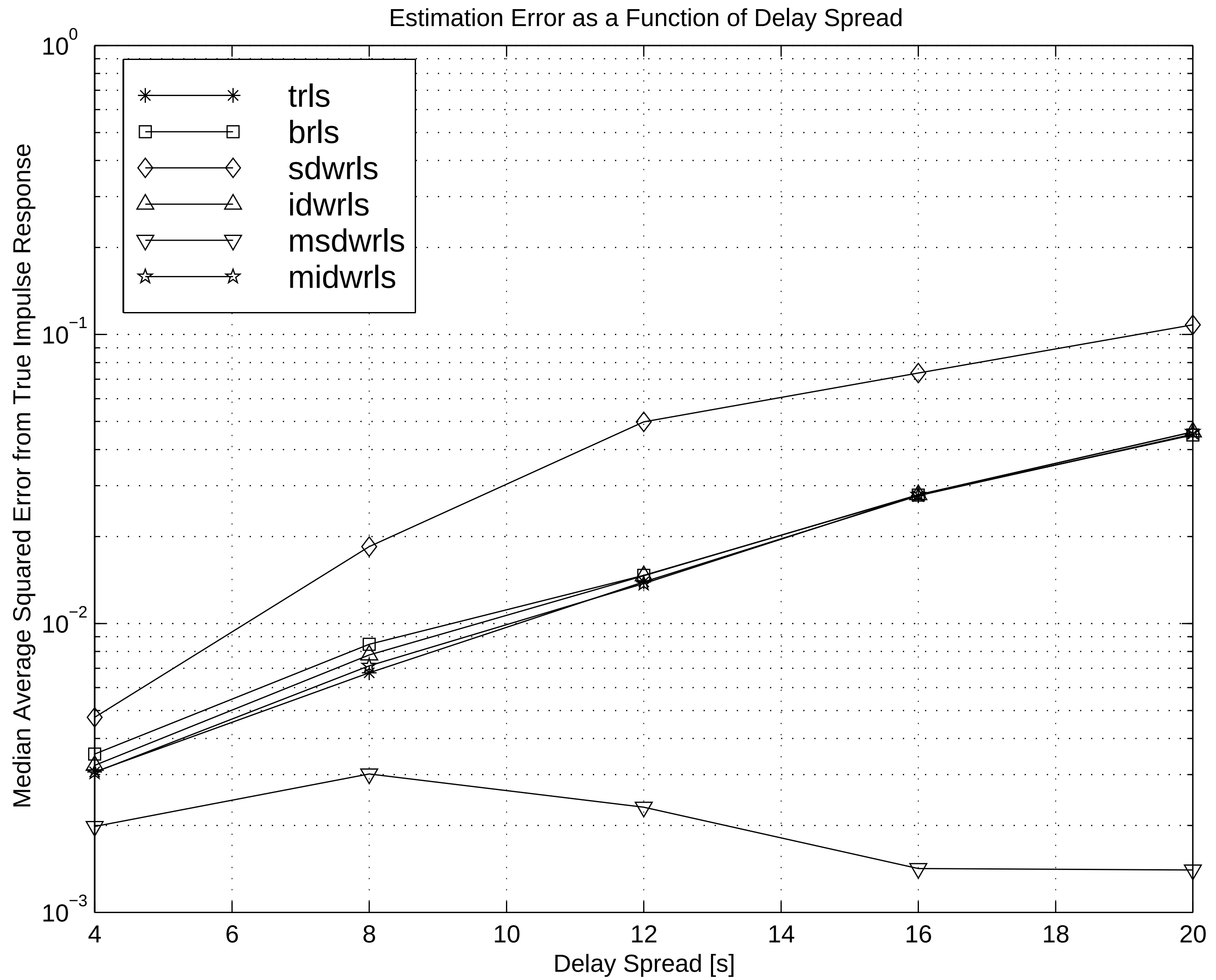


Figure 2: Median of the average squared error of RLS algorithms as a function of delay spread (SNR = 10 dB)

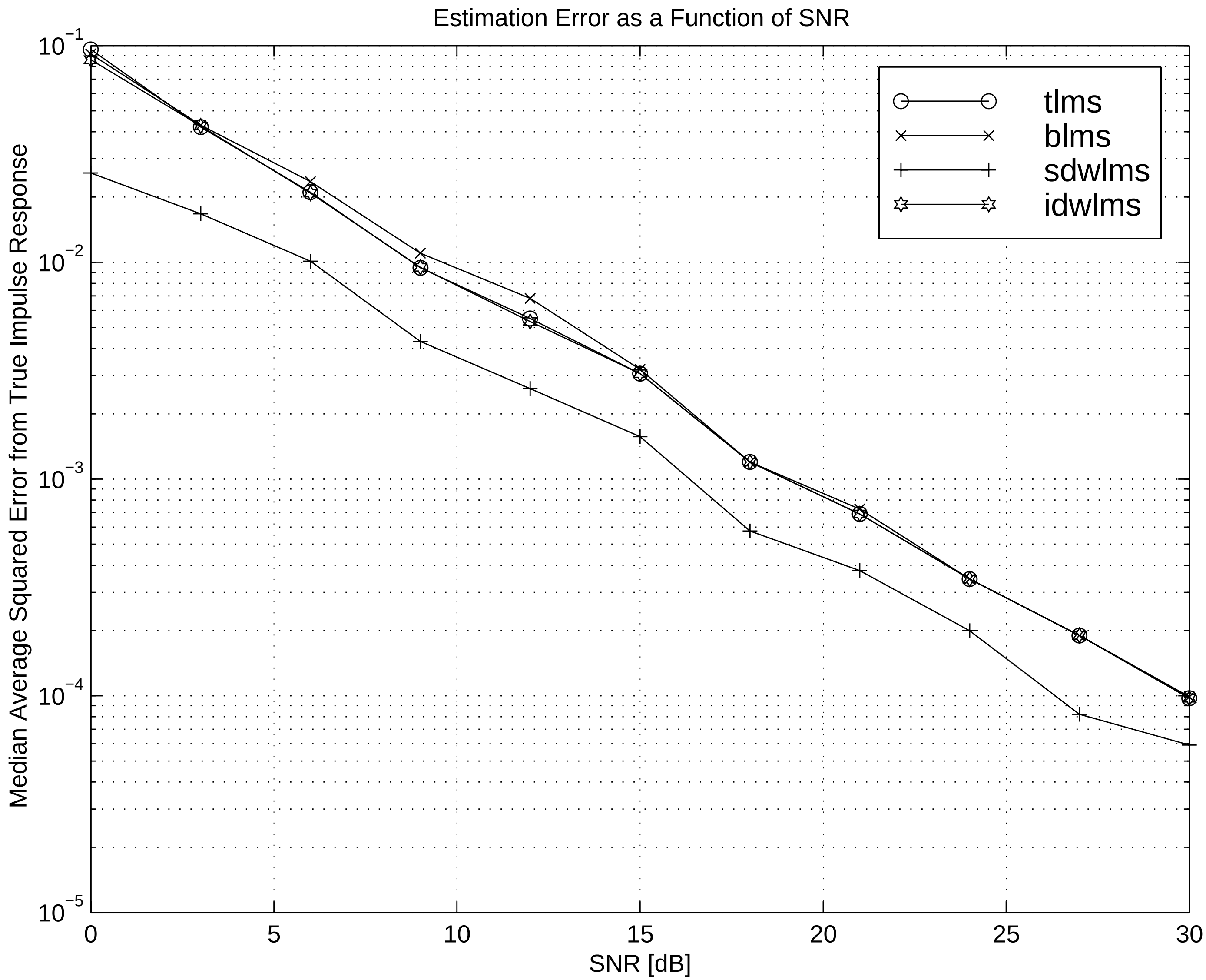


Figure 3: Median of the average squared error of LMS algorithms as a function of SNR (Delay Spread = 1 symbol interval)

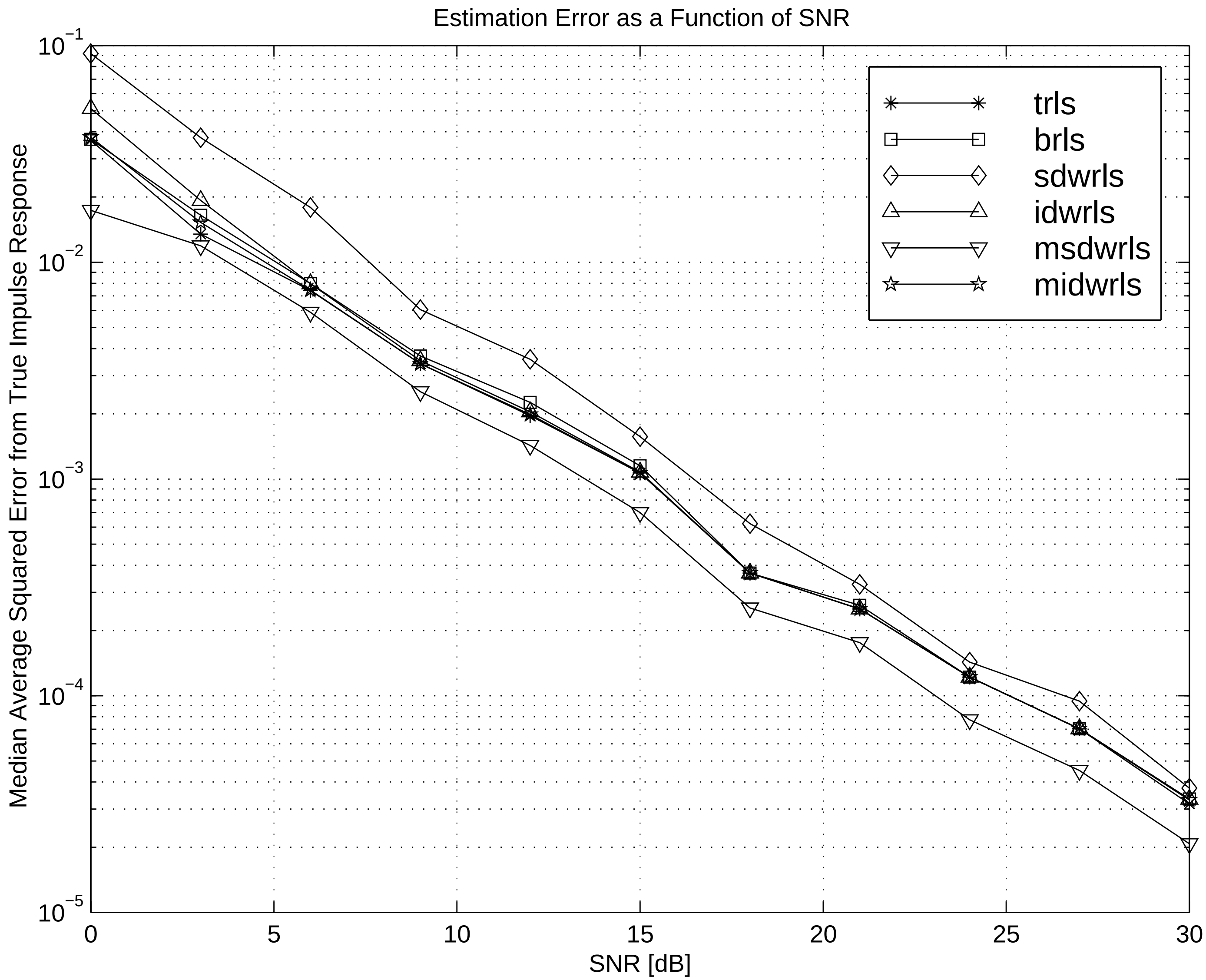


Figure 4: Median of the average squared error of RLS algorithms as a function of SNR (Delay Spread = 1 symbol interval)

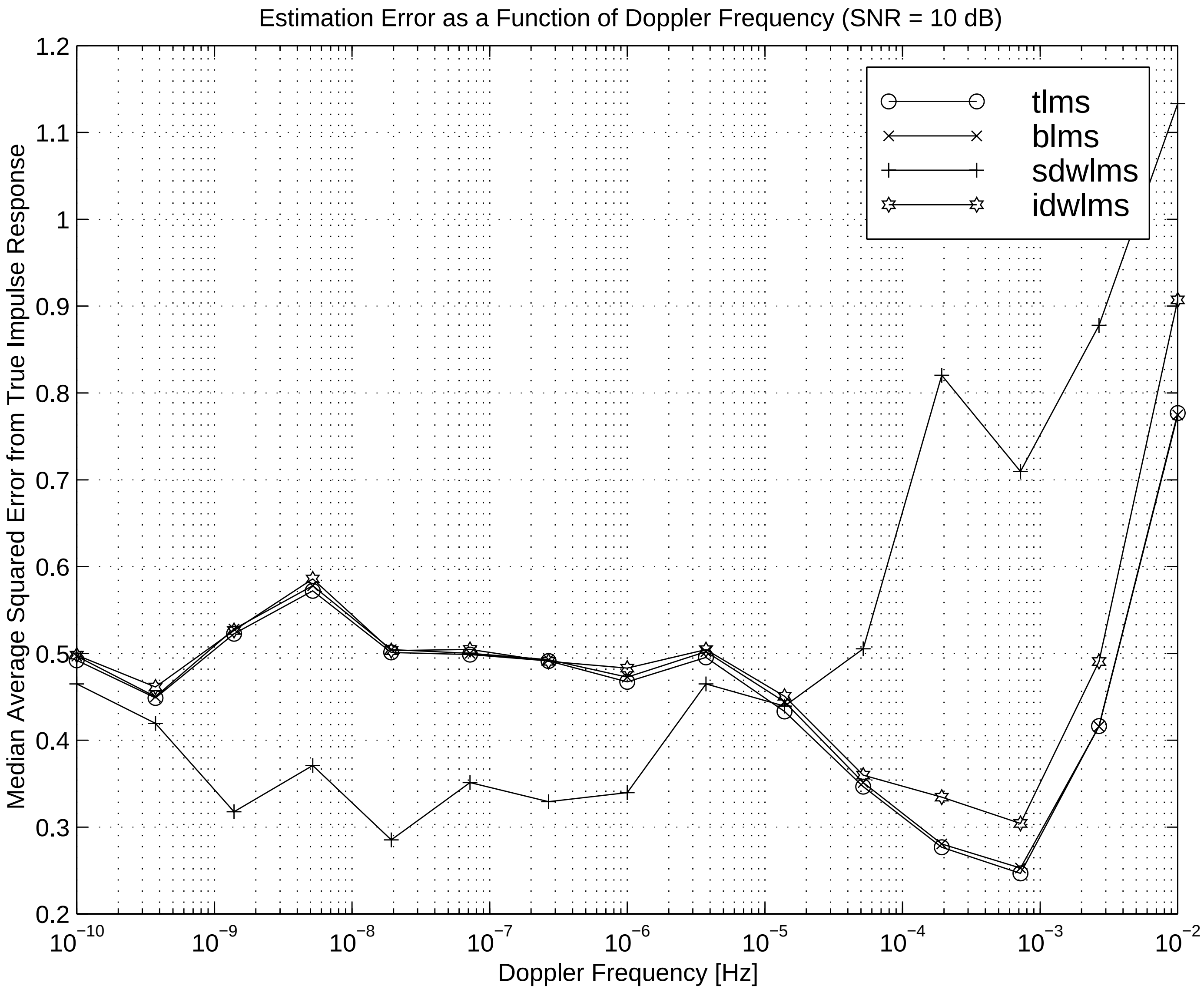


Figure 5: Median of the average squared error of LMS algorithms as a function of Doppler frequency (SNR = 10 dB)

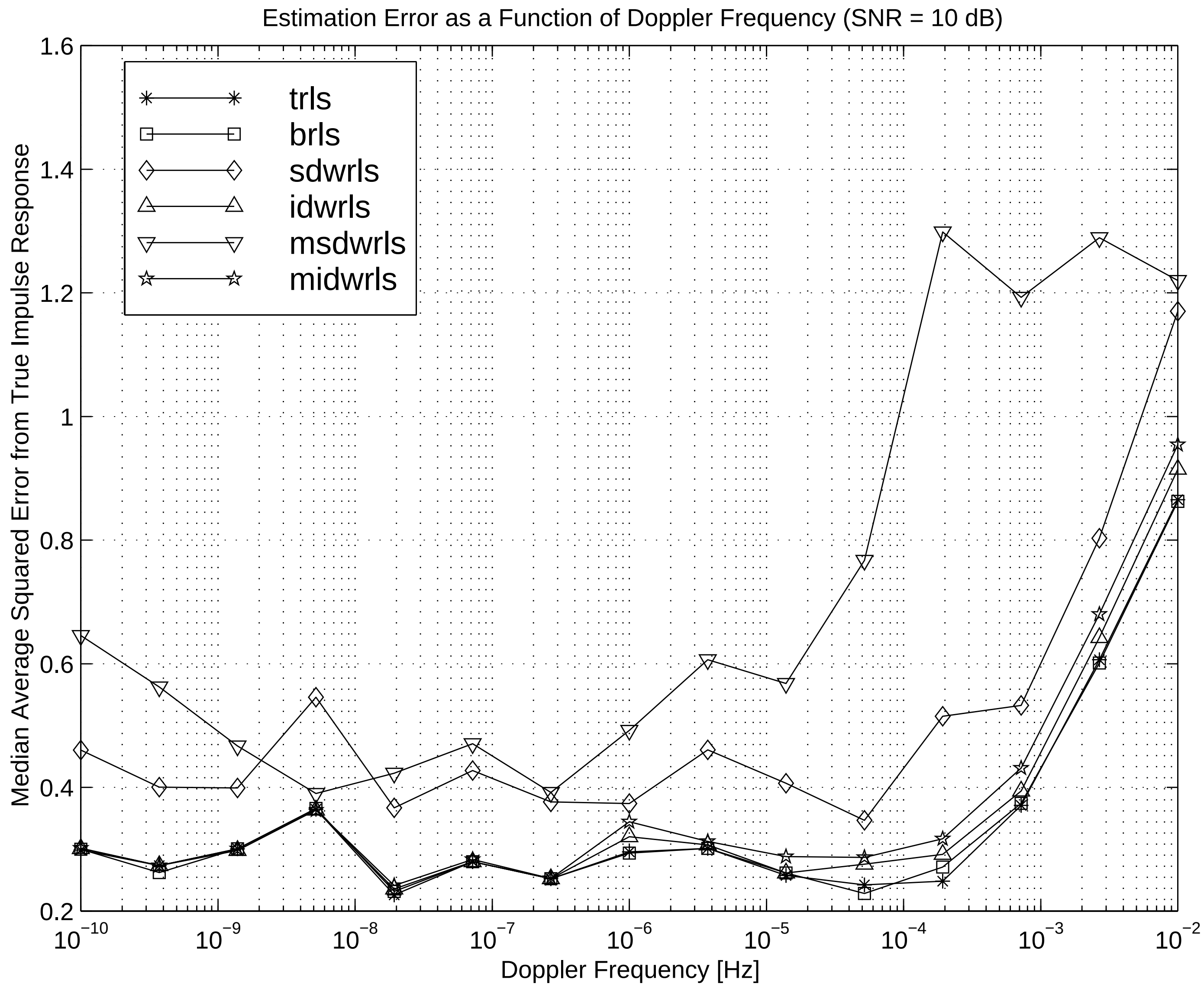


Figure 6: Median of the average squared error of RLS algorithms as a function of Doppler frequency (SNR = 10 dB)

Conclusions

⇒ Summary of Performance Test Results

- Soft decision weighted LMS (SDWLMS) performed *better* than the other LMS algorithms in delay spread (by a factor of 2 to 100, Figure 1) and SNR (by a factor of 2, Figure 3) tests
- Soft decision weighted RLS (SDWRLS) performed *worse* than the other RLS algorithms in delay spread (by a factor of 2, Figure 2) and SNR (by a factor of 3, Figure 4) tests
- Modified soft decision weighted RLS (MSDWRLS) performed *better* than the other RLS algorithms in delay spread (by a factor of 2 to 20, Figure 2) and SNR (by a factor of 2, Figure 2) tests
- SDWLMS performed *better* at normalized Doppler frequencies less than 10^{-5} and *worse* at higher Doppler frequencies than the other LMS algorithms (Figure 5)
- SDWRLS and MSDWRLS performed *worse* over all Doppler frequencies than the other RLS algorithms (Figure 6)

- * Ideal decision weighted LMS and RLS (IDWLMS and IDWRLS) performed similar to their training sequence versions in all tests, and generally better than their ordinary decision-directed counterparts (Figures 1 through 6).

⇒ General Conclusions

- * Decision weighted estimators defined, analyzed, and simulated
- * SDWLMS shows most promise for implementation
- * SDWRLS performed poorly, but MSDWRLS performed well
- * Ideal decision weighted algorithms performed similar to training sequence algorithms