

EECS 861
Homework #13

1. (Concepts: MAP, MSE, and ML estimation)

We want to estimate a received signal A from K observations of Y where Y is modeled as $Y=A+N$. Here the sample mean is

$$\bar{Y} = \frac{1}{K} \sum_{i=1}^K Y_i = 16.$$

N is Gaussian with $E[N]=0$ and $\text{Var}[N] = \sigma_N^2$

X is Gaussian with $E[A]=5$ and $\text{Var}[A] = \sigma_A^2$

N and X are statistically independent. For the following 3 cases:

Case 1: $\sigma_N^2 = 10$ $\sigma_A^2 = 2$ $K = 5$

Case 2: $\sigma_N^2 = 200$ $\sigma_A^2 = 2$ $K = 5$

Case 3: $\sigma_N^2 = 0.1$ $\sigma_A^2 = 2$ $K = 5$

Case 4: $\sigma_N^2 = 10$ $\sigma_A^2 = 2$ $K = 450$

Find

- a. The MAP estimator for A.
- b. The Mean Square (MS) estimator for A.
- c. The Maximum Likelihood (ML) estimator for A.

Hint: check your results with **MAP Estimator with Gaussian Prior and Gaussian Noise**

2. (Concept: unconstrained Wiener filter)

An unconstrained Wiener filter is used to estimate $S(t)$ from $Y(t) = S(t) + N(t)$, where $N(t)$ and $S(t)$ are statistically independent and

$$S_S(f) = \frac{1000}{1 + (40\pi f)^2}$$

and

$$S_N(f) = 10$$

- a. Find the unconstrained Wiener filter $H[f]$
- b. Find the impulse response of the unconstrained Wiener filter $h[f]$. Is the corresponding linear system causal?

3. (Concept: optimum realizable discrete time Wiener filter)

Let the desired signal, $S[k]$, be characterized by a moving average process given by

$$S[k] = X[k] + 0.9X[k-1] + 0.5X[k-2]$$

where $X[k]$ is a white Gaussian random process with zero mean and variance = 0.195

The observed signal is $Y[k] = S[k] + N[k]$ where $N[k]$ is Gaussian with $R_{NN}[n] = 0.4$ for $n=0$ and $R_{NN}[n] = 0$ elsewhere.

- a. Find $R_{SS}[n]$.
- b. Find the optimum realizable Wiener filter $h[k]$ where

$$\hat{S}[k] = \sum_{n=0}^{\infty} h[n] \times Y[k-n]$$

- c. Apply the optimum realizable Wiener filter $h[k]$ to this received signal

http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2025/data_HW13_prob_3_Received_sign

al.xls

d. Calculate the resulting mean square error given the desired signal

http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2025/data_HW13_prob_3_Desired_signal.xls

e. Repeat parts c and d using $h[k] = 1/3, 1/3, 1/3$, that is a three sample moving average and compare the resulting mean square errors.

(Note: $MSE = 1/N \sum_{k=0}^N (S[k] - \hat{S}[k])^2$. In the provided files the length of the received and desired signals is

512. The desired signal, $S[k]$, and the filtered signal, $\hat{S}[k]$, needs to be aligned to calculate the mean square error, i.e., there is a delay going through $h[k]$. Note the first 3 points in $Y[k]$ are used to calculate the first point in $\hat{S}[k]$, also $\hat{S}[k]$ is longer than $S[k]$, in this case the length of $\hat{S}[k]$ is 514)