Radar Pulse Compression

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June 17, 2004
Outline

• Why is pulse compression needed?
• Pulse compression, the compromise
• How it works
• Simplified view of concept
• Pulse coding
  – Phase-coded pulse
  – Chirp (linear FM)
• Receiver signal processing
• Window functions and their effects
Why is pulse compression needed?

Radar range resolution depends on the bandwidth of the received signal.

\[ \rho = \frac{c \tau}{2} = \frac{c}{2B} \]

- \( c \) = speed of light, \( \rho \) = range resolution, \( \tau \) = pulse duration, \( B \) = signal bandwidth

The bandwidth of a time-gated sinusoid is inversely proportional to the pulse duration.
- So short pulses are better for range resolution

Received signal strength is proportional to the pulse duration.
- So long pulses are better for signal reception
More Tx Power??

Why not just get a transmitter that outputs more power?

High-power transmitters present problems
  Require high-voltage power supplies (kV)
  Reliability problems
  Safety issues (both from electrocution and irradiation)
  Bigger, heavier, costlier, …
Pulse compression, the compromise

Transmit a long pulse that has a bandwidth corresponding to a short pulse

Must modulate or code the transmitted pulse
  - to have sufficient bandwidth, $B$
  - can be processed to provide the desired range resolution, $\rho$

Example:

Desired resolution, $\rho = 15$ cm ($\sim 6$")
Required bandwidth, $B = 1$ GHz ($10^9$ Hz)
Required pulse energy, $E = 1$ mJ

**Brute force approach**
Raw pulse duration, $\tau = 1$ ns ($10^{-9}$ s)
Required transmitter power, $P = 1$ MW !

**Pulse compression approach**
Pulse duration, $\tau = 0.1$ ms ($10^{-4}$ s)
Required transmitter power, $P = 100$ W
Simplified view of concept

Energy content of long-duration, low-power pulse will be comparable to that of the short-duration, high-power pulse

\[ \tau_1 \ll \tau_2 \text{ and } P_1 \gg P_2 \]

Goal: \( P_1 \tau_1 \approx P_2 \tau_2 \)
Pulse coding

Long duration pulse is coded to have desired bandwidth. Various ways to code pulse.

Phase code short segments
Each segment duration = 1 ns

Linear frequency modulation (chirp)
\[ s(t) = A \cos(2\pi f_C t + 0.5k t^2 + \phi_C) \]
for \(0 \leq t \leq \tau\)
\(f_C\) is the starting frequency (Hz)
\(k\) is the chirp rate (Hz/s)
\(B = k\tau^2 = 1\) GHz

Choice driven largely by required complexity of receiver electronics
Receiver signal processing
phase-coded pulse generation and compression

Matched Filter Receiver

\[\begin{array}{cccccccc}
- & + & - & - & + & - & - & + \\
+ & + & + & - & - & - & + & - \\
+ & + & + & + & - & - & + & - \\
\end{array}\]

\[\begin{array}{c}
-1 = -1 \\
+1-1 = 0 \\
-1+1-1 = -1 \\
-1-1+1+1 = 0 \\
+1-1-1-1+1 = -1 \\
+1+1-1-1-1 = 0 \\
+1+1+1+1+1+1+1+1 = 8 \\
+1+1+1-1-1 = 0 \\
etc
\end{array}\]
Correlation process may be performed in analog or digital domain. A disadvantage of this approach is that the data acquisition system (A/D converter) must operate at the full system bandwidth (e.g., 1 GHz in our example).

PSL: peak sidelobe level (refers to time sidelobes)
Receiver signal processing
chirp generation and compression

Dispersive delay line is a SAW device
SAW: surface acoustic wave
Stretch chirp processing
Challenges with stretch processing

Received signal (analog) ➔ Low-pass filter ➔ A/D converter ➔ Digitized signal

To dechirp the signal from extended targets, a local oscillator (LO) chirp with a much greater bandwidth is required. Performing analog dechirp operation relaxes requirement on A/D converter.

Echos from targets at various ranges have different start times with constant pulse duration. Makes signal processing more difficult.
Correlation processing of chirp signals

- Avoids problems associated with stretch processing
- Takes advantage of fact that convolution in time domain equivalent to multiplication in frequency domain
  - Convert received signal to freq domain (FFT)
  - Multiply with freq domain version of reference chirp function
  - Convert product back to time domain (IFFT)
Chirp pulse compression and sidelobes

Peak sidelobe level can be controlled by introducing a weighting function -- however this has side effects.
Window functions and their effects

<table>
<thead>
<tr>
<th>Weighting Function</th>
<th>Peak Sidelobe Level</th>
<th>S/N Loss</th>
<th>Relative Mainlobe Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform</td>
<td>-13.2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$0.33 + 0.66 \cos^2(\pi f/\beta)$</td>
<td>-25.7</td>
<td>0.55</td>
<td>1.23</td>
</tr>
<tr>
<td>$\cos^2(\pi f/\beta)$</td>
<td>-31.7</td>
<td>1.76</td>
<td>1.65</td>
</tr>
<tr>
<td>Taylor (n=8)</td>
<td>-40</td>
<td>1.14</td>
<td>1.41</td>
</tr>
<tr>
<td>Dolph Chebyshev</td>
<td>-40</td>
<td>-</td>
<td>1.35</td>
</tr>
<tr>
<td>Hamming</td>
<td>-42.8</td>
<td>1.34</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Time sidelobes are an side effect of pulse compression.

Windowing the signal prior to frequency analysis helps reduce the effect.

Some common weighting functions and key characteristics

<table>
<thead>
<tr>
<th>Window</th>
<th>Rect angle</th>
<th>Hamming</th>
<th>Blackman</th>
<th>Blackman-Harris</th>
</tr>
</thead>
<tbody>
<tr>
<td>Worst Sidelobe (dB)</td>
<td>-13</td>
<td>-43</td>
<td>-58</td>
<td>-61 -67 -74 -92</td>
</tr>
<tr>
<td>3dB beamwidth (bins)</td>
<td>0.89</td>
<td>1.3</td>
<td>1.68</td>
<td>1.56 1.66 1.74 1.9</td>
</tr>
<tr>
<td>Resolution (bin)</td>
<td>1.21</td>
<td>1.81</td>
<td>2.35</td>
<td>2.19 2.2 2.44 2.72</td>
</tr>
<tr>
<td>Scalloping Loss (dB)</td>
<td>3.92</td>
<td>1.78</td>
<td>1.1</td>
<td>1.27 1.13 1.03 0.83</td>
</tr>
<tr>
<td>SNR Loss (dB)</td>
<td>0</td>
<td>1.34</td>
<td>2.37</td>
<td>2.07 2.33 2.53 3.02</td>
</tr>
<tr>
<td>Main Lobe Width (bins)</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>6 8 8</td>
</tr>
<tr>
<td>$a_0$</td>
<td>1</td>
<td>0.54</td>
<td>0.42</td>
<td>0.44959 0.42323 0.40217 0.35875</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.46</td>
<td>0.50</td>
<td>0.44364</td>
<td>0.49755 0.49703 0.48829</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.08</td>
<td>0.05677</td>
<td>0.07922</td>
<td>0.09392 0.14128</td>
</tr>
<tr>
<td>$a_3$</td>
<td>0.00183</td>
<td></td>
<td>0.01168</td>
<td></td>
</tr>
</tbody>
</table>

$W(n) = a_0 - a_1 \cos[2\pi(n-1)/N] + a_2 \cos[4\pi(n-1)/N] - a_3 \cos[6\pi(n-1)/N]$
Window functions

Basic function: \[ c_k = \cos\left(2k\pi(n - \frac{1}{2}N) / N \right) \]
a and b are the \(-6\)-dB and \(-\infty\) normalized bandwidths

Rectangular: \( w(n) = 1 \)
a = 1.21, b = 2
Sidelobe = -13dB

Hanning: \( 0.5 + 0.5c_i \)
a = 1.65, b = 4
Sidelobe = -23dB
Window functions

Hamming: \(0.54 + 0.46c_1\)
\[a=1.81, \ b=4\]
Sidelobe = \(-43\text{dB}\)

Blackman-Harris: 3 term
\(0.423 + 0.498c_1 + 0.079c_2\)
\[a=1.81, \ b=6\]
Sidelobe = \(-67\text{dB}\)

Blackman-Harris: 4 term
\(0.359 + 0.488c_1 + 0.141c_2 + 0.012c_3\)
\[a=2.72, \ b=8\]
Sidelobe = \(-92\text{dB}\)
Detailed example of chirp pulse compression

received signal

\[ s(t) = a \cos(2 \pi f_C t + 0.5 k t^2 + \phi_C) \]

dechirp analysis

\[ s(t) s(t - \tau) = a \cos(2 \pi f_C t + 0.5 k t^2 + \phi_C) \cos[2 \pi f_C (t - \tau) + 0.5 k (t - \tau)^2 + \phi_C] \]

which simplifies to

\[ s(t) s(t - \tau) = \frac{a^2}{2} \left[ \cos(2 \pi f_C \tau + k t \tau - 0.5 k \tau^2) \right. \]

\[ + \cos(2 \pi f_C \tau + k t \tau - 0.5 k \tau^2 - 2 \pi f_C \tau + 2 \phi_C) \left. \right] \]

after lowpass filtering to reject harmonics

\[ q(t) = \frac{a^2}{2} \cos(2 \pi f_C \tau + k t \tau - 0.5 k \tau^2) \]
Conclusions

Pulse compression allows us to use a reduced transmitter power and still achieve the desired range resolution.

The costs of applying pulse compression include:
- added transmitter and receiver complexity
- must contend with time sidelobes

The advantages generally outweigh the disadvantages so pulse compression is used widely.