EECS 361
Homework #7

1. Section 4.1 Participation Activities
4.1.1: Time to phasor domain transform examples.
4.1.2: Transforming to the phasor domain.
4.1.3: Phasor technique to solve differential equations having sinusoidal inputs.

2. (Concept: Applying the phasor-domain technique to find the system output given sinusoidal inputs)
Find the output, y(t), for a system is characterized by the differential equation

$$c_1\, dy(t)/dt + c_2 y(t) = 4\cos(300t - \tfrac{\pi}{6}) - 8\cos(600t - \tfrac{\pi}{4})$$
$$c_1 = 7\text{x}10^{-3} \text{ and } c_2 = 3$$

3. Section 4.2 participation activity
4.2.1: Fourier series analysis technique.

4. Section 4.3 participation activity
4.3.1: Fourier series harmonics

5. Section 4.4 participation activity
4.4.2: Fourier series of triangle wave.

6. Section 4.5 participation activity
4.5.2: Line spectra of triangle wave
4.5.3: Fourier series coefficients.

7. (Concept: directly finding the Fourier series coefficients using Euler's formula and matching terms)
For x(t) = 8cos(300t)-3sin(600t) find

a. Complex exponential Fourier series representation of x(t), x_n
b. Amplitude/phase Fourier series representation of x(t), c_0, c_n, ϕ_n
c. Sine/cosine Fourier series representation of x(t), a_0, a_n and b_n

Hint: $\sin(\theta) = \cos\left(\theta - \frac{\pi}{2}\right)$ and $-\sin(\theta) = \sin(\theta+\pi) = \cos\left(\theta + \frac{\pi}{2}\right)$

8. (Concept: Fourier Series of pulse train)
For $x(t) = \sum_{k=-\infty}^{\infty} \text{rect}\left(\frac{t-kT_o}{\tau}\right)$ were T_o =62.8 ms and τ=15.7ms
a. Plot x(t).
b. Find x_0 and power in x(t) = P_x
c. In the complex exponential Fourier series find the $x_n's$.
(recommend you find the $x_n's$. using T_o and τ as parameters, as the result will be useful for problem 9).
d. Plot $|x_n|$ as a function of n for n={-8...8}, i.e., plot the two-sided magnitude line spectrum as a function of the harmonic index, n.
e. Plot $|x_n|$ as a function of for ω for $-8\,\omega_0 < \omega < +8\,\omega_0$,
i.e., two-sided magnitude line spectrum plot as a function of the frequency in rad/sec.
f. Given the $x_n's$ found in part c, plot $x_{\text{Approx}}(t) = \sum_{n=-N}^{N} x_n e^{jn\omega_0 t}$ for N=8 and compare to the plot obtained in part a.
g. For N finite there will be an error between $x(t) = \sum_{n=-\infty}^{\infty} x_n e^{jn\omega_0 t}$ and $x_{\text{Approx}}(t) = \sum_{n=-N}^{N} x_n e^{jn\omega_0 t}$.
Define a figure of merit as $P_{\text{Error}} = P_x - P_{x_{\text{Approx}}}$ where P_x= power of x(t) and $P_{x_{\text{Approx}}}$ is power of $x_{\text{Approx}}(t)$.
$P_{x_{\text{Approx}}} = \sum_{n=-N}^{N} |x_n|^2$. Find P_{Error} for N=5.

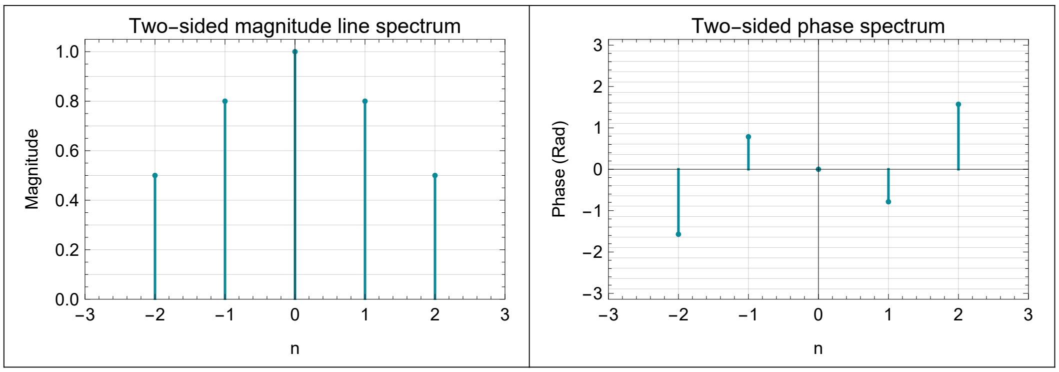
9. (Concept: Impact of changing the period and pulse width on the line spectrum.)
For $x(t) = \sum_{k=-\infty}^{\infty} \text{rect}\left(\frac{t-kT_o}{\tau}\right)$.
Plot the two-sided magnitude line spectrum (label the x-axis in frequency-Hz) for
a. T_o = 4ms and τ=1ms.
b. T_o = 8ms and τ=1ms.

c. T_o = 8ms and τ=0.5ms.
d. Comment on the how fixing T_o and adjusting τ changes the two-sided magnitude line spectrum.
And comment on the how fixing τ and adjusting T_o changes the two-sided magnitude line spectrum.

10. (Concept: Finding the Fourier Series of trigonometric functions)
Find the amplitude/phase Fourier Series for x(t)=cos(2π500t) + 2cos^2(2π500t) and plot the two-sided magnitude line spectrum in Hz. Hint: Use trigonometry identity for cos$^2(\theta)$.

11. (Concept: Finding a time signal, x(t) given the two-sided magnitude line spectrum and two-sided phase spectrum)
Find x(t) given two-sided magnitude line spectrum and two-sided phase spectrum shown below, assume T_0 = 1 ms.

Magnitude and Two−sided Phase Spectrum of $x(t)$



12. Section 4.6 Participation Activities
4.6.1: Complex exponential Fourier series representation for sawtooth wave.
4.6.2: Conversions between Fourier series representations.

13. Section 4.8 Participation Activities
4.8.2: Fourier analysis of RL circuit squarewave.
4.8.3: Fourier analysis of RC circuit, half-wave rectified sine input.

14. Section 4.9 Participation Activities
4.9.1: Average power of sinusoidal signals.
4.9.2: Average power of sum and product of sinusoids.

15. (Concept: Finding power using Parseval's theorem)
For x(t) = 5cos(600πt) - 2sin(1200πt) find P_x using the Parseval's theorem.

16. (Concept: Finding power in a frequency band)
Given x(t) =2 $\sum_{k=-\infty}^{\infty} \text{rect}\left(\frac{t-kT_o}{\tau}\right)$ were T_o =1ms and τ=0.25ms.
a. That is the average power in x(t).
b. What is the % average power in a bandwidth of 5000 Hz.