EECS 562

Homework 1

1. Let

a. z_1 = 4+j6, Find Re(z_1), Im(z_1), $|z_1|$, $|z_1|^2$ and α and β in $z_1 = \alpha e^{j\beta}$

b. Repeat a for $z_1 = -4 - j6$

c. Find $x(t) = \mathrm{Re}[z_1 e^{-j2\pi f_c t}]$ where f_c = 100 Hz

2. Let z_1 = 3.7+j1.5, z_2 = -3.7+ j1.5, z_3 = -3.7-j1.5, z_4 = 3.7-j1.5

a. Plot z_i for i=1..4 putting the real part of z_i on the x-axis and the imagery part of z_i on the y-axis.

b. For f_c = 10 MHz find $x_i(t) = \mathrm{Re}[z_i e^{-j2\pi f_c t}]$ for i=1..4

(This problems leads to the mathematical model for quadrature phase shift keying - QPSK; for each symbol time sending one of these 4 signals to send 2 bits)

3. Let $x_1(t) = 10\cos(2\pi 1000(t - 62.5\mu s))$ and $x_2(t) = 10\cos(2\pi 1000t - \frac{\pi}{8})$, $x_1(t) \mp x_2(t)$ TRUE or FALSE

4.

For

$x_1(t) = 10\,\mathrm{sinc}(10t)$,

$x_2(t) = 20\,\mathrm{sinc}(20\,t)$

$x_3(t) = 50\,\mathrm{sinc}(50\,\pi)$

With $\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$

a. Find the Fourier transform of $x_1(t)$, i.e. find $X_1(f)$.

b. Plot x_1(t), x_2(t), and x_3(t)

c. Rank order the signals from lowest bandwidth to highest bandwidth.

5. Find the power and energy in $4\cos(2\pi 1000t) + 8\sin(2\pi 2000t)$

6. A bit is transmitted as

x(t) = $A\cos(2\pi f_c t)$ if bit = “1” for T_b

or

x(t) = $-A\cos(2\pi f_c t) = A\cos(2\pi f_c t - \pi)$ if bit = “0” for T_b

For $A = 1 \times 10^{-4}$ and T_b=10μs and f_c = 100 MHz

a. Find the energy and power in x(t).

b. What is the bit rate in Mb/s?

(This problem provides the basis for Binary Phase Shift Keying-BPSK.)

7. Let $x_1(t) = \cos(2\pi f_1 t)\,\mathrm{rect}(\frac{t-0.05}{0.01})$ and $x_2(t) = \cos(2\pi f_2 t)\,\mathrm{rect}(\frac{t-0.05}{0.01})$

For f_1=2000 and f_2=2100

a. Find $\int_{-\infty}^{\infty} \cos(2\pi f_1 t)\sin(2\pi f_1 t)\,dt$

b. Find $\int_0^{0.01} \cos(2\pi f_1 t)\cos(2\pi f_2 t)\, dt$

c. Define the orthogonality property of signals

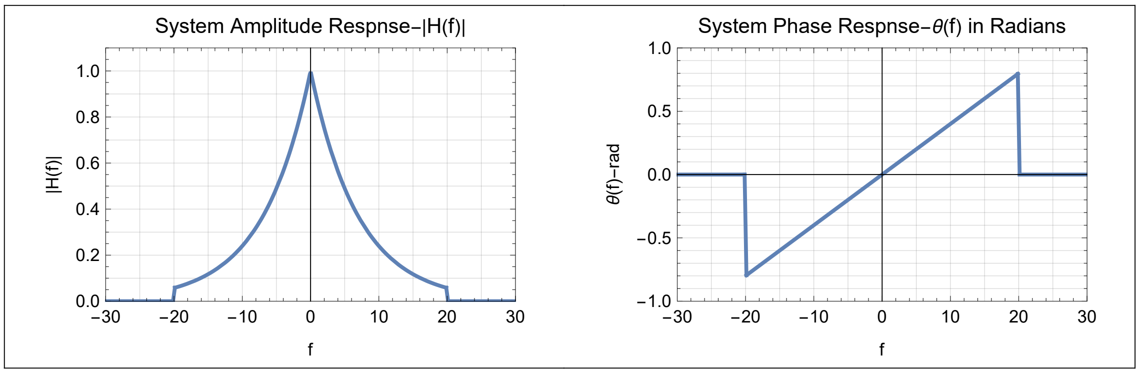
8. For x(t) = 4cos(2π1000t) + 8sin(2π2000t)

a. Plot the double-sided phase and magnitude spectrum for x(t).

b. What is the bandwidth of x(t)?

9. A system with $H(f) = |H(f)|\, e^{-j\theta(f)}$

where amplitude $|H(f)| = \text{rect}\left(\frac{f}{40}\right) e^{-\left|\frac{f}{7}\right|}$ and phase $\theta(f) = -\frac{f}{25}$ response is given below.



a. The signal x(t)=cos(2π2.5t)+cos(2π12.5t) is the input to the system with the frequency response H(f) given above. Find the system output, y(t).

b. A signal g(t) with a bandwidth of B_g rad/sec is the input to a system with H(ω) with a bandwidth of B_{sys} rad/sec, the system output is y(t). If $B_{sys} \gg B_z$ then y(t)≈g(t). TRUE or FALSE.

10. A signal x(t)=δ(t-0.1) is the input to a linear time invariant system with an impulse response of h(t) = $e^{-10|t|}$. Find and sketch the system output y(t).

11. A filter has an impulse response of h(t) = sinc(100t)

a. Is this an ILPF? Yes or NO

b. What the filter bandwidth?

c. With an input signal x(t)=2δ(t) + δ(t- τ) where τ=10ms input to this filter, plot the output signal y(t) in the time domain.

d. What is y(0) and y(τ)? How are they related to x(0) and x(τ)?

12. Solve the following.

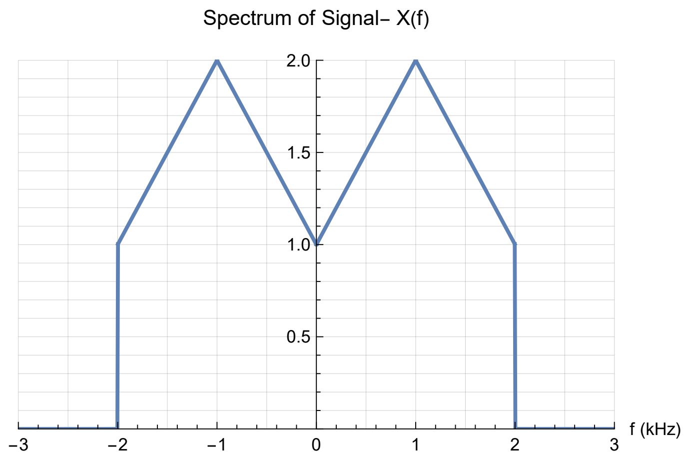
a. $\int_{-\infty}^{\infty} \delta(\tau)\, e^{-5\tau}\, d\tau$

b. $\int_{-\infty}^{\infty} \delta(\tau - .01)\, u(\tau)\, e^{-5\tau}\, d\tau$

c. $\int_{-\infty}^{\infty} \delta(\tau - 1)\, u(\tau)\, e^{-5\tau}\, d\tau$

d. $\int_{-\infty}^{\infty} \delta(\tau - t)\, u(\tau)\, e^{-5\tau}\, d\tau$

13. The spectrum of x(t) is given by:



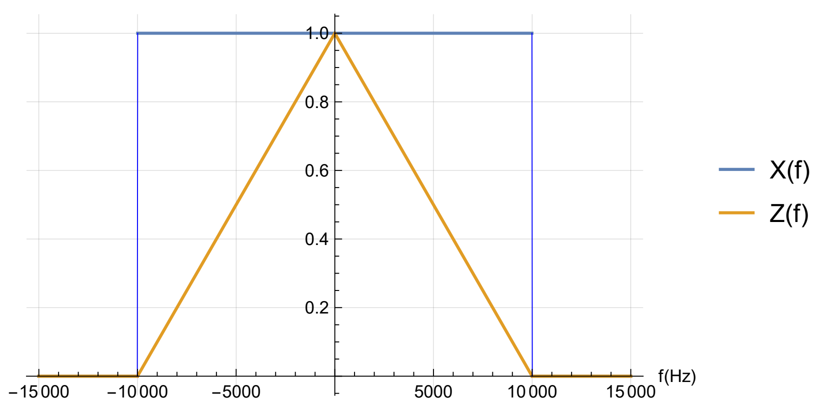
a. The signal x(t) is sampled at 5000 samples/sec to form x_s(t). Plot the spectrum of x_s(t).

b. For x(t) given above, what is the minimum sample rate required to prevent aliasing?

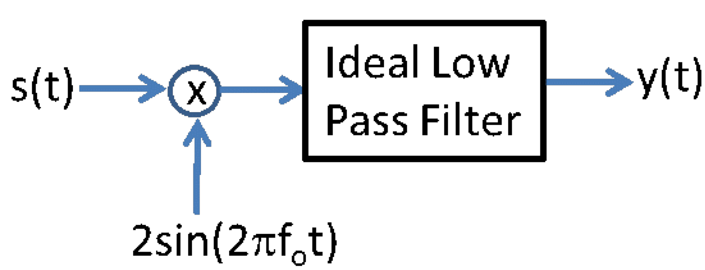
c. If no aliasing is present, describe how x(t) is recovered from x_s(t).

14. Let s(t) = $x(t)\cos(2\pi f_0 t)+z(t)\sin(2\pi f_0 t)$ with f_0=100khz and Z(f) = $\text{rect}\left(\frac{f}{20\,000}\right)$ and X(f)= $\Lambda\left(\frac{f}{10\,000}\right)$ as shown below.

Out[#]=



a. Plot the Magnitude spectrum at the input to the LPF.



b. Find the output y(t) in terms of x(t) and z(t) of the system above. The bandwidth of the ILPF is 11 kHz. [Hint: use the trigonometry identities for $\sin^2(\theta)$ and $\cos^2(\theta)$]

(The solution to this problem provides the basis for quadrature modulation.)

15. LTE (4G/5G) systems use the DFT. Here a DFT of length 2048 is used with a sample frequency of f_s = 30.72Msamples/s.

a. What is the record length in seconds?
(In LTE this is the symbol time.)
b. What is the spacing in kHz between frequency components of this DFT?
(In LTE this is the carrier spacing)

16. Show that the impulse response $h_R(t)$ for a system defined as $y(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau)\, d\tau$ is $\frac{1}{T}\text{rect}\left(\frac{t-T/2}{T}\right)$.

The impulse response $h_R(t)$ for a system defined as $y(t) = \frac{1}{T}\int_{t}^{t-T} x(\tau)\, d\tau$ is $\frac{1}{T}\text{rect}\left(\frac{t-T/2}{T}\right)$.
Let x(t)=δ(t) then $h_R(t) = \frac{1}{T}\int_{-\infty}^{\infty}\text{rect}\left[\frac{t-\tau}{T}\right]\delta(\tau)\, d\tau = \frac{1}{T}\int_{t}^{t-T}\delta(\tau)\, d\tau = \frac{1}{T}\text{rect}\left(\frac{t-T/2}{T}\right)$.
The output of a LPF with a impulse response $h_R(t) = \frac{1}{T}\text{rect}\left(\frac{t}{T}\right)$ at time T is the same as the output of an integrator (averager) that integrates for T sec.

17.

a. Convert 10 Watts to dB_W
b. Convert 10 Watts to dB_m
c. Convert 30 dB_m to watts
d. The path loss between a transmitter and receiver is 30 dB, The transmitter power is 27 dB_W, what is the receiver power in dB_W and watts.
e. The received power from part d. is reduced by 0 dB_W What is the resulting received power in Watts.