EECS 861

Homework #10

1. (Concept: Discrete time white noise and autocorrelation of function of equal weighted moving average)
A discrete time R.P. X[n] has an autocorrelation function of

$$R_{xx}[k] = \begin{matrix} 1 \text{ for } k = 0 \\ 0 \text{ for } k \neq 0 \end{matrix}$$

For Y[n]=$\frac{1}{4}$(X[n]+ X[n-1]+ X[n-2]+X[n-3]) find

a. E[Y[n]] and Var[Y[n]]

b. Find R_{YY}(k)

2. (Concept: Noise power and variance of bandpass white Gaussian noise.)
N(t) is bandpass white Gaussian noise with $\eta/2 = 10^{-10}$, centered at 50 Ghz with a bandwidth $B_N = 1$ Ghz.

a. Find the noise power.

b. Find P(|N(t)|>0.6).

3. (Concept: Time averaged white noise)
Let Y(t)= $\frac{1}{T}\int_{t-T}^{t} X(t)\, dt$, this system is an integrator.

a. Find the transfer function, H(f), for this system, i.e., Y(f)=X(f)H(f), hint find the impulse response first.

b. Let X(t) be white Gaussian noise with $S_X(f) = \eta/2$, find E[Y(t)].

c. Let X(t) be white Gaussian noise with $S_X(f) = \eta/2$, find Var[Y(t)].

Hint: See Homework 1 problem 6.

4. (Concept: Properties of RP at the output of low pass RC filter)
The transfer function of a linear time invariant filter is

$$H(f) = \frac{200}{1 + \frac{j2\pi f}{f_0}} \text{ with } f_0 = 10\,000 \text{ Hz}$$

The input to this filter is zero-mean white Gaussian Noise with $S_x(f) = 5 * 10^{-10}$

a. Find the PSD of the filter output Y(t), S_Y(f).

b. Find E[Y(t)]

c. Find Var[Y(t)]

d. Find the noise power at the filter output Y(t).

e. What is P(Y(t)>0.5)

f. P(Y(t)>0.6|Y(t-100μs)=0). Hint: first find $R_{yy}(\tau)$.

5. (Concept: Applying large TB product to finding properties of the output of a time average.)
Using Y(t) from problem 4, let

$$Z = \frac{1}{T}\int_0^T Y(\tau)\, d\tau \quad \text{with T = 53.35 ms}$$

Find P(Z>0.02), apply appropriate approximations.

6. (Concept: Output signal-to-noise ratio)
A signal X(t) is corrupted by statistically independent additive white Gaussian noise, N(t) with a band-

width B_N and is input to a linear time-invariant filter H(f).

$$S_N(f) = \begin{cases} \frac{10^{-11}}{2} & \text{for } |f| < 50\text{ kHz} \\ 0 \end{cases}$$

$X(t) = A\cos(2\pi f_i t)$ where $A = 0.005$ and f_i is a constant given below.

$H(f) = \frac{1}{1+\frac{j2\pi f}{f_0}}$ with $f_0 = 10\,000$ Hz

a. Find the input signal-to-noise ratio (in dB) for f_i=5kHz

b. Find the output signal-to-noise ratio (in dB)for for f_i=5kHz

c. Find the output signal-to-noise ratio (in dB)for for f_i=10kHz

d. Find the output signal-to-noise ratio (in dB) for for f_i=20kHz

e. Why does the output signal-to-noise ratio change as f_i changes?

7. (Concept: Properties of ARMA RP’s)

A discrete time process is defined as $Y[n] = a_1$Y[n-1] + N[n]. Where N[n] is zero-mean white Gaussian Noise with a variance of σ^2=0.75. Let a_1=-0.8.

a. Is this an autoregressive or moving average process?

b. Find E[Y[n]], $E[Y^2[n]]$, Var[Y[n]], and R_{YY}[k].

8. (Concept: Properties of an MA(2) RP)

One way to define a second order moving average process, MA(2) is defined by

$X[n] = e[n] + b_1\, e[n-1] + b_2\, e[n-2]$

where e[n] is zero-mean white Gaussian Noise with a variance of σ^2.

a. Find the covariance matrix for X[n], X[n-1], and X[n-2]. For b_1 = 0.95 & b_2=0.95 and σ^2=0.15.

b. Plot three member functions given the data in sheet 1 in the file specified below. Find the autocorrelation function for the data given in this file. Would the second order moving average process model given in this problem be a good representation for this data?

http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2025/data_HW_5_Prob_2.xls

9. (Concept: Properties of ARMA RP’s)

Use the data in given file.

a. Find and plot the autocorrelation function of this data.

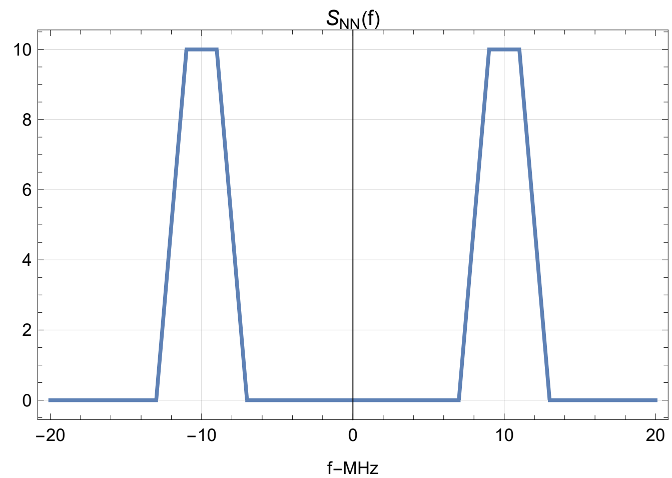
http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2025/data_HW_10_Prob_9.xls

b. Could this data be a sample function from an MA(2) process, justify your answer.

c. Assuming this data is a sample function from an AR(1) process suggest a value for α_1. Hint experiment with the AR(1) example in **http://www.ittc.ku.edu/~frost/EECS_861/Mathematica_-files/ARMA_study.cdf**

9. (Concept: Quadrature Representation of Bandpass Gaussian Signals)

The spectral density of a narrowband Gaussian process N(t) is shown in Below. Find the following spectral densities associated with the quadrature representation of N(t) using f_c = 10 MHz.



a. $S_{N_c N_c}(f)$

b. $S_{N_c N_s}(f)$

c. If f_c is changed to f_c=10.5 MHz, does $S_{N_c N_c}(f)$, and $S_{N_c N_s}(f)$ change, if so why?