EECS 361
Homework #2

1. Section 2.5 Participation activities
- 2.5.2: Unit step function.
- 2.5.4: Ramp function.
- 2.5.5: Triangle waveform synthesized from ramps.
- 2.5.6: Rectangular (rect) function.
- 2.5.8: Impulse function.
- 2.5.9: Sampling property of the impulse function.
- 2.5.11: Exponential function.
- 2.5.12: Exponential functions.

2. (Concept: Property of decaying exponentials)
The half-life of a decaying exponential is the time it takes for a quantity to decrease to half of its original value.
Given $x(t)=4u(t)e^{\frac{-t}{8}}$ find how long it takes x(t) to be reduced to 50% of its peak value, that is find the half-life value.

3. (Concept: Relationship between unit step and rectangle functions)
Plot the following functions over a range of t from -10 to 10.

a. $x_1(t) = 2u(t+1)$

b. $x_2(t) = 2u(t-1)$

c. $x_2(t)-x_1(t)$

4. (Concept: Properties of the delta function.)
Solve the following.

a. $\int_{-\infty}^{\infty} \delta(\tau)\, 2\, e^{-4\tau}\, d\tau$

b. $\int_{-\infty}^{\infty} \delta(\tau - 0.2)\, u(\tau)\, 2\, e^{-4\tau}\, d\tau$

c. $\int_{-\infty}^{\infty} \delta(\tau - .8)\, u(\tau)\, 2\, e^{-4\tau}\, d\tau$

d. $\int_{-\infty}^{\infty} \delta(\tau - t)\, u(\tau)\, 2\, e^{-4\tau}\, d\tau$

c.

5. (Concept: Time delay is equivariant to a phase shift)
Plot and compare these three signals. Explain their similarities and differences.
$x_1(t) = \cos(2\pi 2000t)$, $x_2(t) = \cos\left(2\pi 2000\left(t - \frac{1}{16000}\right)\right)$, $x_3(t) = \cos\left(2\pi 2000t - \frac{\pi}{4}\right)$

6. (Concepts: Mathematical modeling of a periodic function using a finite duration prototype signal and DC value of a signal)
Let a finite duration prototype signal be $x(t) = 4\,\text{rect}\left(\frac{t}{2}\right)$

$$x_p(t) = \sum_{k=-n}^{n} x(t - kT_0) \text{ With } T_0 = 4$$

a. For n=2 plot $x_p(t)$

b. Sketch x(t) as n⟶∞, in this case is x(t) periodic; what is the period?

c. What is the average (DC) value of x(t)?

7. (Concept: Weighted sum of harmonically related sines produces a periodic signal)
a. Plot

$$x(t) = \sum_{n=1}^{6} \frac{1}{n\pi} \sin(2\pi nt) \quad \text{for } 0<t<6$$

b. Plot

$$y(t) = \sum_{n=1}^{6} \frac{1}{n\pi} \cos\left(2\pi n t - \frac{\pi}{2}\right) \text{ for } 0<t<6$$

c. Why are x(t) and y(t) the same?

8. (Concept: Constructing binary signal from a binary sequence and a modulated digital signal from a binary sequence)

Given a sequence of 4 bits {1,0,1,0} are represented by voltage levels a_i= {1, -1, 1, -1},
here a logical “0” maps into a voltage level -1 and a logical “1” maps into a voltage level +1;

a. The levels are mapped into a binary signal. Plot the binary signal x(t),

$$x(t) = \sum_{n=1}^{4} a_n \operatorname{rect}(t - n + 0.5)$$

b. Binary Phase Keyed (BPSK) modulation is used to modulate binary information onto a radio frequency (RF) carrier.

A BPSK signal is given as $x(t)\cos(2\pi f_c t)$. Here set f_c=5 and plot $x_{\text{RF-BPSK}}(t)$.

$$x_{\text{RF-BPSK}}(t) = \sum_{n=1}^{4} a_n \operatorname{rect}(t - n + 0.5)\cos(2\pi 5t)$$

c. A modulation using amplitude shift keying (ASK) signal is generated by changing the a_i to b_i mapping to b_i = {1, 0, 1, 0}. Plot $x_{\text{RF-ASK}}(t)$.

$$x_{\text{RF-ASK}}(t) = \sum_{n=1}^{4} b_n \operatorname{rect}(t - n + 0.5)\cos(2\pi 5t)$$

9. (Concept: Introduction to the $\frac{\sin(\alpha t)}{\alpha t}$ function and its properties. Later $\frac{\sin(t)}{t}$ is defined as sinc(t) $\triangleq \frac{\sin(t)}{t}$)

Note that Using L’Hôpital’s rule $\frac{\sin(t)}{t}\Big|_{t=0}$=1)

Plot for -2<t<2

a. $\frac{\sin(\alpha t)}{\alpha t}$ for $\alpha = \pi$

b. $\frac{\sin(\alpha t)}{\alpha t}$ for $\alpha = 2\pi$

c. $\frac{\sin(\alpha t)}{\alpha t}$ for $\alpha = 4\pi$

d. What are the zero crossing times, i.e., at what times are these signals zero?
What is the relationship between α and the width of the signal?