EECS 861

Homework #3

1. (Concept: Relationship between the variance, mean square, and the square of the mean)

Show that $\sigma_X^2 = \text{Var}[X]=E[X^2]-(E[X])^2$

2. (Concept: Discrete random variable, total probability, finding mean, variance, and mean square)

X is a discrete random variable with

P(X=-2) =0.3, P(X= -1) = 0.2, P(X=1) = 0.2, P(X= 2) = a

a. Find "a"

Given "a" find:

b. Find P(X< -3)

c. Find P(X=0)

d. Find E[X]

e. Find $E[X^2]$

f. Find Var[X]

3. (Concept: calculating mean and mean square from data; comparing empirical data to a model)

Using 200 samples $\{x_1 \ldots x_{200}\}$ of a discrete random variable X is given in http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2025/data_HW3_Prob_2.csv

a. Given this data what is an estimate for $p_k = P(X=k)$ for k=-2, -1, 1, 2?

b. Find the sample mean of X using $\overline{X} = \frac{1}{200}\sum_{i=1}^{200} x_i$

c. Estimate the mean of X using $\hat{X} = \sum_{k=-2}^{2} k p_k$

d. Find the sample mean square of X using $\overline{X}^2 = \frac{1}{200}\sum_{i=1}^{200} x_i^2$

e. Estimate the mean square of X using $\hat{X}^2 = \sum_{k=-2}^{2} k^2 p_k$

f. Is the pmf given in problem 1 a "good" probabilistic model for this data?

4. (Concept: properties of the pdf, and expected value and variance)

X is a random variable with $f_X(x) = 0.1\delta(x)+ 0.1\delta(x-1)+ K*u(x-1)e^{-(x-1)}$ where u(x)= unit step function

a. Sketch $f_x(x)$.

b. Find K.

c. What is P(X= 0)?

d. What is P(X= 2)?

e. What is P(.5<X< 3)?

f. Find E[X]

g. Var[X]

5. (Concept: Gaussian and Uniform random variables)
X is a Gaussian random variable X with μ_X = 0 and σ_X=0.5

a. What is P(-0.5<X<.5)?

b. Plot $P(X < x_i)$ for x_i= -4.0,-1.0, -0.5, -0.25, 0.0, 0.25, 0.5, 1.0, 4.0

c. Confirm your answers using
https://www.mathportal.org/calculators/statistics-calculator/normal-distribution-calculator.php

d. Assuming X is a Uniform random variable [-1.5, 1.5] repeat part b.

e. Is a Uniform random variable [-1.5, 1.5] "good" probabilistic model for the data given in Homework 2-Problem 2?

6. (Concept: joint and conditional distribution functions; correlation coefficient; and SI)
X and Y have the following joint distribution function

□	X = −2	X = 0	X = 2
Y = −2	1 / 8	1 / 8	0
Y = 0	0	0	1 / 4
Y = 2	1 / 8	1 / 8	1 / 4

a. Find P(X=0).

b. Find P(Y=2).

c. Find P(X=0|Y=2).

d. Find Covar[X,Y]

e. Find ρ_{XY}.

f. Are X and Y SI random variables?

7. (Concept: Properties of expected value and variance)
Given a and b are constants and X and Y are RV's.
Find:

a. E[a+bX]

b. E[aX+bY]

c. Var[aX]

d. Var[aX+bY]

e. Assuming X and Y are SI find Var[aX+bY]

f. Assuming N is a constant, Var[X_i]=σ_X^2 ∀i, and the X_i's are SI. Define $Z = \frac{1}{N}\sum_{i=1}^{N} X_i$. Find Var[Z]