EECS 861

Homework #5

1. (Concepts: Member functions and an ensemble of member functions)

Define a random process X(t) based on the outcome k of tossing a fair 6 sided die as:

$$X(t) = \begin{cases} -2 & k = 1 \\ -1 & k = 2 \\ 1 & k = 3 \\ 2 & k = 4 \\ t & k = 5 \\ -t & k = 6 \end{cases}$$

a. Find the joint probability mass function of X(0) and X(2).

b. Find the marginal probability mass functions of X(0) and X(2).

c. Find E{X(0)}, E{X(2)}, and E{X(0)X(2)}.

2. (Concepts: Discrete time RP, the two-dimensional nature of RP's)

For this problem use the data in this file.

http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2025/data_HW_5_Prob_2.xls

Each Sheet contains data from one discrete time random process,

Case 1 X[n],

Case 2 Y[n],

Case 3 Z[n].

Each row is a sample function of that discrete time random process.

a. For Sheet 1 create 3 plots, one plot per row for the first 3 rows.

b. For Sheet 1 create 3 plots, one plot per column for the first 3 columns.

c. For Sheet 1 calculate the average and variance of all the values in each row, plot the row averages.

d. For Sheet 1 calculate the average and variance of all the values in each column, plot the column averages.

e. For Sheet 1 consider column 2 and 3 as a pair of random samples; estimate the correlation coefficient between these samples.

f. For Sheet 1 repeat part e. for column 2 and 4.

g. For Sheet 1 repeat part e. for column 2 and 5.

h. Repeat e.-g. for Sheet 2

i. Repeat e.-g. for Sheet 3

h. Discuss the differences in the estimate the correlation coefficient for the three discrete time random processes.

3. (Concept: Analytical description using random variables, time varying pmf)

$X(t) = A\sin(2\pi t+\varphi)$

For $\varphi=0$ and P(A=-1)= P(A=1)= P(A=-2)= P(A=2)=0.25.

a. Sketch all possible sample functions of X(t)
b. What is P(X(1)=0)?
c. What is P(X(0.25=0)?
d. What is the PMF for the RV X(1.25)?
e. What is the PMF for the RV X(1.0)?
f. Find E[X(t)].

For A=1 and P(φ=+π/4)= P(φ=- π/4)=0.5

g. Sketch 2 sample functions of X(t)
h. Find E[X(t)].

4. (Concept: Random walk)
X[n] is a discrete random sequence.

$$X[n]=\sum_{i=1}^{n} J_i \text{ with } P(J_i=1)=P(J_i=-1)=\frac{1}{2} \text{ and } J_i\text{'s are S.I and X[0]=0}$$

a. Sketch two sample functions of X[n] for n=1....10
b. Find P(X[3]=1)
c. Find E[X[3]]
d. Find P(X[6]=0| X[3]=1)

5. (Concept: Random process with time varying variance with an infinite number of member functions)
A random process is described by X(t)= Yt+5, where Y is a Gaussian zero mean unit variance random variable, i.e., Y ~N(0,1).

a. Find E[X(t)]
b. Find Var[X(t)]
c. Find P(X(2)>7)

6. (Concept: Random process with time varying variance with finite number of member functions)
A random process X(t) has 4 member functions that occur with equal probability:

X_1(t) = t^2
X_2(t) = cos(2πt)
X_3(t) = $-t^2$
X_4(t) = - cos(2πt)

a. Plot the sample functions.
b. Find PMF for X(0)
c. Find P[X(1)=-1]
d. Find E[X(t)]
e. Find Var[X(0.5)]
f. For t>1 find P(X(t)>1)
g. Find Var[X(t)]