EECS 562

Homework 1

1. (Concept: complex numbers and functions)
Let

a. z_1 = 2+j3, Find Re(z_1), Im(z_1), $|z_1|$, $|z_1|^2$ and α and β in $z_1 = \alpha e^{j\beta}$

b. Repeat a. for $z_1 = -2 - j3$

c. Find $x(t) = \mathrm{Re}[z_1 e^{-j2\pi f_c t}]$ where f_c = 100 Hz

2. (Concept: mathematical model for quadrature phase shift keying - QPSK modulation; for each symbol time sending one of 4 signals to send 2 bits and signal constellations, aka, IQ diagrams)
Let z_1 = 1.57+j2.35, z_2 = -1.57-j2.35, z_3=2.35-j1.57, z_4 = -2.35+j1.57

a. Plot z_i for i=1..4 putting the real part of z_i on the x-axis and the imagery part of z_i on the y-axis.

b. For f_c = 10 MHz find $x_i(t) = \mathrm{Re}[z_i e^{-j2\pi f_c t}]$ for i=1..4

3. (Concept: Equivalence between time delay and phase shift)
Let $x_1(t) = 10\cos(2\pi 1000(t - 125\mu s))$ and $x_2(t) = 10\cos(2\pi 1000t - \frac{\pi}{4})$, $x_1(t) \neq x_2(t)$ TRUE or FALSE

4. (Concepts: Properties of the sinc function, bandwidth, and inverse time duration-bandwidth relationship)
For

$x_1(t)$ = 100 sinc(100 t),

$x_2(t)$ = 200 sinc(200 t)

$x_3(t)$ = 500 sinc(500 t)

With sinc(x) = $\frac{\sin(\pi x)}{\pi x}$

a. Find the Fourier transform of $x_1(t)$, i.e. find $X_1(f)$.

b. Plot x_1(t), x_2(t), and x_3(t)

c. Rank order the signals from lowest bandwidth to highest bandwidth.

5. (Concept: Signal power and energy)
Find the power and energy in $2\cos(2\pi 1000t) + 4\sin(2\pi 2000t)$

6. (Concepts: Energy per bit and Binary Phase Shift Keying-BPSK)
A bit is transmitted as
$x(t) = A\cos(2\pi f_c t)$ if bit = "1" for T_b
or
$x(t) = -A\cos(2\pi f_c t) = A\cos(2\pi f_c t - \pi)$ if bit = "0" for T_b
For $A = 1\times10^{-3}$ and T_b=1μs and f_c = 100 MHz

a. Find the energy in x(t).

b. What is the bit rate in Mb/s?

7. (Concept: The orthogonality property of signals)
Let $x_1(t) = \cos(2\pi f_1 t)\,\mathrm{rect}(\frac{t-0.05}{0.01})$ and $x_2(t) = \cos(2\pi f_2 t)\,\mathrm{rect}(\frac{t-0.05}{0.01})$
For f_1=2000 and f_2=2100

a. Find $\int_{-\infty}^{\infty} \cos(2\pi f_1 t)\sin(2\pi f_1 t)\, dt$

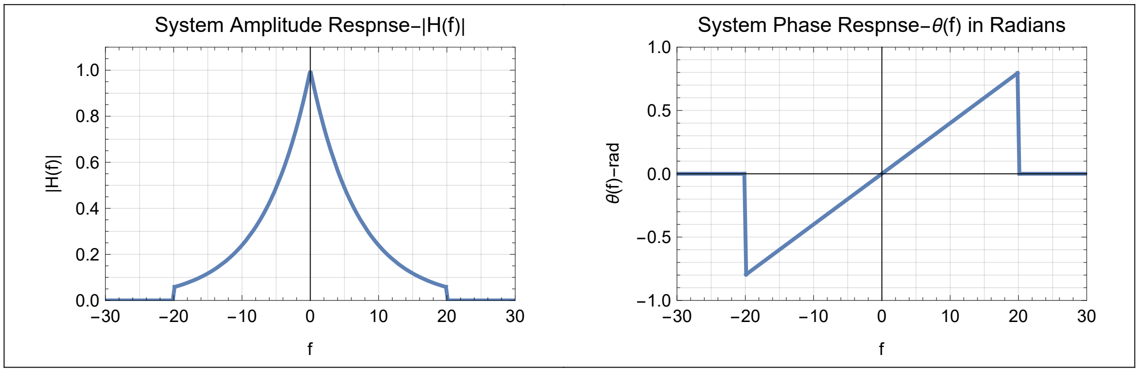
b. Find $\int_{-\infty}^{\infty} x_1(t)\, x_2(t)\, dt$

c. $x_1(t)$ and $x_2(t)$ are orthogonal. True or False

8. (Concept: double-sided phase and magnitude spectrum)
For x(t) = 4cos(2π1000t) + 8sin(2π2000t)

a. Plot the double-sided phase and magnitude spectrum for x(t).
b. What is the bandwidth of x(t)?

9. (Concept: Sinusoidal response of LTI system)
A system with $H(f)= |H(f)|\, e^{-j\theta(f)}$
where amplitude $|H(f)|= \text{rect}\left(\frac{f}{40}\right) e^{-\left|\frac{f}{7}\right|}$ and phase $\theta(f)=-\frac{f}{25}$ response is given below.



a. The signal x(t)=cos(2π2.5t)+cos(2π12.5t) is the input to the system with the frequency response
H(f) given above. Find the system output, y(t).

b. A signal g(t) with a bandwidth of B_g is the input to a system with H(f) with a bandwidth of B_{sys}, the system output is y(t).

If $B_{sys} >> B_g$ then y(t)≈g(t). TRUE or FALSE.

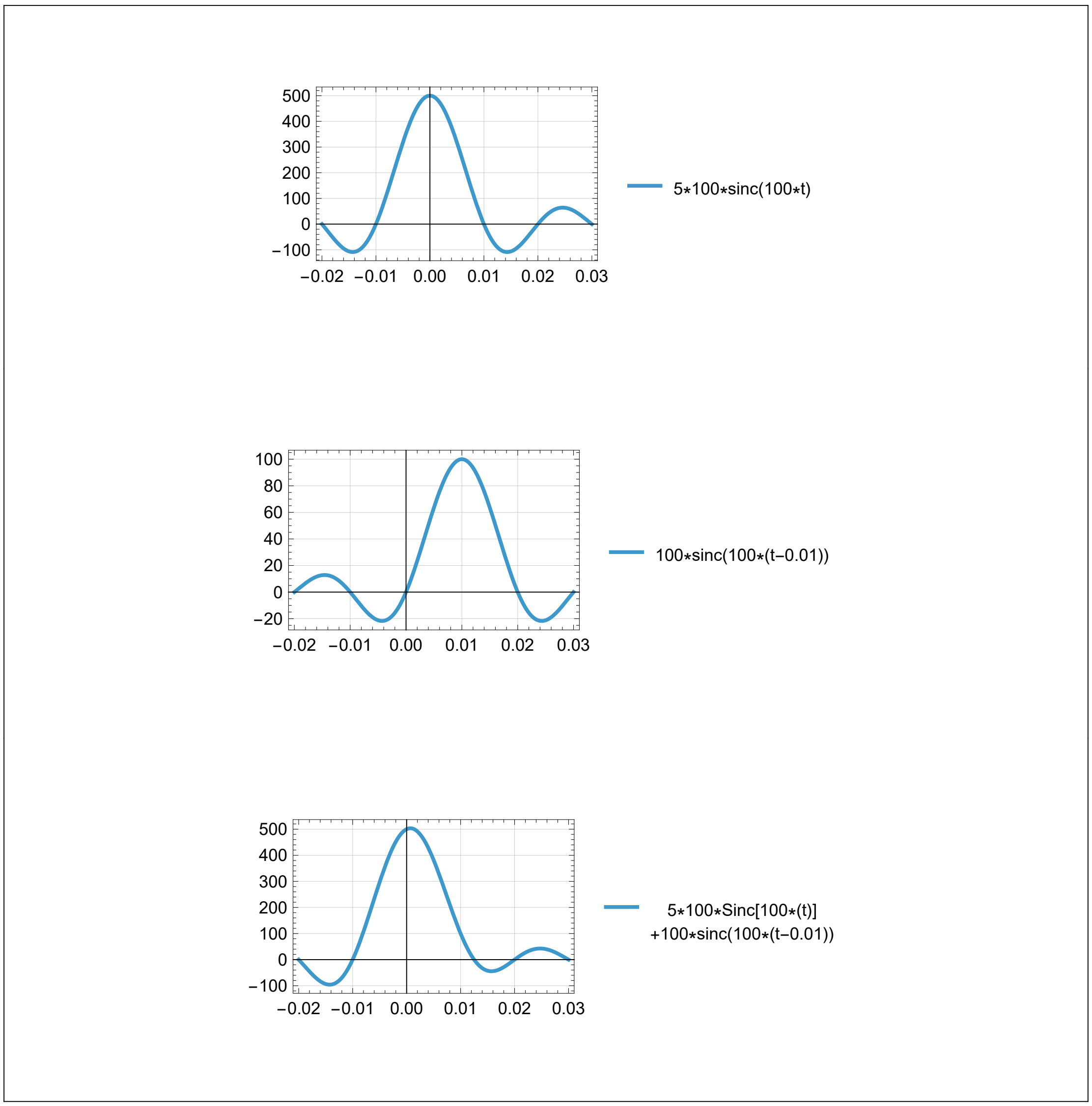
10. (Concept: Impulse response of LTI system)
A signal x(t)=10δ(t-0.4) is the input to a linear time invariant system with an impulse response of h(t) = $e^{-10|t|}$. Find and sketch the system output y(t).

11. (Concepts: Bandwidth of ILPF and zero crossing properties of impulse response of ILPF)
A filter has an impulse response of h(t) = 100sinc(100t)

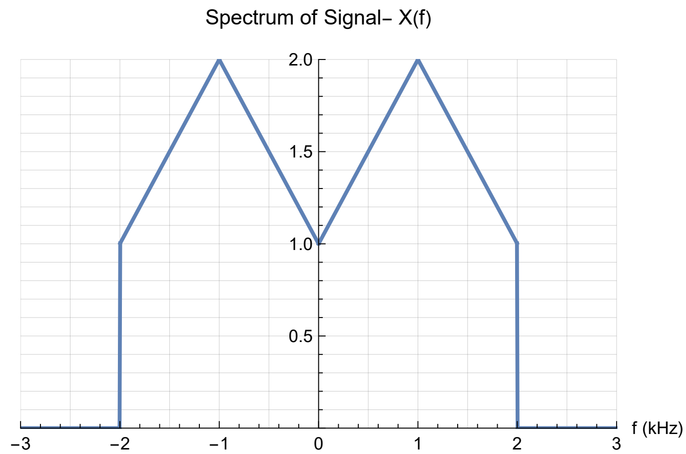
a. Is this an ILPF? Yes or NO
b. What the filter bandwidth?
c. With an input signal x(t)=5δ(t) + δ(t- τ) where τ=10ms input to this filter, plot the output signal y(t) in the time domain.
d. What is y(0) and y(τ)?
e. Define $y_1(t)$ = 5δ(t) ∗h(t) and $y_2(t)$ = δ(t-τ) ∗h(t) what is $y_1(0)$ and $y_2(\tau)$?
Compare $y_1(0)$ and $y_2(\tau)$ to y(0) and y(τ)

In[•]:=
```
Plot1 = Plot[5 * 100 * Sinc[π * 100 * (t)], {t, -0.02, 0.03}, GridLines → Automatic,
    PlotRange → All, PlotLegends → {"5*100*sinc(100*t)"}, Frame → True];
Plot2 = Plot[100 * Sinc[π * 100 * (t - tau)], {t, -0.02, 0.03}, GridLines → Automatic,
    PlotRange → All, PlotLegends → {"100*sinc(100*(t-0.01))"}, Frame → True];
Plot3 = Plot[y11[t], {t, -0.02, 0.03}, GridLines → Automatic, PlotRange → All,
    PlotLegends → {"5*100*Sinc[100*(t)]\n+100*sinc(100*(t-0.01))"}, Frame → True];
GraphicsColumn[{Plot1, Plot2, Plot3}, Frame → True]
```

Out[•]=



12. (Concept: Sampling)
The spectrum of x(t) is given by:



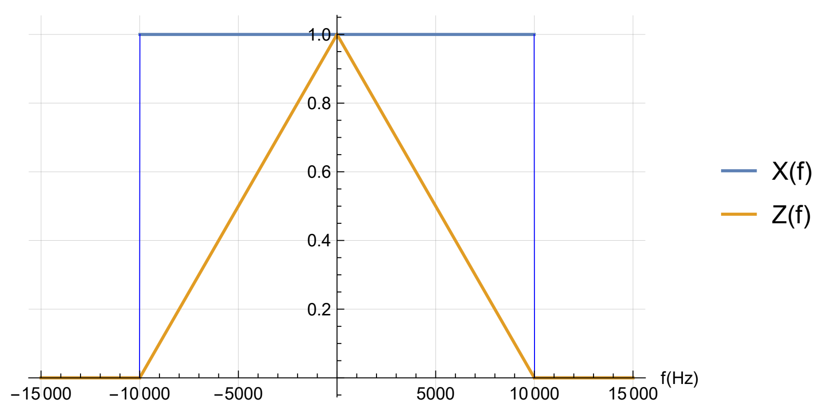
a. The signal x(t) is sampled at f_s = 4500 samples/sec to form x_s(t). Plot the spectrum of x_s(t).

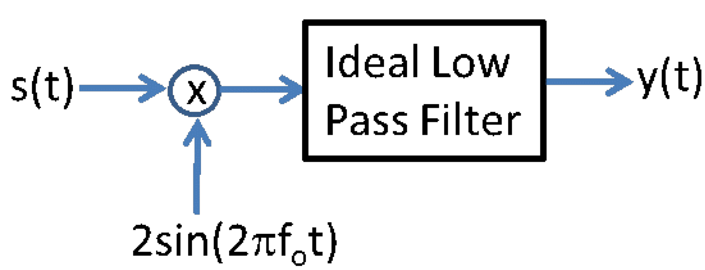
b. For x(t) given above, what is the minimum sample rate required to prevent aliasing?

c. With f_s = 4500 samples/sec, describe how x(t) is recovered from x_s(t).

13. (Concept: Quadrature modulation)

Let s(t) = $x(t)\cos(2\pi f_0 t)+z(t)\sin(2\pi f_0 t)$ with f_0=100khz and $Z(f) = \text{rect}\left(\frac{f}{20\,000}\right)$ and $X(f) = \Lambda\left(\frac{f}{10\,000}\right)$ as shown below.





Find the output y(t) in terms of x(t) and z(t) of the system above. The bandwidth of the ILPF is 11 kHz. [Hint: use the trigonometry identities for $\sin^2(\theta)$ and $\cos^2(\theta)$]

14. LTE (4G/5G) systems use the DFT. Here a DFT of length 2048 is used with a sample frequency of f_s = 30.72Msamples/s.

a. What is the record length in seconds?
(In LTE this is the symbol time.)
b. What is the spacing in kHz between frequency components of this DFT?
(In LTE this is the carrier spacing)

15. (Concept: Equivalence between an integrator and a filter)

a. Find the transfer function for a filter with an impulse response $h_R(t)=\frac{1}{T}\text{rect}\left(\frac{t-T/2}{T}\right)$.
b. Show that the impulse response $h_R(t)$ for a system defined as
$y(t) = \frac{1}{T}\int_{t-T}^{t} x(\lambda)\, d\lambda$ is $\frac{1}{T}\text{rect}\left(\frac{t-T/2}{T}\right)$.

16. (Concepts: dB_W and dB_m)

a. Convert 10 Watts to dB_W
b. Convert 10 Watts to dB_m
c. Convert 30 dB_m to watts
d. The path loss between a transmitter and receiver is 30 dB.
The transmitter power is 33 dB_W, what is the receiver power in dB_W and watts.