EECS 861

Homework #1

1. (Concept: convolution)

a. Let $f_1(x) = 2\,e^{-2x}\,u(x)$ and $h_1(x) = 3\,e^{-3x}\,u(x)$, find and plot $y_1(t) = f_1(t) * h_1(t)$ where $*$ indicates a convolution.

b. Find the area under the curve of $y_1(x)$, i.e., $\int_{-\infty}^{\infty} y_1(x),\, dx$=?

c. Let $f_2(x) = \text{rect}(x)$ and $h_2(x) = 0.25\,\text{rect}(\frac{x-2}{4})$, find and plot $y_2(t) = x_2(t) * h_2(t)$ where $*$ indicates a convolution.

c. Find the area under the curve of $y_2(t)$, i.e., $\int_{-\infty}^{\infty} y_2(x),\, dx$=?

2. (Concept: pulse duration, bandwidth, time-bandwidth product)

In general a functions's width in the time domain can be defined using the mean square duration T_d^2 then the root mean square (RMS) duration = $T_d = \sqrt{T_d^2}$ where

$$T_d^2 = \frac{\int_{-\infty}^{\infty} t^2\, |x(t)|^2\, dt}{\int_{-\infty}^{\infty} |x(t)|^2\, dt}$$

a. Let $x_1(x) = \frac{1}{2\tau} e^{-|x|/\tau}$. Find T_d.

b. Find the Fourier transform of $x_1(x)$, i.e., $x_1(x) \longleftrightarrow X_1(f)$

c. The root mean square bandwidth of a function defined $\sqrt{B_{\text{rms}}^2}$ as where B_{rms}^2 is given by

$$B_{\text{rms}}^2 = \frac{\int_{-\infty}^{\infty} f^2\, |X(f)|^2\, df}{\int_{-\infty}^{\infty} |X(f)|^2\, df}$$

. Find B_{rms}.

d. What is $T_d\, B_{\text{rms}}$?

e. What is the significance of a constant time-bandwidth product.

Hint: [Reference: **Table of Integrals, Series, and Products,** I.S. Gradshteyn and I.M. Ryzhik Seventh Edition, Academic Press, 2007, page 3.238 equation 3.241 #4]

4.[11] $$\int_0^{\infty} \frac{x^{\mu-1}\, dx}{(p + qx^{\nu})^{n+1}} = \frac{1}{\nu p^{n+1}} \left(\frac{p}{q}\right)^{\mu/\nu} \frac{\Gamma\left(\frac{\mu}{\nu}\right)\Gamma\left(1+n-\frac{\mu}{\nu}\right)}{\Gamma(1+n)}$$

$$\left[0 < \frac{\mu}{\nu} < n+1, \quad p \neq 0, \quad q \neq 0\right]$$

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Specific forms of this general equation are:

$$\int_0^{\infty} \frac{1}{(q\,x^2+1)^2}\, dx = \frac{\pi}{4\sqrt{q}}$$

$$\int_0^{\infty} \frac{x^2}{(1+qx^2)^2}\, dx = \frac{\pi}{4\,q^{3/2}}$$

3. (Concept: Signal energy, Energy/bit, bit rate)

A bit is transmitted as

$x(t) = A\cos(2\pi f_c t)$ if bit = "1" for T_b

or

x(t) = −Acos (2 πf_ct)= −Acos (2 πf_ct-π) if bit = “0” for T_b

In another form x(t) = ±Acos(2 πf_c t) rect$\left(\frac{t-\frac{T_b}{2}}{T_b}\right)$

For A=0.01 and T_b=10ms, f_c=100kHz

a. Find the energy in x(t), that is the energy / bit = E_b.

b. What is the bit rate in b/s?

4. (Concept: Impulse response, transfer function, output of linear time invariant system with cosine input)

A linear time-invariant system with input signal x(t) produces an output signal y(t) =4x(t-1ms) - 2x(t-1.125ms) + x(t-1.5ms)

a. Find the impulse response.

b. Find the system transfer function.

c. Let x(t) = cos(2π500t)+cos(2π1500t), find y(t).

5. (Concept: Ideal filter)

A filter has an impulse response of h(t) = 100sinc(100t); here sinc(x)=$\frac{\sin(\pi x)}{\pi x}$.

a. Is this an ILPF? Yes or NO

b. What the filter bandwidth?

6. (Concept: Averaging)

A LTI system has an impulse response h(t) = $\begin{cases} \frac{1}{T} & \text{for } 0 \le t \le T \\ 0 & \text{elsewhere} \end{cases}$

a. Find the transfer function, H(f), for this filter. Use sinc(x) = $\frac{\sin(\pi x)}{\pi x}$

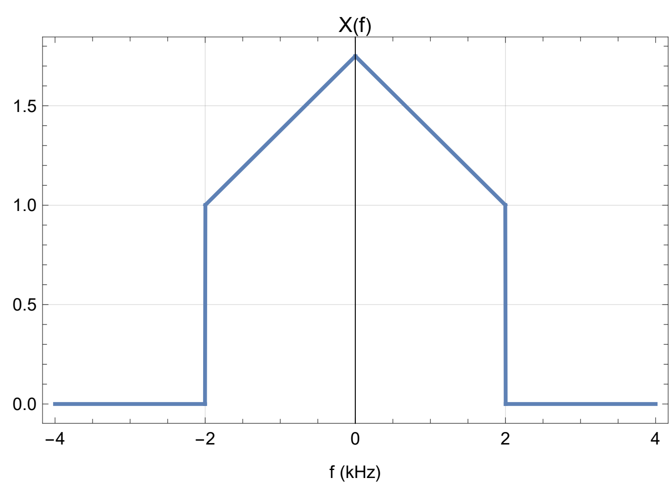
b. Plot the magnitude transfer function for this filter for T=2ms.

c. Explain why this filter is called an “averager”.

d. Does H(0) change as T changes?

7. (Concept: Sampling)

A signal x(t) has a Fourier Transform given as



a. The signal x(t) is sampled at a rate of f_s=6000 samples/sec to create $x_s(t)$. Sketch the spectrum of the sampled signal $x_s(t)$.

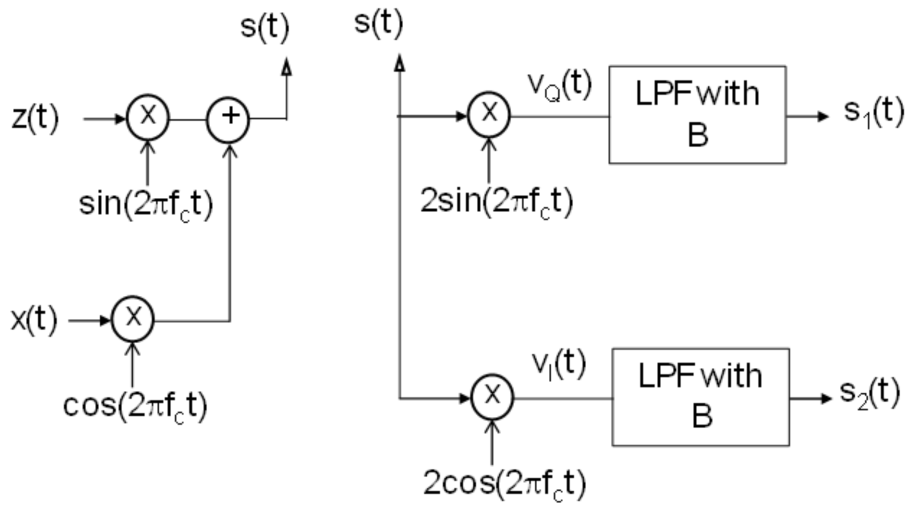
b. The signal x(t) is sampled at a rate of 6000 samples/sec. Describe, e.g., draw a block diagram, a system to recover x(t) from $x_s(t)$.

c. What is the minimum value of f_s where x(t) can be recovered from $x_s(t)$?

d. What is the advantage of over sampling?

8. (Concept: Quadrature Modulation)

Let s(t) = $x(t)\cos(2\pi f_c t) + z(t)\sin(2\pi f_c t)$ with f_c=100khz. The baseband signals x(t) and z(t) have a bandwidth of 10 kHz.



Find the output $s_1(t)$ and $s_2(t)$ in terms of x(t) and z(t) of the system above. The bandwidth of the ILPF is 11 kHz. [Hint: use the trigonometry identities for $\sin^2(\theta)$ and $\cos^2(\theta)$]

9. (Difference equations and frequency response of discrete time systems, ARMA filters)

Given y[n] = x[n]+0.5y[n-1].

a. Find the transfer function, H(z).

b. Find the frequency response, $H(e^{j2\pi f})$, $|f| < \frac{1}{2}$

c. Plot $|H(e^{j2\pi f})|$ for $|f| < \frac{1}{2}$

d. Is this difference equation called a moving average (MA), and autoregressive (AR), or ARMA.