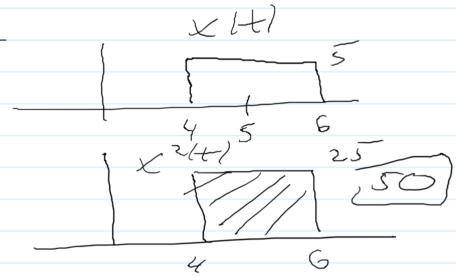
- Special function $\delta_{T_0}(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0)$

$x_p(t) = \sum_{\forall k} x(t - kT_0)$

- $\Pi(t)$
- $\delta(t)$
- $E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$

$x(t) = 5\,\mathrm{rect}\left(\frac{t-5}{2}\right)$



- $P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt$

T_0 = period

if periodic $P_x = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt$

$x(t) = 5\cos(\omega_1 t)$ $P_x = 25/2 = 12.5$ $\frac{A^2}{2}$

$x_p(t) = \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$

$P_{x_p} = \sum_{n=1}^{\infty} \frac{c_n^2}{2}$

- Energy signal E_x finite $P = 0$ Power Signal P finite $\infty = E$

- Linear System $x(t) \to \boxed{} \to y(t)$

$\alpha x_1(t) \to y_1(t)$ $\beta x_2(t) \to y_2(t)$

$\alpha x_1 + \beta x_2 \to y_3(t)$

If $y_1(t) + y_2(t) = y_3(t)$ Then Linear

Time invariance $x(t) \to \boxed{} \to y(t)$

$x(t-\tau) \to \boxed{} \to y(t-\tau)$

LTI

Let System be LTI $\delta(t) \to \boxed{} \to h(t)$

$h(t)$ = impulse response

$u(t) \to \boxed{} \to y_{step}(t)$ Step response

$y_{step}(t) = \int_{-\infty}^{t} h(\lambda)\, d\lambda$

$y_{step}(t)=\int_{-\infty}^{t} h(\lambda)\,d\lambda$ step response

$y(t)=\int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)\,d\lambda$

$y(0)=\int_{-\infty}^{\infty} x(\lambda)h(-\lambda)\,d\lambda$

$h(\lambda)\to h(-\lambda)$ flip

multiply $x(\lambda)h(-\lambda)$

integrate $\to y(0)$

$y(1)=\int_{-\infty}^{\infty} x(\lambda)h(1-\lambda)\,d\lambda$

$h(\lambda)$ flip $h(-\lambda)$

$h(-\lambda)$ shift $\to h(1-\lambda)$

multiply

$x(\lambda)h(1-\lambda)$

integrate $\to y(1)$

$x(t)$ is causal then $x(t)=0$ $t<0$

or $x(t)=x(t)u(t)$

$y(t)=\int_{-\infty}^{\infty} h(\lambda)x(t-\lambda)\,d\lambda$

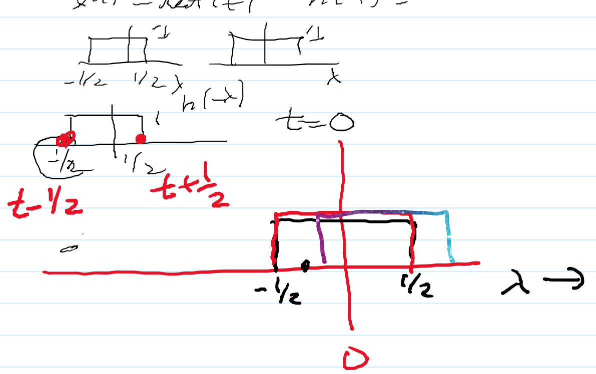
$\int_{-\infty}^{\infty} h(\lambda)u(t-\lambda)x(t-\lambda)\,d\lambda$

$u(t-\lambda)=1$ $t>\lambda$

$\int_{-\infty}^{t} h(\lambda)x(t-\lambda)\,d\lambda$

$h(t)=u(t)h(t)$

$x(t)=rect(t)$ $h(t)=rect(t)$



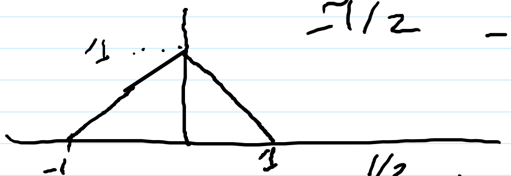
case 1 $y(t)=0$ $-\infty<t<-1$

$t=-1$

Case 1 $y(t) = 0 \quad -\infty < t < -1$

$t + 1/2 = -1/2 \qquad t = -1$

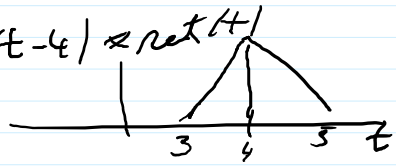
Case 2 $y(t) = \int_{-1/2}^{t+1/2} (1)(1)\, d\lambda \quad d\lambda \qquad -1 < t < 0$



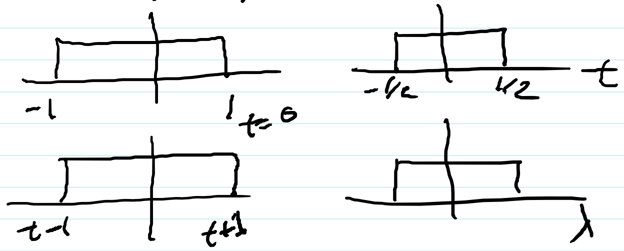
Case 3 $y(t) = \int_{t-1/2}^{1/2} (1)(1)\, d\lambda \qquad 0 < t < 1$

Case 4 $y(t) = 0 \qquad t > 1$

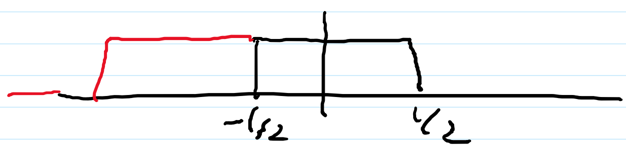
$y(t) = \text{rect}(t-4) * \text{rect}(t)$



$\text{rect}(t/2) \qquad \text{rect}(t)$

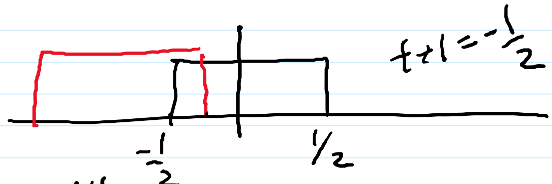


Case 1 $y(t) = 0 \qquad -\infty < t < -3/2$

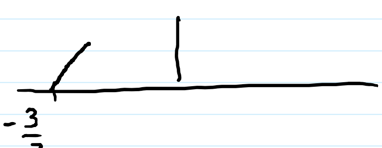


$t+1 = -1/2 \qquad t = -3/2$

Case 2

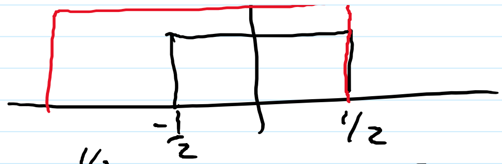


$y(t) = \int_{-\frac{1}{2}}^{t+1} (1)(1)\, d\lambda \qquad -\frac{3}{2} < t < ?$



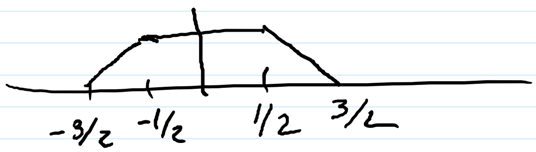
Case 3

Case 3

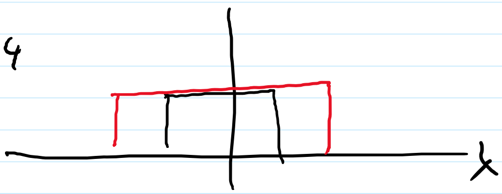


$$y(t) = \int_{-1/2}^{1/2} (1)(1)\, d\lambda = 1 \qquad -\frac{3}{2} < t < -\frac{1}{2}$$

$t+1 = \frac{1}{2} \qquad t = -\frac{1}{2}$

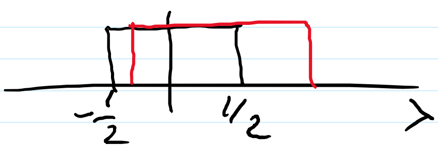


Case 4



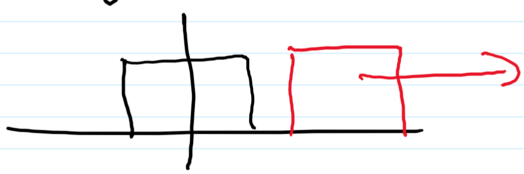
$$y(t) = 1 \qquad -\frac{1}{2} < t < \frac{1}{2}$$

Case 5



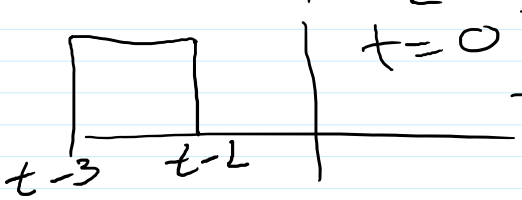
$$\int_{t-1}^{3/2} (1)(1)\, d\lambda$$

Case 6 $\quad y(t) = 0 \qquad t > 5/2$



$t = 0$

flipped



$R^2/2 \qquad \frac{(\sqrt{2})^2}{2} + \frac{2^2}{2} = 3$

$A\cos(\omega_1 t) + B\cos(\omega_2 t) \qquad$ Powers add

$\sqrt{2}\cos(\omega_1 t + 0.7)$ phase change

1

HW 3 #4 $100\times10^{-3}\cos(2\pi f_c t)$ "1" $T_b = \frac{1}{10\times10^9}$

BPSK $-100\times10^{-3}\cos(2\pi f_c t)$ "0" $r_b = 10\times10^9$

$$\int_0^{T_b} (100\times10^{-3})^2 \cos^2(2\pi f_c t)\,dt$$

$$\approx 5\times10^{-13} \qquad \cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos(2\theta)$$

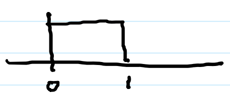
5. $10\cos(2\pi f_m t)\cos(2\pi f_c t)$ m = message, c = carrier

$$\left.\begin{array}{l} 5\cos(2\pi(f_c+f_m)t) \\ +5\cos(2\pi(f_c-f_m)t) \end{array}\right\} \quad \frac{5^2}{2} + 5^2/2$$

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = \int_{-\infty}^{\infty} x(\lambda)\,h(t-\lambda)\,d\lambda$$

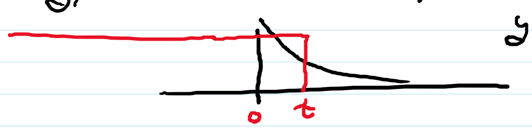
Example $x(t) = \mathrm{rect}(t - 1/2)$ $h(t) = u(t)e^{-t}$

$y(t) = x * h$



$x(t) = u(t) - u(t-1)$ → $y(t) = y_1(t) - y_1(t-1)$

$y_1(t) = u(t) * h(t)$

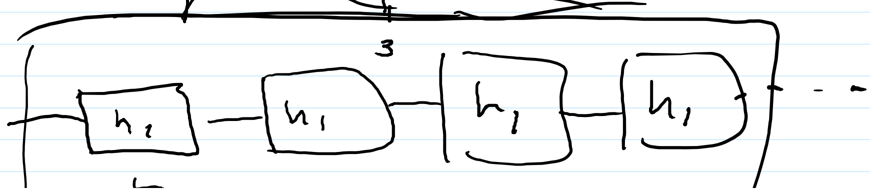


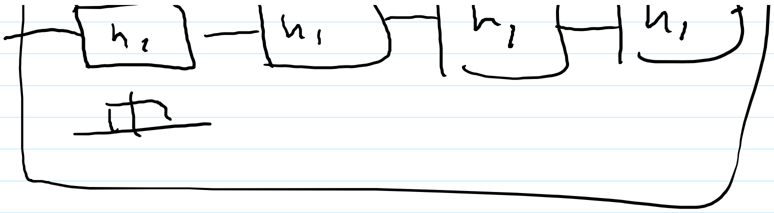
$$y(t) = \int_0^t e^{-t}\,dt = u(t)(1-e^{-t})$$

$$y(t) = u(t)(1-e^{-t}) + u(t-1)\left(1 - e^{-(t-1)}\right)$$

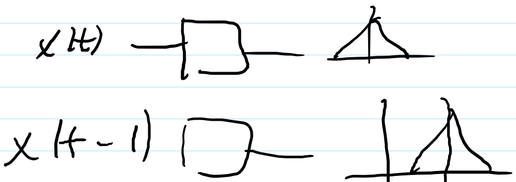
$(1-e^{-1})$

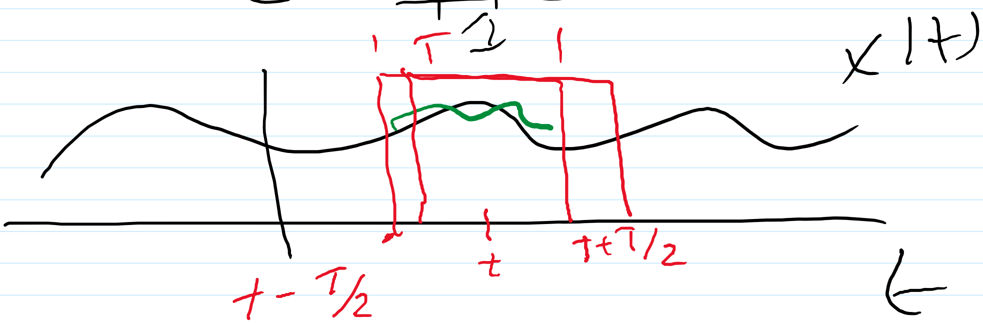
$x_1(t) = \mathrm{rect}(t - 1/2) + \mathrm{rect}(t-3)$



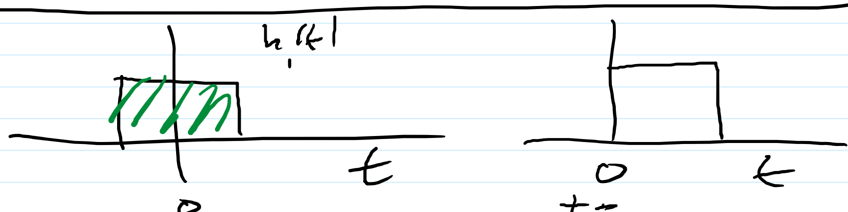


$$h(t) = h_1 * h_2 * h_1 \ldots$$





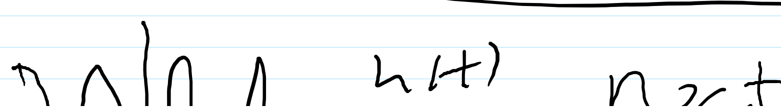
$\boxed{\text{If } h(t) = 0 \quad t < 0 \text{ then called} \quad h(t) = u(t)h(t)}$



$$\underline{y(t)} = u(t)h(t) * x(t) = \int_{-\infty}^{t} x(\lambda)h(t-\lambda)$$

$$\int_{-\infty} |h| \, d\lambda$$

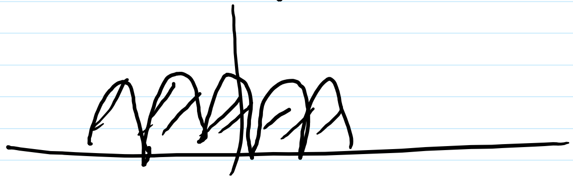
$\boxed{\text{If } \int_{-\infty}^{\infty} |h(\lambda)| \, d\lambda \text{ is finite then BIBO stable}}$

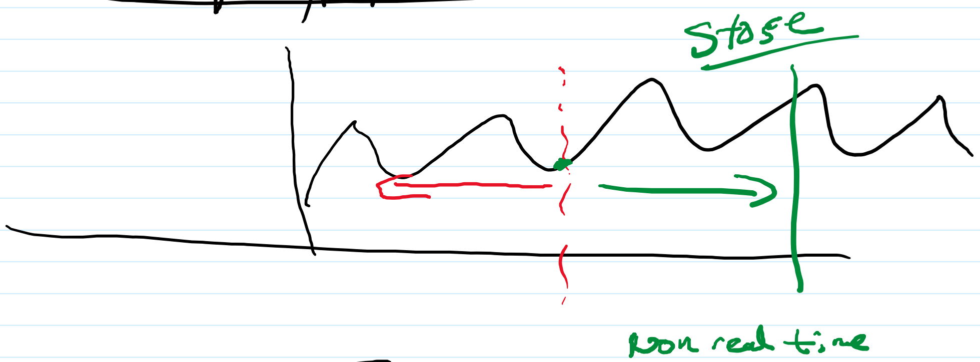


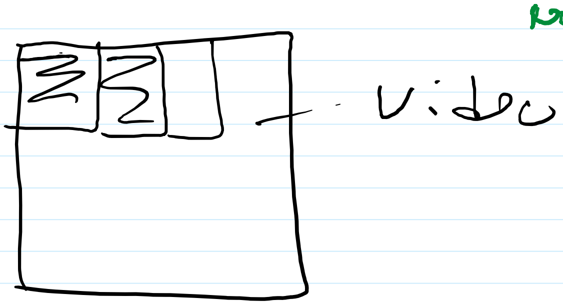
$h(t)$ next

Not causal

Is Stable







$$\boxed{x(t) = e^{j\omega t}} \qquad \boxed{y(t) = \underbrace{H(\omega)}_{\text{no function of } t} e^{j\omega t}}$$

$$\frac{dy}{dt} = j\omega H(\omega) e^{j\omega t}$$

$$j\omega H(\omega)\cancel{e^{j\omega t}} + \frac{1}{RC} H(\omega)\cancel{e^{j\omega t}} = \frac{1}{RC}\cancel{e^{j\omega t}}$$

Solve for $H(\omega)$

$$H(\omega)\left(j\omega + \frac{1}{RC}\right) = \frac{1}{RC}$$

$$H(\omega) = \frac{\frac{1}{RC}}{j\omega + \frac{1}{RC}} = \frac{1}{j\omega RC + 1}$$

$$x(t) = \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$$

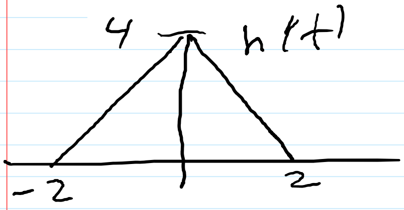
$H(\omega)$

$$y(t) = \sum^{\infty} c_n |H(n\omega_0)| \cos(n\omega_0 t + \theta_n + \phi_n)$$

$H(\omega)$

$$y(t) = \sum_{n=1}^{\infty} c_n \left|H(n\omega_0)\right| \cos(n\omega_0 t + \theta_n + \phi_n)$$

1. Let the system input be x(t) = 5δ(t-4) and the system impulse response of an LTI system be h(t) = 4tri($\frac{t}{2}$). Find the system output y(t) = x(t)∗h(t).



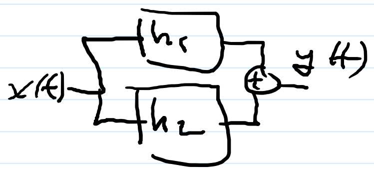
$\delta(t) \to \boxed{h} \to h(t)$

$5\delta(t) \to \boxed{} \to 5h(t)$

$5\delta(t-4) \to \boxed{} \to 5h(t-4)$

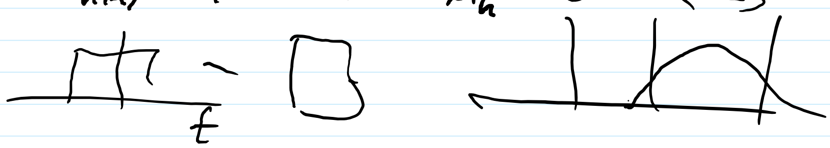
Properties of conv

- $x(t) \to \boxed{h_1} \to \boxed{h_2} \to y(t)$ $\quad x(t) \to \boxed{h_1 * h_2} \to y(t)$



$x(t) \to \boxed{h_1(t) + h_2(t)}$

x(t) has width W_x $\quad y \to W_y = W_x + W_h$
h(t) " " W_h



- causality LTI: If $h(t) = u(t)h(t)$ then causal
- BIBO stability: If $\int_{-\infty}^{\infty} |h(t)|$ is finite then stable
- $x(t) = e^{j\omega t} \to \boxed{} \to H(\omega)e^{j\omega t} = y(t)$
- Find $H(\omega)$ for LCCDE
- $x(t) = A\cos(\omega_1 t + \theta)$

$$y(t) = A\left|H(\omega_1)\right| \cos(\omega_1 t + \theta + \angle H(\omega_1))$$

$$x(t) = \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n)$$

$$y(t) = \sum^{\infty} c_n \left|H(n\omega_0)\right|$$

$$x(t) = \sum_{n=1} C_n \cos(n\omega_0 t + \theta_n)$$

$$y(t) = \sum_{n=1}^{\infty} C_n \left|H(n\omega_0)\right| \cos\left(n\omega_0 t + \theta_n + \angle H(n\omega_0)\right)$$

Example: $C\frac{dy(t)}{dt} + \frac{y(t)}{R} = C\frac{dx(t)}{dt}$

$$Cj\omega H(\omega)\cancel{e^{j\omega t}} + \frac{H(\omega)\cancel{e^{j\omega t}}}{R} = Cj\omega \cancel{e^{j\omega t}}$$

Solve for $H(\omega)$ Not function of $\underline{t}$

$H(\omega)$ is complex $\quad H(-\omega) = H^*(\omega)$

$|H(\omega)|$ even $\quad \angle H(\omega)$ is odd

$$H(\omega) = \frac{j\omega}{j\omega + \frac{1}{RC}} \qquad RC = \frac{1}{2\pi}$$

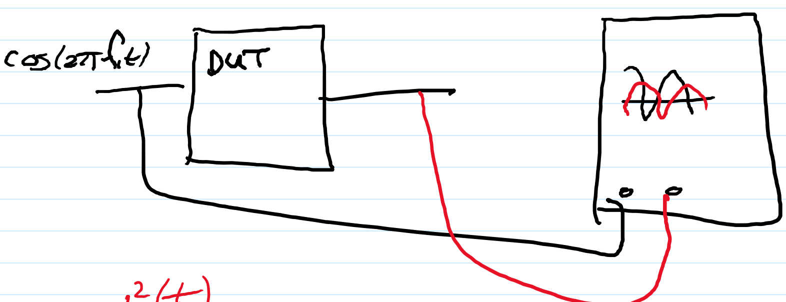
$x(t) = \cos(2\pi t)$ find $y(t)$ $\quad \omega = 1$

$|H(1)| = \frac{1}{\sqrt{2}} \qquad \angle H(1) = \pi/4$

$$y(t) = \frac{1}{\sqrt{2}}\cos(2\pi t + \pi/4)$$

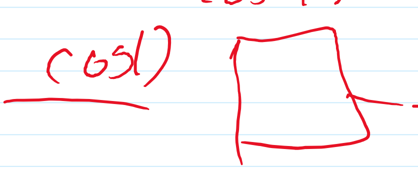
$P_x = \frac{1}{2} \qquad P_y = \frac{(1/\sqrt{2})^2}{2} = 1/4$

$P_y/P_x = \frac{1}{2} \qquad 10\log(P_y/P_x) = -3\text{dB}$



$x^2(t)$

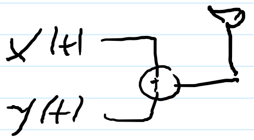
$\cos^2(t) = \frac{1}{2} + \frac{1}{2}\cos(2\omega t)$

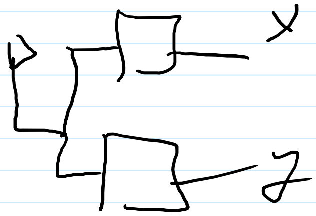


$x_p(t) \qquad T_0 \qquad f_0 = \frac{1}{T_0}$ Hz

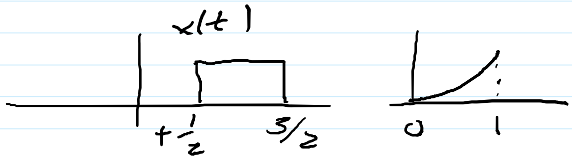
$\rightarrow \omega_0 = \frac{2\pi}{T_0}$

$n\omega_0 \qquad n > 1 \qquad$ harmonics

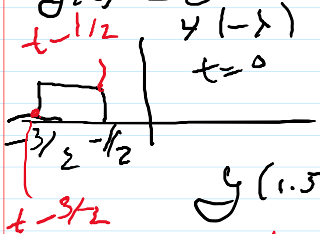




February 10, 2026



$y(0) = 0$



λ^2 $t = 1.5$

$y(1.5)$

$y(1.5) = \int_0^1 \lambda^2 d\lambda = \frac{1}{3}$

- $H(\omega) \rightarrow$ given LCCDE

  $x(t) = \sum C_n \cos(n\omega_0 t + \theta_n)$ ω_0 = fundamental freq

  $y(t) = \sum |H(n\omega_0)| c_n \cos(n\omega_0 t + \theta_n + \angle H(n\omega_0))$

- Phasor $\bar{X} = |X| e^{j\phi}$ LCCDE

  solve for $\bar{Y}$ @ ω_1 $y(t) = Re[\bar{Y} e^{j\omega t}]$

- Orthogonality x & y are real

  $\int_0^T x(t) y(t) dt = 0$

Example: $f_c = 100\,MHz$ $\Delta f = 20\,kHz$

$T = \frac{1}{\Delta f} = 50\,\mu s$

$\int_0^T \cos(2\pi f_c t) \cos(2\pi (f_c + \Delta f) t) = 0$

$\frac{f_c}{\Delta f}$ = integer

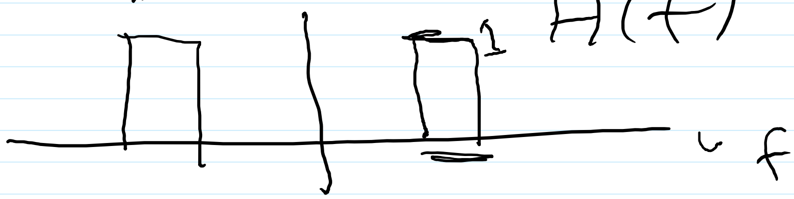
- $x_p(t)$ periodic T_0 $f_0(Hz) = \frac{1}{T_0}$

  $\omega_0 = \frac{2\pi}{T_0}$

$x_p(t) = a_0$

$$\omega_0 = \frac{2\pi}{T_0}$$

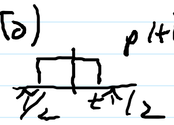
$$x_p(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \cos(n\omega_0 t)$$



$$y(t) = c_0|H(0)| + \sum_{n=1}^{\infty} c_n |H(nf_0)| \cos(2\pi n f_0 t + \theta_n + \angle H(nf_0))$$

Feb 12, 2026

Fourier Series $x_p(t) = \sum_{k=-\infty}^{\infty} p(t - kT_0)$

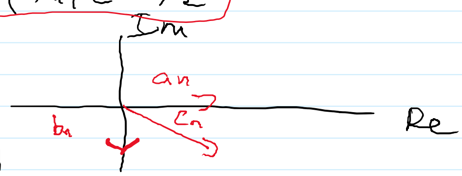


$$x_p(t) = c_0 + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 t + \theta_n) \qquad c_n \geq 0 \quad n \neq 0$$

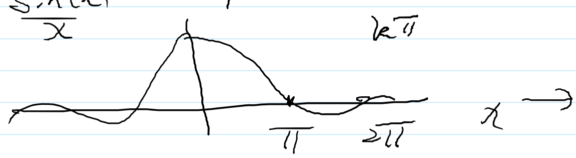
$$a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$$

$$\sum_{n=-\infty}^{\infty} x_n e^{jn\omega_0 t} \qquad x_n = |x_n| e^{j\theta_n}$$

$$x_n = \frac{1}{2}(a_n + j b_n) \qquad \boxed{|x_n| = c_n/2}$$



- $\text{sinc}(x) = \dfrac{\sin(x)}{x}$



- $p(t) = \text{rect}(t/\tau)$

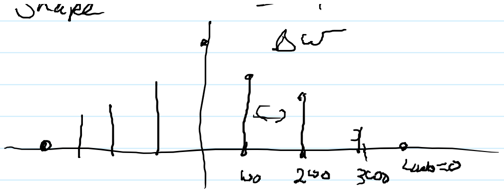
$$\underset{n=0}{\frac{\tau}{T_0}} + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} (\tau/T_0)\, \text{sinc}\left(\frac{n\pi\tau}{T_0}\right)$$

Line spacing $= 1/T_0$ (Hz) $\qquad \frac{2\pi}{T_0}$ rad/sec

Shape $\rightarrow \tau \qquad \tau/\underline{T_0} = 1/4$

BW

Shape $\quad T_0 = 14$



$\frac{n\pi}{4} \quad n = \frac{4}{\pi}$

— If $p(t)$ is real then $|X_n|$ real ϕ_n odd

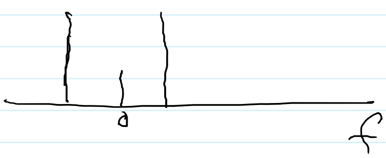
— Magnitude plots

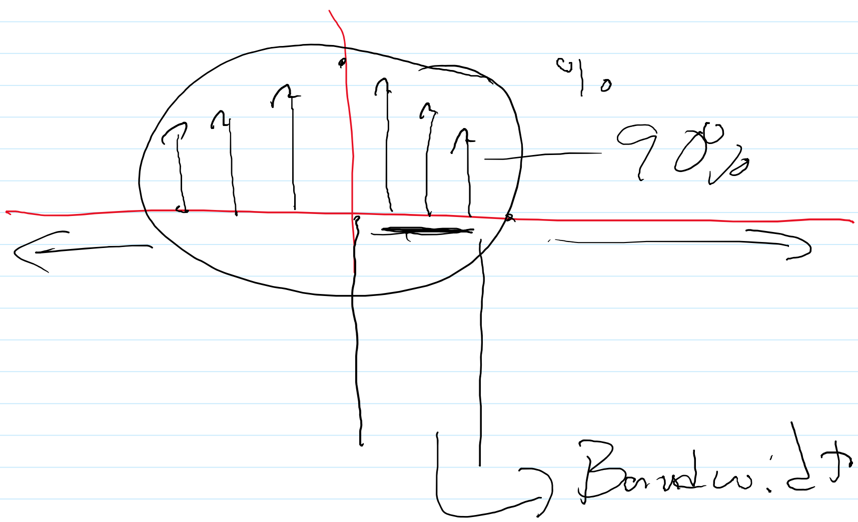
— Given LCCDE $\longrightarrow H(\omega)$

$x_p(t) \longrightarrow$ F.S

$x_p(t)$ —— $\boxed{\quad}$ —— $y(t)$

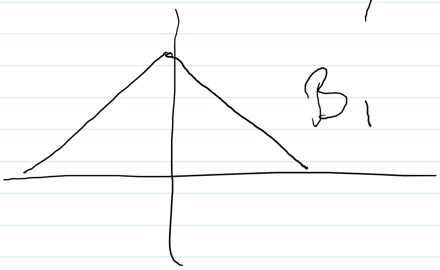
$$= c_0 |H(0)| + \sum_{n=1}^{\infty} c_n |H(n\omega_0)| \cos(n\omega_0 t + \phi_n)$$





$$B \propto \frac{1}{T}$$

$B \sim \frac{1}{T}$



B_1

B_2

$B_2 > B_1$

Feb 17, 2026 HW 6 #9

$h(t) = u(t)e^{-t} - 16e^{-2t}u(t) + 13e^{-3t}u(t)$

$x(t) = \cos(2t) \quad \omega_1 = 2$

Section 3.7 $\quad H(\omega) = \int_{-\infty}^{\infty} h(\tau) e^{-j\omega\tau} d\tau$

Ex 3.7.1 $\quad \int_0^{\infty} e^{-a\tau} e^{-j\omega\tau} d\tau = \frac{1}{a+j\omega}$

$H(\omega) = \frac{5}{1+j\omega} - \frac{16}{2+j\omega} + \frac{13}{3+j\omega}$

$$\boxed{y(t) = |H(\omega_1)| \cos(2t + \angle H(\omega_1))}$$

$H(\omega_1) = \underbrace{\frac{5}{1+2j}}_{1-2j} - \underbrace{\frac{16}{2+2j}}_{-4+4j} + \underbrace{\frac{13}{3+2j}}_{3-2j} = 0$

$\omega_1 = 2$

HW 6 #7 $\quad \frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 7y = 5\frac{dx}{dy}$

$H(\omega) \quad x(t) = e^{j\omega t} \quad y(t) = H(\omega)e^{j\omega t}$

$H(\omega) = \frac{5j\omega}{(j\omega)^2 + 2j\omega + 7} = \frac{5j\omega}{(7-\omega^2) + 2j\omega}$

b. Find ω_1 such that $\angle H(\omega_1) = 0$

$7 - \omega_1^2 = 0 \quad H(\omega_1) = 5/2$

$\omega_1 = \sqrt{7}$

— F.S. $\quad x_p(t) = \sum X_n e^{jn\omega_0 t}$

$H(\omega) \quad y(t) = |X_n||H(n\omega_0)|$

– F.S. $x_p(t) = \sum X_n e$

$H(\omega)$ $y(t) = |X_n||H(n\omega_0)|$
$\cos(n\omega_0 t + \angle X_n + \angle H(n\omega_0))$

$H(\omega)$ weighting on each ω

– Parseval's Theorem $\sum_{n=-\infty}^{\infty} |X_n|^2 = \frac{1}{T_0}\int_0^{T_0} |x(t)|^2 dt$

– Example! $x(t) = 10\,\text{rect}(t/0.1)$ $x_p(t) = \sum_{k=-\infty}^{\infty} x(t - kT_0)$

$T_0 = .5$ Find power between 1Hz – 5Hz

$x_p(t) = \frac{A\tau}{T_0} + \sum_{n=1}^{\infty} \frac{A\tau}{T_0}\,\text{sinc}\left(\frac{n\pi\tau}{T_0}\right)\cos(n\omega_0 t)$

$A = 10$ $\tau = .1$ $T_0 = .5$ $\frac{\tau}{T_0} = .2$ $f_0 = 2\text{Hz}$

$\text{sinc}\left(\frac{5\pi}{.2}\right)$
$\text{sinc}(\pi) = 0$

$50\,\text{sinc}\left(\frac{\pi}{5}\right)\cos(2\pi 2t) + 50\,\text{sinc}\left(\frac{2\pi}{5}\right)\cos(2\pi 4t)$

$P = 1803.$

End for Test 1

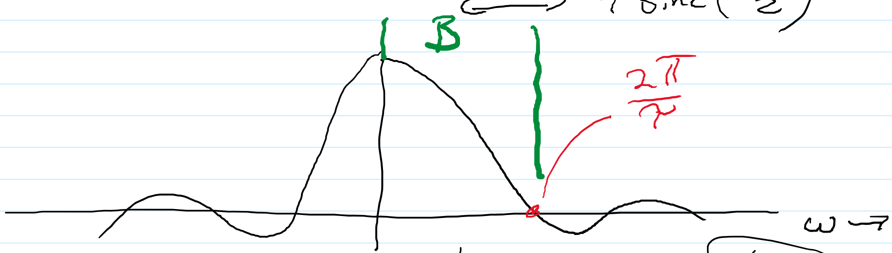
Fourier Transform $X(\omega) = \int_{-\infty}^{\infty} x(\tau) e^{j\omega\tau} d\tau$

$H(\omega) = \int_{-\infty}^{\infty} h(\tau) e^{j\omega\tau} d\tau$

$H(\omega) = \mathcal{F}\{h(t)\}$

$x(t) = \mathcal{F}^{-1}(X(\omega)) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{-j\omega t} d\omega$

Example $x(t) = \text{rect}(t/\tau) \longleftrightarrow \tau\,\text{sinc}\left(\frac{\omega\tau}{2}\right)$

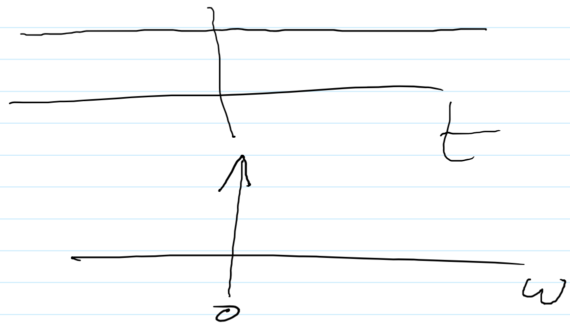


$B \propto \frac{1}{\tau}$ Hz $\boxed{\$/\text{Hz}}$

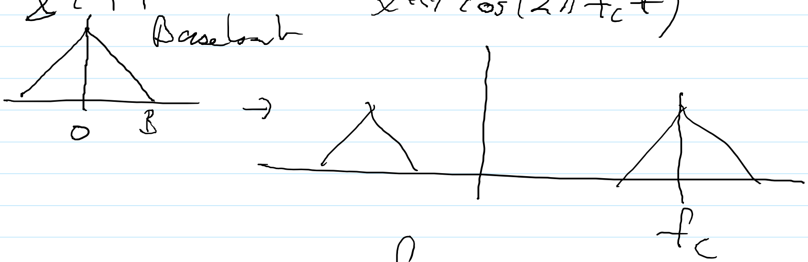
bits/sec $\neq$ Hz

bits/sec $\neq$ Hz

D.C.



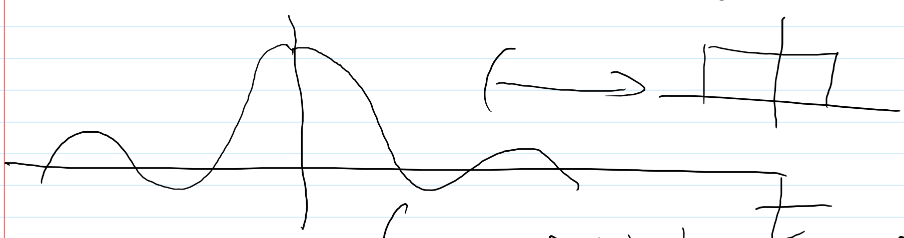
$X(f)$ $\quad x(t)\cos(2\pi f_c t)$



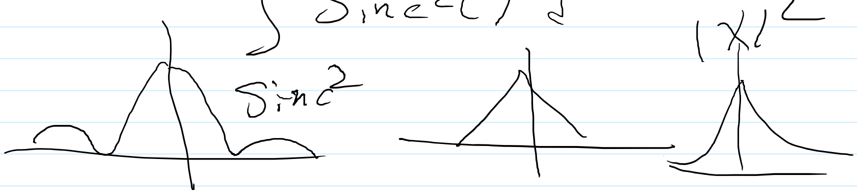
C = carrier freq.

9.5 MHz

$B = 3$ kHz $\quad$ AM $\quad f_c = 610$ kHz



$\int \text{sinc}^2(\ )\, d$ $\qquad |X|^2$



February 19, 2026

$$\frac{dy}{dt} + y(t) = x(t) \qquad x(t) = e^{j\omega t}$$
$$y(t) = H(\omega) e^{j\omega t}$$

$$j\omega H(\omega)\cancel{e^{j\omega t}} + H(\omega)\cancel{e^{j\omega t}} = \cancel{e^{j\omega t}}$$

$$H(\omega) = \frac{1}{1+j\omega} \qquad x(t) = 1 \rightarrow 1$$
$$\omega = 0$$

$$\cos(t) \rightarrow \omega = 1 \qquad H(1) = \frac{1}{1+j\omega} = \frac{1}{\sqrt{2}e^{j\pi/4}}$$

$$\sqrt{2}\cos(t - \pi/4)$$

$\sqrt{2}\cos(t - \pi/4)$

Fourier Transform — properties

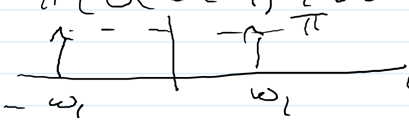
- Additive
- Time scaling $B \propto \frac{1}{\tau}$
- time shift $x(t-t_0)$ linear phase
- freq shift
- modulation
- duality $x(t) \Leftrightarrow X(\omega)$, $X(t) \Leftrightarrow x(-\omega)$
- conv in time is multiply freq
  $x(t) * h(t) \longleftrightarrow X(\omega)H(\omega)$
- multiply time
  $x(t)\cdot y(t) \longleftrightarrow \frac{1}{2\pi} X(\omega) * Y(\omega)$?
- If $x(t)$ is real then $|X(\omega)|$ is even, $\angle X(\omega)$ is odd

FT pairs

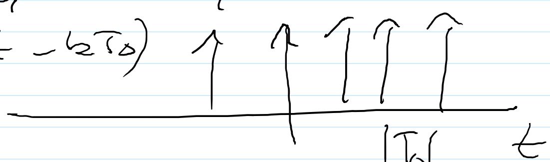
$\delta(t) \Leftrightarrow 1 \qquad 1 \Leftrightarrow 2\pi\delta(\omega)$

$A\,\mathrm{rect}(t/\tau) \Longleftrightarrow A\tau\,\mathrm{sinc}\left(\frac{\omega\tau}{2}\right)$

$\cos(\omega_1 t) \Longleftrightarrow \pi\left(\delta(\omega+\omega_1) + \delta(\omega-\omega_1)\right)$



$$\delta_{T_0}(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$$



$$\updownarrow$$

$$\frac{1}{T_0}\sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0) \qquad \omega_0 = \frac{2\pi}{T_0}$$

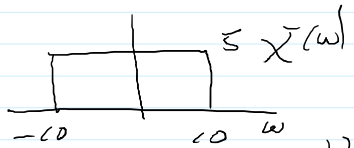
Example $x(t) = \frac{50}{\pi}\,\mathrm{sinc}(10t)$

a) E_x

b) % energy $B = 2.5$ rad/sec

$$\int_{-\infty}^{\infty} \left(\frac{50}{\pi}\,\mathrm{sinc}(10t)\right)^2 dt$$

$$X(\omega) = 5\,\mathrm{rect}\left(\frac{\omega}{20}\right)$$



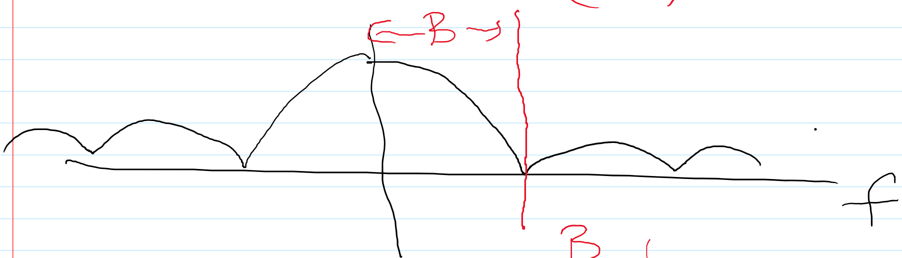
$$\frac{1}{2\pi}\int |X(\omega)|^2 d\omega$$

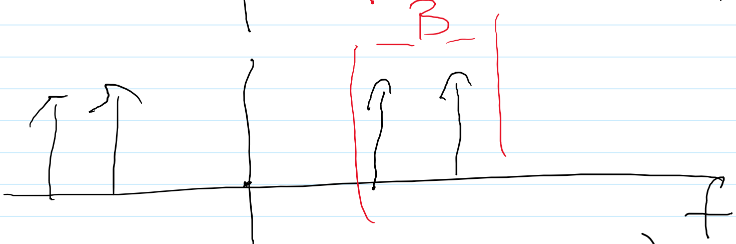
$$\frac{20\cdot 25}{2\pi} = 79.6$$

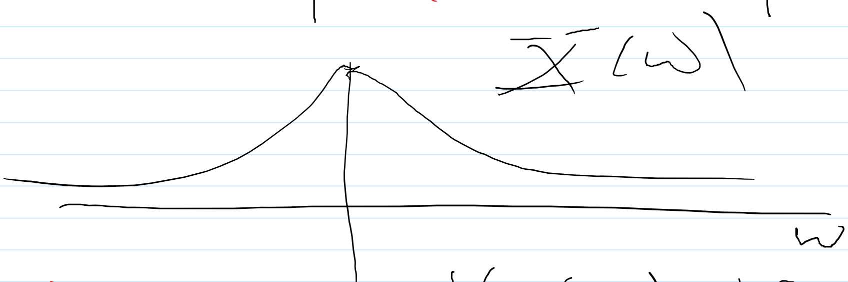
$$\% = 100\left(\frac{5\cdot 25}{2\pi}\right) = 25\%$$

$$\frac{20\cdot 25}{2\pi} = 79.6$$

$$\% = 100\left(\frac{\frac{5\cdot 25}{2\pi}}{\frac{20\cdot 25}{2\pi}}\right) = 25\%$$





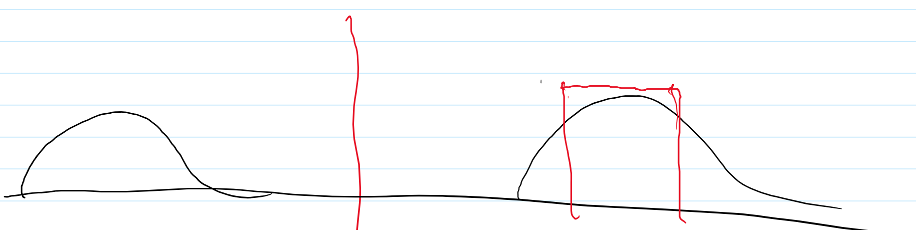


$$\Psi_x(f) = |X(f)|^2$$

$$\int_{-\infty}^{\infty} \Psi_x(f)\,df = \Psi_x(0)\,2B_e$$

$$B_e = \frac{\int_{-\infty}^{\infty} \Psi_x(f)\,df}{2\Psi_x(0)}$$

$$= \frac{\int_0^{\infty} \Psi_x(f)\,df}{\Psi_x(0)}$$

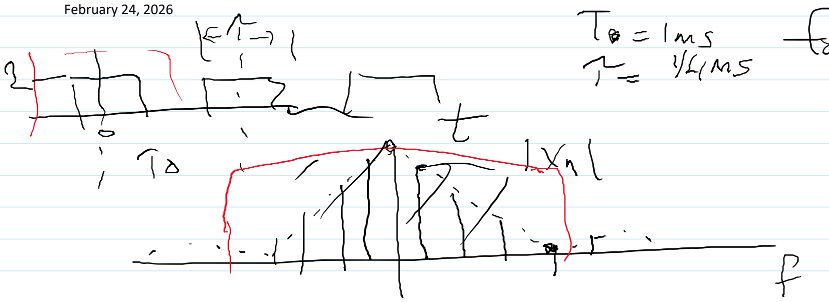


Bandpass case

Lowpass case

$$\boxed{B \propto \frac{1}{\tau}}$$

February 24, 2026

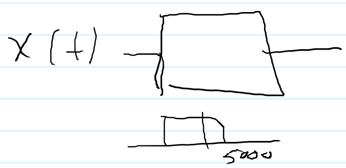


$T_0 = 1ms \quad f_0 = \underline{1000}$

$\tau = 1/4 ms$

$1/\tau = 4000 Hz$

$$P_x = \int_{-T_0/2}^{T_0/2} (2\,rect(t/\tau))\,dt = \int_{-\tau/2}^{\tau/2} 4\, d\tau = 1$$



$y(t) = \quad n = -5 \cdots 5$

$$P_y = \sum_{n=-5}^{5} |X_n|^2$$

$x_0 = 1/2$

$X_1 = 0.45 \quad X_4 = 0$

$X_2 = .35 \quad X_5 = 0.09$

$X_3 = .15$

$$X_n = A \frac{\tau}{T_0} \operatorname{sinc}\left(\frac{n\pi\tau}{T_0}\right)$$

$P_y = 0.91 \quad 91\%$

$$X_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\, e^{jn\omega_0 t}\, dt$$

$$\frac{1}{T_0}\int_{-\tau/2}^{\tau/2} 2\, e^{jn\omega_0 t}\, dt$$

$$x(t) = \cos(2\pi 500 t) + \frac{1}{2} + \frac{1}{2}\cos(2\pi 1000 t)$$

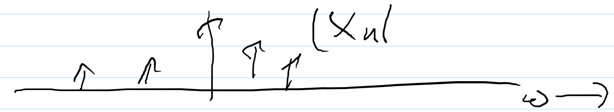
$a_0 = \frac{1}{2} \quad a_1 = 1 \quad a_2 = 1/2$

- $E_x = \int_{-\infty}^{\infty} |x(t)|^2\, dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2\, d\omega$

$x(t) \longleftrightarrow X(\omega)$

- $x_p(t) = \sum X_n e^{jn\omega_0 t} \longleftrightarrow \sum 2\pi X_n \delta(\omega - n\omega_0)$

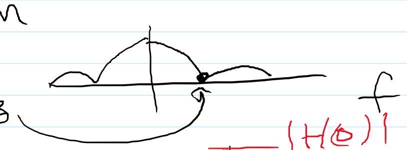
- $x_p(t) = \sum X_n e^{jn\omega_0 t} \longleftrightarrow \sum 2\pi X_n \delta(\omega - n\omega_0)$



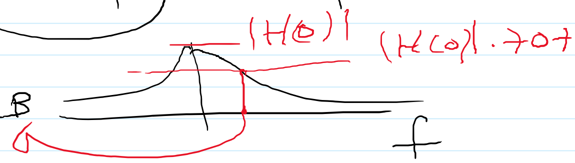
- $x_p(t) = \sum x(t - kT_0)$ $\qquad \omega_0 = \frac{2\pi}{T_0}$

$$X_n = \frac{1}{T_0} X(n\omega_0)$$

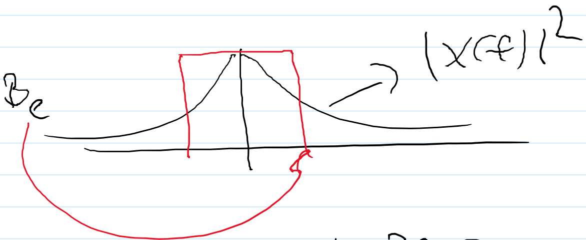
- Bandwidth
  - B_{fz}



  - B_{3dB}



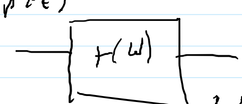
  - B_e

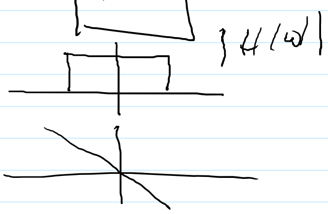


  - B_{rms}
  - $B\tau \rightarrow$ constant

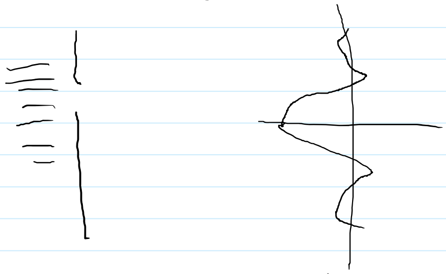
$$B \propto \frac{1}{\tau}$$

$x_p(t)$





$y(t) = A|H(\omega)| \cos(\omega t + \theta \angle H(\omega))$



$K \neq 0$

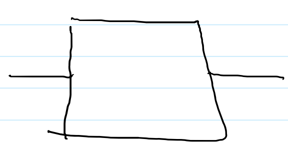
$K\delta(t - t_0)$

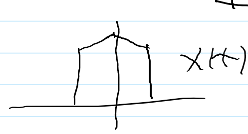
$\delta(t)$

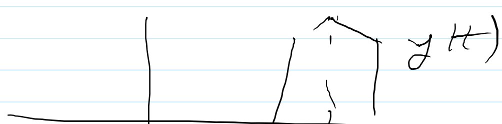
$Kx(t - t_0)$

Ideal filter

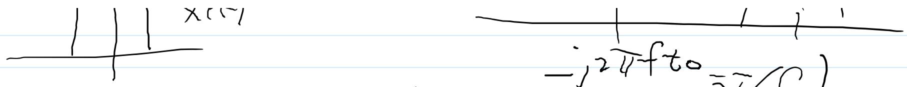
$x(t)$





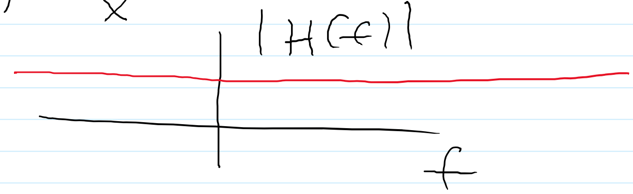


$K e^{-j2\pi f t_0} X(f)$

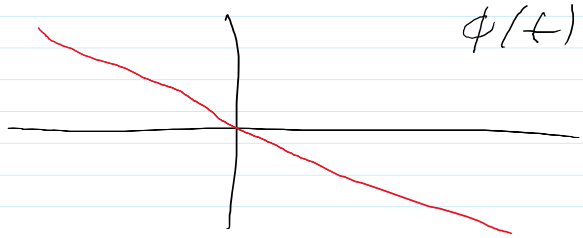


$X(f)$ $\qquad Y(f) = K e^{-j2\pi f t_0} X(f)$

$$H(f) = \frac{Y}{X} = K e^{j2\pi f t_0}$$



Constant Magnitude



Linear Phase

$$h(t) = K\delta(t - t_0)$$

dB $\qquad$ dB_m $\qquad$ Power wrt 1mS

1 mW $\qquad$ $0\,dB_m$

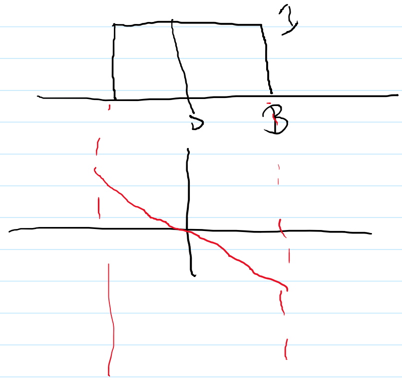
10 mW $\qquad$ $= 10\,dB_m$

dB_W $\qquad$ 1 watt

dB_{staft}

ILPF $\qquad K e^{-j2\pi f t_0}$ $\qquad |f| < B$

$\qquad = 0$ $\qquad |f| > B$



February 26, 2026

HW 7 $\quad$ 15 $\qquad 5\cos(2\pi 300t) - 2\sin(2\pi 600t)$

$$5\cos(2\pi 300t) - 2\cos(2\pi 600t - \pi/2)$$

$$5\cos(2\pi 300t) + 2\cos(2\pi 600t - \pi/2 - \pi) \quad -\frac{3\pi}{2}$$

$$\underbrace{\frac{2}{2} e^{-j3\pi/2}}_{x_{-2}} e^{-j2\pi 600t} + \underbrace{\frac{5}{2}}_{x_{-1}} e^{-j2\pi 300t} + \underbrace{\frac{5}{2}}_{x_1} e^{+j2\pi 300t} + \underbrace{\frac{2}{2} e^{+j\frac{3\pi}{2}}}_{x_2} e^{+j2\pi 600t}$$

$$P = \sum |x_i|^2 = 1^2 + \left(\frac{5}{2}\right)^2 + \left(\frac{5}{2}\right)^2 + 1^2$$

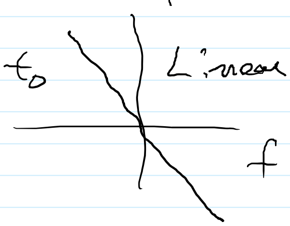
$$P = \sum |x_i|^2 = 1^2 + \left(\tfrac{1}{2}\right) + \left(\tfrac{1}{2}\right) + 1^2$$
$$= 5^2/_2 + 2^2/_2$$

- $H(\omega) = |H(\omega)| e^{j\phi(\omega)} \longrightarrow \phi(\omega)$ is important
- LPF, BPF, BRF, HPF $\rightarrow$ passband $\rightarrow$ BW
- $x(t)$ B_{signal} — $\boxed{B_{system}}$ — $y(t)$ If $B_{system} >> B_{signal}$ then $y(t) \simeq x(t)$
- RLC bandpass & bandreject filter $\omega_0 = \frac{1}{\sqrt{LC}}$

  tune by change $\underline{C}$ $\boxed{Q\text{-factor}}$
- Distortionless transmission Ideal

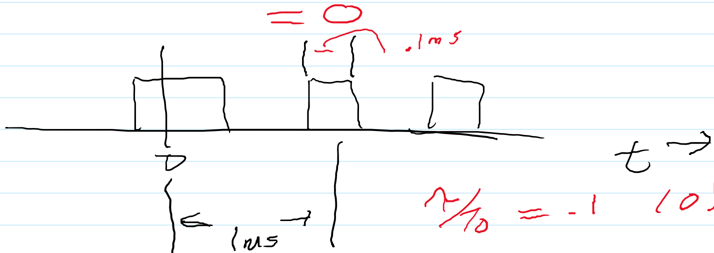
  $x(t)$ — $\boxed{}$ — $y(t) = K x(t - t_0)$ $\forall f$

  $H(f) = K e^{-j2\pi f t_0}$ $\phi(f) = -2\pi f t_0$

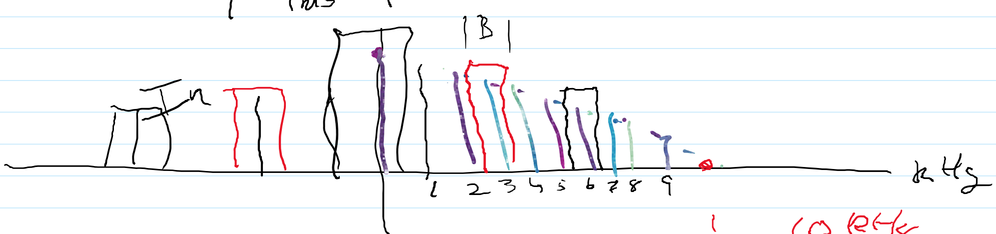


ILPF $H(f) = K e^{-j2\pi f t_0}$ $|f| < B$

$= 0$



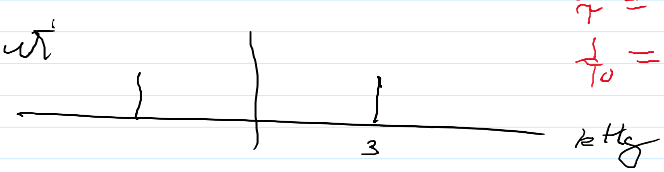
$\tau/_{T_0} = .1$ 10%



$\frac{1}{\tau} = 10$ kHz

$\frac{1}{T_0} = 1$ kHz

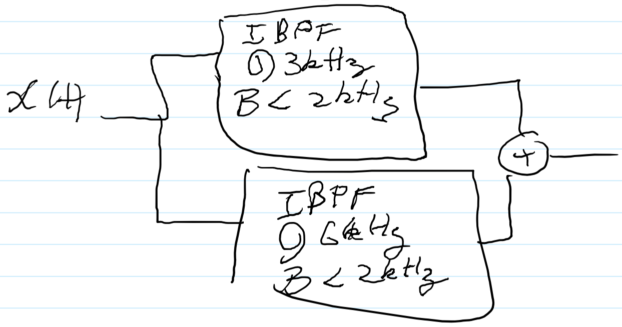
Out



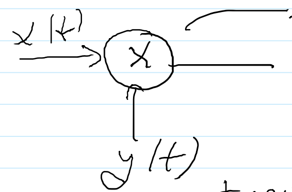
a) IBPF centered at 3 kHz $B < 2$ Hz

b) — $\boxed{\text{IBPF} \; @ \; 3\text{kHz}}$

b)



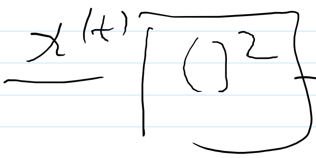
c) ILPF $B < 1kHz$



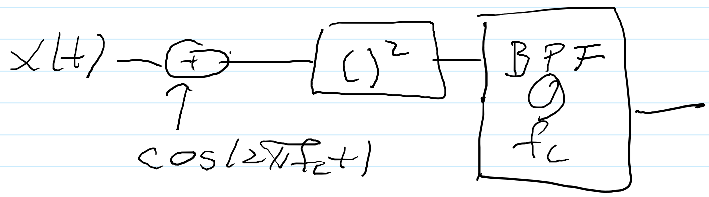
→ mixer product multiplier

typically $y(t) = \cos(2\pi f_c t)$

C = Carrier

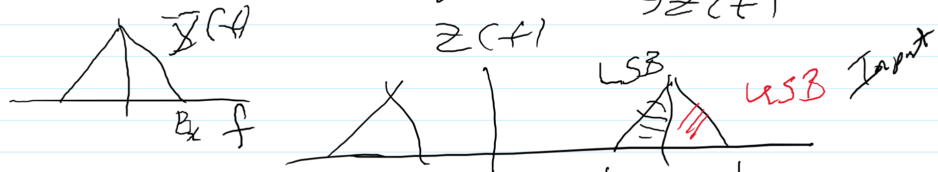


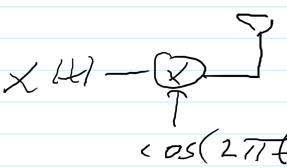
$y(t) = x^2(t)$



$x(t)$ is a LP signal

$K\, x(t)\cos(2\pi f_c t)$
$\to z(t)$



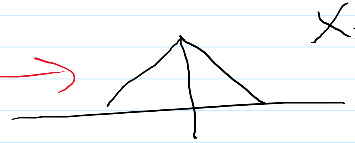


DSB-SC

$B_{RF} = 2B_x$

$X_o(f)$

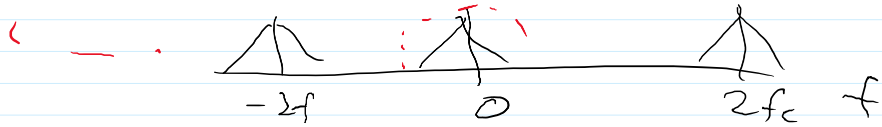
work →

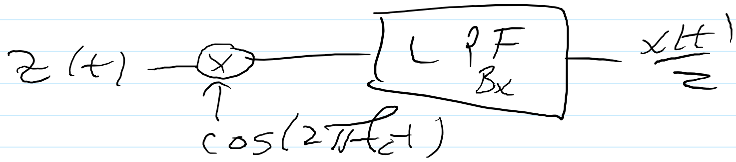


$z(t)\cos(2\pi f_c t) = [x(t)\cos(2\pi f_c t)]\cos(2\pi f_c t)$

$x(t)\cos^2(2\pi f_c t) = \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos(2\pi 2 f_c t)$

$$x(t)\cos^2(2\pi f_c t) = \frac{1}{2}x(t) + \frac{1}{2}x(t)\cos(2\pi 2f_c t)$$





March 3, 2026

$x_p(t)$ $P = \frac{1}{T_0}\int_{T_0} |x(t)|^2 dt$

$$\mathcal{E}_x = \int |x(t)|^2 dt$$

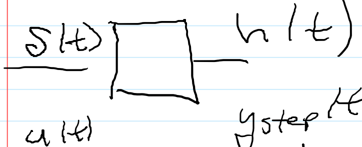
$x(t)$

$x\left(\frac{t-t_0}{\tau}\right)$

$A^2/2$

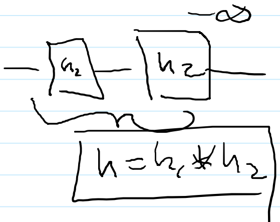
$A_n \cos(n\omega_0 t + \theta_n)$

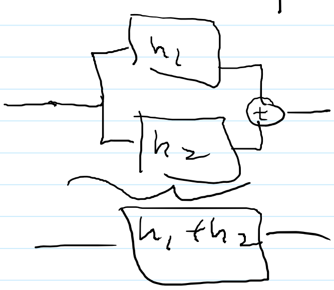
LTI



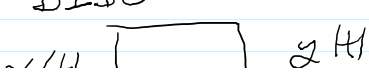
$$y_{step}(t) = \int_{-\infty}^{t} h(\lambda)\, d\lambda$$

series





BIBO



BIBO

If $\int |h(t)| dt$ finite

then BIBO

If $h(t) = 0 \quad t < 0$

then causal

If $h(t) = h(t)u(t)$

- $x(t) = e^{j\omega t}$ LTI

$H(\omega) e^{j\omega t}$

$H(\omega)$

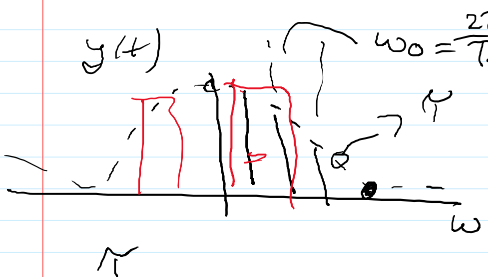
If $x(t) = A\cos(\omega_1 t + \theta)$

$y(t) = A |H(\omega_1)| \cos(\omega_1 t + \theta + \angle H(\omega_1))$

Find $H(\omega)$

Find C_n & ϕ_n

$y(t)$ $\quad \omega_0 = \frac{2\pi}{T_0}$



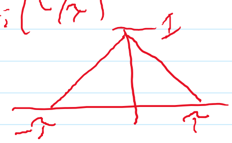
$\sum A_n \cos(n\omega_0 t + \phi_n)$

$P = \sum_{n=0}^{\infty} A_n^2 / 2$

$\sum_{n=-\infty}^{\infty} |X_n|^2$

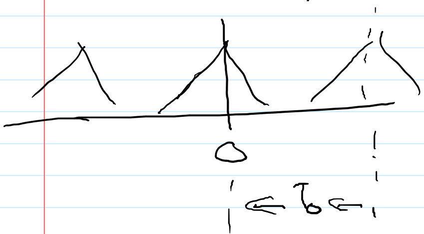
% power in Band of frequencies

$tri(t/\tau)$

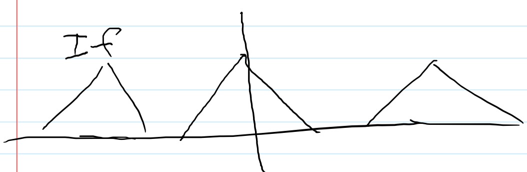


$(t - nT_0)$

$$\sum \text{tri}\left(\frac{t-nT_0}{\tau}\right)$$

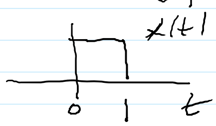


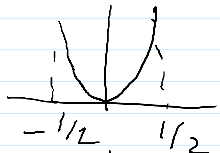
If

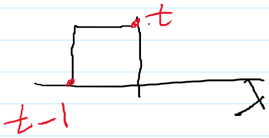


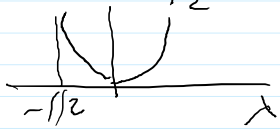
$b_n = 0$

$a_n = 0$

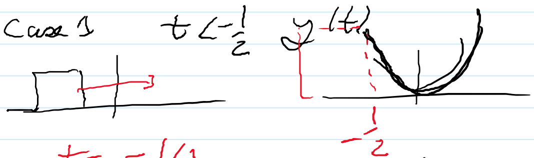




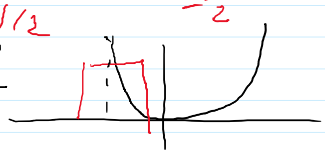




Case 1 $t < -\frac{1}{2}$

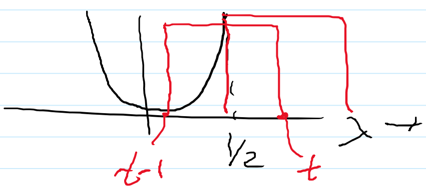


case 2



$$\int_{-1/2}^{t} \lambda^2 \, d\lambda \qquad -\frac{1}{2} < t < \frac{1}{2}$$

Case 3



$$\int_{t-1}^{1/2} \lambda^2 \, d\lambda \qquad \frac{1}{2} < t < 3/2$$

$t-1 = 1/2$

Dynamic $y(t) = x(t) - 5x(t+3)$

$y(3) = x(3) - 5x(8)$

Causal → not causal

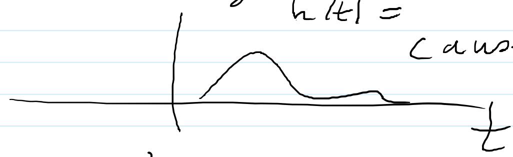
$y(3) = x(3) - 5x(8)$

Causal → not causal

LTI for $h(t) \neq K$

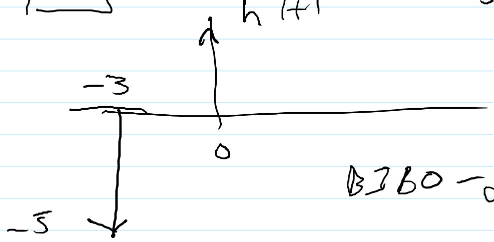
dynamic

$h(t) =$ causal



Find $h(t)$

$\delta(t) \to \boxed{} \to h(t)$ $\qquad h(t) = \delta(t) - 5\delta(t+3)$



BIBO - stable

$a x_1(t) \longrightarrow a x_1(t) - 5a x_1(t+3) = y_1$

$b x_2(t) \to b x_2(t) - 5b x_2(t+3) = y_2$

$a x_1 + b x_2 \to y_1 + y_2$ so linear

$x(t-\tau) = x_1(t-\tau) - 5x_1(t+3-\tau)$

$= y_1(t-\tau)$ TI

$y(t) = x(t) - 5x(t+3) + 3$

NL

$\text{sinc}(x) = \frac{\sin(x)}{x}$

$c_1 \frac{dy}{dt} + c_2 y(t) = x(t)$

$c_1 j\omega H(\omega) \cancel{e^{j\omega t}} + c_2 H(\omega) \cancel{e^{j\omega t}} = \cancel{e^{j\omega t}}$

$H(\omega) = \frac{1}{c_1 j\omega + c_2}$

$= |H(\omega)| e^{j\phi(\omega)}$

$x_1(t) = 4\cos(300t - \pi/6)$

$y_1(t) = 4|H(300)| \cos(300t - \frac{\pi}{6} + \phi(300))$

$x_2(t) = -8\cos(600t - \pi/4)$

$y_2(t) = 8|H(600)| \cos(600t - \pi ... \phi(600))$

$x_2(t) = -8\cos(600t - \pi/4)$

$y_2(t) = -8\,|H(600)|\cos(600t - \frac{\pi}{4} + \phi(600))$

$y(t) = y_1(t) + y_2(t)$

HW 7 #7

$n=1 \qquad n=2$

$x(t) = 8\cos(300t) - 3\sin(600t)$

$\omega_0 = 300$

$a_0 = 0 \quad a_1 = 8 \quad b_1 = 0$

$a_2 = 0 \quad b_2 = -3$

$C_n \qquad c_0 = 0$

$8\cos(300t) - 3\cos(600t - \frac{\pi}{2})$

$8\cos(300t) + 3\cos(600t - \frac{\pi}{2} - \pi)$

$c_1 = 8 \quad \phi_1 = 0 \qquad c_2 = 3 \quad \phi_2 = -\frac{3\pi}{2}$

$\frac{3}{2} e^{+j\frac{3\pi}{2}} e^{-j600t}$) $n = -2$

$+ \frac{8}{2} e^{-j300t}$) $n = -1$

$+ \frac{8}{2} e^{+j300t}$) $n = +1$

$3/2\, e^{-j\frac{3\pi}{2}} e^{+j600t}$) $n = +2$

$X_{-2} = 3/2\, e^{+j3\pi/2} \qquad X_{-1} = 8/2$

$X_1 = 8/2 \qquad X_2 = \frac{3}{2} e^{-j\frac{3\pi}{2}}$

$X_0 = 0$