EECS 361
Homework #6

1. Section 3.7 Participation Activities
  3.7.1: Frequency response function H(ω) and output from sinusoidal input.
  3.7.2: Time-domain response and frequency response pairs
  3.7.3: Is this system sinusoidal response LTI (Linear Time Invariant)?

2. (Concepts: Cosine input to a LTI system has output of a cosine of same frequency with possibly a different amplitude and phase and 3dB frequency)
Given $H(\omega) = \frac{8}{8+j\omega}$ and input x(t) find the system output, y(t), for the following cases.
Hint: Look at Homework 1 Problem 4
  a. x(t) = 1
  b. $x(t)$ =cos(2t)
  c. $x(t)$ =cos(8t)
  d. $x(t)$ =cos(16t)
  e. For $x(t)$ =cos(8t) what is the ratio of the power in y(t), P_y to the power in x(t), P_x, i.e., P_y / P_x and what is $10\log(P_y / P_x)$.

3. (Concept: Finding frequency response from LCCDE for first and second order systems)
Find the frequency response, H(ω), for a system described by the following LCCDE's.

  a. $\frac{dy(t)}{dt} + ay(t) = x(t)$ where a is a real constant
  b. $\frac{d^2 y(t)}{dt^2} + (2a\omega_u)\frac{dy(t)}{dt} + \omega_u^2 y(t) = \omega_u^2 x(t)$ where a and ω_u are real constants

4. (Concept: relating the impulse response to the frequency response function)
For $h(t) = u(t)ae^{-at}$
  a. Find H(ω). Hint: see Example 3.7.1
  b. Put H(ω) in the form of H(ω)=R(ω)+jI(ω) where R(ω) and I(ω) are real functions. Hint: see Homework 1, Problem 4
  c. Put H(ω) in the form of $H(\omega)=A(\omega)e^{j\theta(\omega)}$ where A(ω) and $\theta(\omega)$ are real functions. Hint: see Homework 1, Problem 4
  d. For a=8 plot $|H(\omega)|$, $|H(\omega)|^2$, $20\text{Log}(|H(\omega)|)$ and $\theta(\omega)$.
  e. With a= 8 what is the value of $H(\omega_1)$, $\frac{|H(\omega_1)|}{|H(0)|}$, $10\text{Log}(\frac{|H(\omega_1)|^2}{|H(0)|^2})$ at $\omega_1 = 8$?
  Compare the $10\text{Log}(\frac{|H(\omega_1)|^2}{|H(0)|^2})$ at $\omega_1 = \frac{1}{8}$ to the result from Problem 2e above.

5. (Concept: Methods to describe an LTI system)
Given the results of problems 2-4, what are three ways to model (describe/characterize) an LTI system.

6. (Concept: Cosine input to a LTI system has output of a cosine of same frequency with possibly a different amplitude and phase)
The input to an LTI system is $x(t)=A\cos(\omega_1 t+\theta)$ write the system output, y(t), in terms of H(ω).

7. (Concept: Cosine input to a LTI system has output of a cosine of same frequency with possibly a different amplitude and phase and finding conditions where the input signal is in-phase with the output signal)
Exercise 3.7.2

8. (Concept: Sum of cosine inputs to a LTI system has output of that is the sum of the cosines at the same frequencies with possibly a different amplitude)
x(t) = 1+6cos(2t)+12cos(8t) is the input to a LTI system with $H(\omega) = \frac{8}{8+j\omega}$ find the output signal y(t).

9. (Concept: Cosine input to a LTI system has output of a cosine of same frequency with possibly a different amplitude and phase)
An LTI system has an impulse response of $h(t)=5u(t)e^{-t} - 16u(t)e^{-2t} + 13u(t)e^{-3t}$. Find the system output for an input of cos(2t).

10. (Concept: Given an input of a sum of cosines and the corresponding output finding the frequency transfer function at those frequencies)
Exercise 3.7.8

11. (Concept: Given an input of cosines at specific frequencies and the corresponding output, find the frequency transfer function at those frequencies.)

a. Plot $|H(\omega_i)|$ for $\omega_i = \{0, 1, 2, 4, 8\}$ given

Input = x (t)	Output = y (t)
1	4
cos (t)	2 cos (t)
cos (2 t)	0.8 cos (2 t)
cos (4 t)	0.235 cos (4 t)
cos (8 t)	0.0615 cos (8 t)

b. Would these observations be consistent with $H(\omega) = \frac{2}{1+\omega^2}$? Yes or No, and justify your answer.