EECS 861

Homework #11

1. (Concept: MAP detection)

Under H_0 the observed signal is N and H_1 the observed signal is 2 + N where N is a Gaussian with zero mean and a standard deviation of 0.75.

a. Assuming $P(H_1)= P(H_0)=0.5$ derive the MAP decision rule.

b. Find the P_D, P_M, and P_{fa} given the MAP decision rule. Verify your results using **Binary Detection with Gaussian Noise**

c. Find the P_e.

d. Repeat part b. with under H_0 the observed signal is -1+N and H_1 the observed signal is +1+ N.

Hint: Sketch the conditional distributions for this part and part b.

2. (Concept: Bayes' Decision Rule)

For the parameters given in problem 1 parts a-c and the costs are given as $C_{10}= C_{01}=2$, $C_{00}=0$, and $C_{11}=1$. Find the decision rule that minimizes the average cost.

3. (MAP and N-P detectors)

In a target detection problem, the target is present for 0.01 μs. When the target is not present only noise N(t) is received. N(t) is additive zero mean Gaussian WSS random process, N(t) has the following PSD

$$S_N(f) = \begin{cases} \frac{\eta}{2} = 10^{-8} & |f| < 2 \text{ GHz} \\ 0 & elsewhere \end{cases}$$

When the target is present the received signal is Y(t)=A+ N(t) where A=10. Y(t) is sampled every 0.001 μs. One sample of Y(t) is used to detect the presence of the target.

a. Assuming P(target is present)= P(target is not present)=0.5 derive the MAP decision rule.

b. Find P_D, P_M, and P_{fa} given the MAP decision rule.

Verify using **Binary Detection with Gaussian Noise**

c. Design an N-P detector is to obtain a P_{fa} =0.01

d. Find P_D, P_M, and P_{fa} given the N-P detector.

4. Repeat Problem 3 part c and d using a decision variable Z where assume T=0.0025μs and that you know the target is present starting at t=0 or not present starting at t=0. $Z= \frac{1}{T}\int_0^T Y(t)\,dt$

5. (Concept: Bit detection with additive zero mean WSS random noise)

A digital signal X(t) has a bit rate of 150 b/s where X(t) is -A V (bit=0) or +A V (bit=1) and A=1.5 and bits are transmitted with equal probability. T_B is the bit duration. The transmitted signal is corrupted by additive zero mean WSS random process, where N(t) has the following PSD. The received signal is Z(t)=X(t)+N(t). Assume the receiver is in bit synchronization.

$$S_N(f) = \begin{cases} \frac{\eta}{2} = \frac{1}{250} & |f| < 5000 \\ 0 & elsewhere \end{cases}$$

The decision variable, Y is given by $Y = \frac{1}{T_B} \int_0^{T_B} Z(t)\, dt$

a. Find the distribution of Y|0 bit is transmitted.

b. Find the distribution of Y|1 bit is transmitted.

c. Derive the MAP decision rule.

d. Find the probability of bit error, P_e.

Verify using **Binary Detection with Gaussian Noise**

6. (Concept: System trade-offs as E_b, $S_n(f)$, and bit rate change)

Trade-offs using system specified in Problem 3 and Problem 5.

a. Using system specified in Problem 3 will the P_D increase or decrease or stay the same as A increases with the N-P detector?

b. Using system specified in Problem 3 will the P_{fa} increase or decrease or stay the same as A increases with the N-P detector?

c. Using system specified in Problem 3 will the P_e increase or decrease as η increases in Problem 3?

d. Using system specified in Problem 5 will the P_e increase or decrease or stay the same as the bit rate increases in Problem 5?

7. (Concept: Receiver Operating Characteristic)

Under H_1 (target present) the observed signal is A+N and under H_0 (target absent) the observed signal is N where N is a Gaussian with zero mean and a standard deviation of σ.

Define S/N (dB) = 10 Log$\left(A^2/\,\sigma^2\right)$.

On the same graph plot the ROC for S/N = 0.5dB, 1.0dB, 3.0dB, 6dB. Verify your answer using **http://www.ittc.ku.edu/~frost/EECS_861/Mathematica_files/ROC.cdf**

8. (Concept: MAP decision rule with non-gaussian RVs)

Under H_1 (target present) the observed signal has a pdf of

$$f_{Y|H_1}(y \mid H_1) = \frac{u(y)}{5}\, e^{-y/4}$$

Under H_0 (target not present) the observed signal has a pdf of

$$f_{Y|H_0}(y \mid H_0) = u(y)\, e^{-y}$$

Two S.I. samples of the observed signal are collected. Assume $P(H_0)$=0.5.

a. Derive the MAP decision rule.

b. Find the pdf of the decision variable given H_0.

c. Find the pdf of the decision variable given H_1.

d. Find P_M

e. Find P_{fa}