EECS 361
Homework #9

1. Section 4.11 Participation Activities
4.11.1: Time scaling property of the Fourier transform.
4.11.2: Fourier transform time scaling.
4.11.3: Fourier transforms of shifted impulse functions.
4.11.4: Time shift property of the Fourier transform, and phase.
4.11.6: Deriving the Fourier transform modulation property, using the frequency-shift property.
4.11.7: Fourier transform properties: derivative, modulation, and convolution.
4.11.8: Reviewing Fourier transform properties.

2. (Concept: Using the use the relationship between Fourier Transform and Fourier series, $\boldsymbol{x}_n = \frac{1}{T_0} X(n\omega_0)$)

a. Find the complex exponential Fourier series coefficients, $\boldsymbol{x}_n$ of $x_p(t)$ shown below, that is,

$x(t) = 0.5\,\mathrm{rect}(\frac{t}{4}) + \mathrm{tri}(\frac{t}{2})$ and $x_p(t) = \sum_{k=-\infty}^{\infty} x(t + T_0 k)$ where $T_0 = 10$.

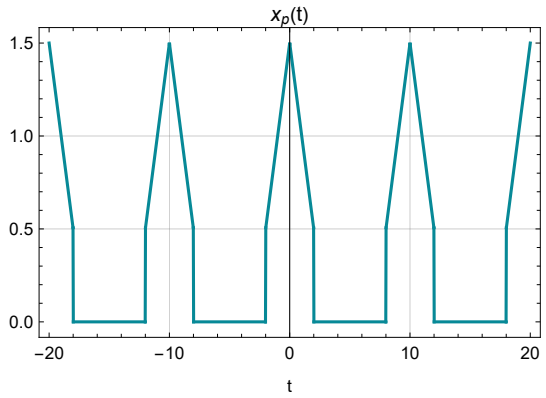
Check your result by plotting complex exponential Fourier series of $x_p(t)$, i.e., $x_p(t) = \sum_{n=-M}^{M} x_n e^{j2\pi n\omega_0 t}$, select an appropriate M to confirm your result.

b. Plot $\frac{1}{T_0}|X(\omega)|$, two-sided magnitude line spectrum of $x_p(t)$ and compare, try to put $\frac{1}{T_0}\left|X(\omega)\right|$ and the line spectrum on the same plot.

c. Change T_0 to 5 and repeat part b.

d. Comment on the impact of changing the period, is it the same as observed in Homework 7 Problem 9.

Hint: See Homework 8 Problem 3a and use the relationship between Fourier Transform and Fourier series, $\boldsymbol{x}_n = \frac{1}{T_0} X(n\omega_0)$



3. (Concept: Using the use the relationship between Fourier Transform and Fourier series, $\boldsymbol{x}_n = \frac{1}{T_0} X(n\omega_0)$)

Repeat Problem 2a and 2b with $x(t) = 0.5\,\mathrm{rect}(t) + \mathrm{tri}(2t)$. Comment on the impact of changing the pulse width, is it the same as observed in Homework 7 Problem 9.

4. (Concept: Using Fourier transform pairs)
Find the Fourier transform of:

a. $\cos(2\pi 2000t)$
b. $\cos(2\pi 2000t + \frac{\pi}{8})$
c. $e^{-0.1t}\cos(2t)\,u(t)$ Hint: See Figure 4.12.3: Fourier transform pairs.

5. (Concept: Multiplication in the time domain is convolution in the frequency domain)
Find the Fourier Transform of $x(t)g(t)$ where $x(t) = \frac{30}{\pi}\mathrm{sinc}(30t)$ and $g(t) = \frac{50}{\pi}\mathrm{sinc}(50t)$.

6. Section 4.13 Participation Activities
4.13.1: Time-domain energy computations for non-periodic signals.
4.13.2: Parseval's theorem and energy spectral density.

7. (Concept: Convolution in the time domain is multiplication in the frequency domain and applying Parseval's theorem)
Find the energy in y(t) where y(t)= x(t)*h(t) and x(t)=u(t)4$e^{-0.5\,t}$ and h(t)=$\frac{3}{\pi}$sinc(3t)

8. Section 4.16 Participation Activities
4.16.1: Fourier circuit analysis.
4.16.2: RC circuit analysis via Fourier transform, with exponential input.
4.16.3: RC circuit analysis via Fourier transform, for cosine input

9. (Concepts: Inverse pulse duration - bandwidth relationship and modulation property.)
Find the continuous time Fourier Transform (CTFT) and plot the magnitude of the continuous time Fourier Transform (CTFT) of the following signals:

a. $10^3 \text{rect}\left(\frac{t}{\tau}\right)$ for τ=1ms and 4ms
b. 10^3 $\text{tri}\left(\frac{t}{\tau}\right)$ for τ=1ms and 4ms
c. $e^{\frac{-\pi t^2}{\tau^2}}$ for τ=1 and 4
d. 4000rect(t/τ) cos$(2\,\pi f_c\,t)$ for τ= 1ms and f_c = 25 000

10. (Concept: Finding the inverse Fourier Transform and duality property)
Find the inverse continuous time Fourier Transform of the following signals:

a. $0.002\ \text{sinc}\left(\frac{0.001\,\omega}{2}\right)$
b. $0.001\left(\text{sinc}\left(\frac{0.001\,(\omega-\omega_c)}{2}\right)+\text{sinc}\left(\frac{0.001\,(\omega+\omega_c)}{2}\right)\right)$
c. $\frac{8}{4+\omega^2}$

11. (Concept: The Fourier transform of a periodic signal has delta functions)
Find the Fourier transform of x(t) for τ=1.0 ms and T_0 =4.0ms (See Homework 7 Problem 9)

$$x(t) = \sum_{k=-\infty}^{\infty} \text{rect}\left(\frac{t - \text{kT}_0}{\tau}\right)$$

12. (Concept: Convolution in the time domain is multiplication in the frequency domain and filtering)
Exercise 4.11.7 Hint: put $\frac{\sin(4\,t)}{\pi t}$ into the form of a sinc() function and plot the Fourier transform of x(t), 3 $e^{-|t|}$, and $\frac{\sin(4\,t)}{\pi t}$. Then plot the Fourier transform of y(t).