EECS 861

Homework #12

1. (Concept: Detection with multiple observations in correlated noise)

A decision is based on 2 samples, Y_1 and Y_2. Y is a multivariate Gaussian random vector

$$Y = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} \quad E[Y \mid H_0] = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \text{ and } E[Y \mid H_1] = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ with } \Sigma = \begin{pmatrix} 1.3575 & 0.675 \\ 0.675 & 1.3575 \end{pmatrix}$$

a. Design the optimum detector. Assume $P(H_0)= P(H_1)=1/2$.

b. Find P_e.

c. Repeat parts a. and b. with

$$E[Y \mid H_0] = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \text{ and } E[Y \mid H_1] = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ with } \Sigma = \begin{pmatrix} 1.3575 & -0.675 \\ -0.675 & 1.3575 \end{pmatrix}$$

d. Explain why the results, i.e., P_e, from b. and c. are different.

e. The 500 bits in the file below (sheet labeled Volts maps 0->-1 and 1->+1) are sampled at two sample/bit, the resulting transmitted signal is corrupted by additive Gaussian correlated noise with $\Sigma = \begin{pmatrix} 1.3575 & 0.675 \\ 0.675 & 1.3575 \end{pmatrix}$, apply optimum detector found in part a. to the received signal given in the file below and estimate the P_e of your detector, then compare to P_e found in part b.

Here are the transmitted bits

http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2025/data_HW12_Prob_1_bits.xls

The received signal is given in

http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2025/data_HW12_Prob_1.xls

2. (Concept: Detection with multiple observations in white noise with different pulse shapes)

Let $s_1[k] = -2, -2, -2$ for k =0, 1, 2 and $s_0[k] = -s_1[k]$, i.e., $s_0[k] = 2, 2, 2$ for k =0, 1, 2

Assume

$P(s_1) = 0.5 = P(s_0)$

Y[k] = S[k] + N[k] for k =0…2 where

S[k] & N[k] are statistically independent

N[k] is white Gaussian noise with a zero mean and variance= 4, i.e., σ_N=2.

a. Find the MAP decision algorithm.

b. Find the probability of error.

c. Apply the MAP decision algorithm for the follow observations

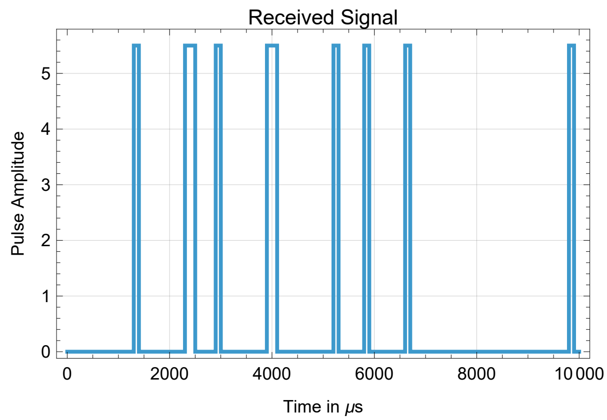
y[k] = {-0.4, 0.1, -0.1}, is the receiver output is $s_1[k]$ or $s_2[k]$?

d. Repeat a)-b) for $s_1[k] = 1, -3.162, 1$

e. Why is P_e from part b. and part d. the same.

3. (Concept: Pulse detection)

Time is slotted into 10 μs time slots, in a received signal X(t) a time slot of 10 μs may contain a pulse of amplitude +5.5, as shown below.



The received signal is R(t)= X(t) +N(t) where N(t) is bandlimited white Gaussian noise. The noise PSD is

$$S_N(f) = \begin{cases} 10^{-4} & |f| < 1\text{MHz} \\ 0 & elsewhere \end{cases}$$

The received signal, R(t), is sampled at a rate of 10 Msamples/sec. Samples are collected in time synchronization with the pulses. The received signal is N(t) if no pulse for each pulse duration of 10 μs or X(t)+N(t) for each pulse duration of 10 μsis a pulse is present.

a. Design a MAP pulse detector assuming Prob(pulse)=0.5

b. For your pulse detector and given the parameters above calculate the probability of detection and false alarm.

c. Design a pulse detector using a Neyman-Pearson (N-P) rule with a $P_{\text{fa}} = 0.01$ and find probability of detection.

d. Apply the MAP detector with Prob(pulse)=0.5 to the data set given below. How many pulses are in this record. This file contains a record of collected samples.

http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2025/data_HW12_Prob_3.xls

4. (Concept: Biased/unbiased estimators)

Let $f_X(x;\theta) = \frac{1}{\sqrt{5\pi}} e^{\frac{-(x-\theta)^2}{5}}$. Given $x_1 \ldots\ x_N$ be N statistically independent samples from f_X(x)

Is $\overline{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$ an biased estimator for θ; yes or no and justify.