EECS 861

Homework #6

1. (Concepts: Autocorrelation function, autocovariance function, conditional probability as function of time differences)

Z(t) = Xt+Y

Where X and Y are jointly Gaussian random variables with $E[X] = \mu_X=0$, $E[Y] = \mu_Y=0$, $\sigma_x=2$, $\sigma_Y=4$, and $\rho_{XY}=0.8$.

a. Find E[Z(t)]

b. Find Var[Z(t)]

c. What is the pdf of Z(t), name of pdf and its parameters?

d. Find P(Z(1)>13.3)

e. Find autocorrelation function, $R_{ZZ}(t_1, t_2)$

f. Find autocovariance function, $C_{ZZ}(t_1, t_2)$

g. Find the joint pdf of Z(0) and Z(1).

h. Find P(Z(1)>13.3|Z(0)=10)

2. (Concepts: strict and wide sense stationarity)

Given Z(t) in problem 1.

a. Is Z(t) strict sense stationary?

b. Is Z(t) wide sense stationary?

3. (Concept: Combinations of RP's)

X(t) and Y(t) are SI WSS zero mean random process with $R_{XX}(\tau)$ and $R_{YY}(\tau)$. Find the autocorrelation function of Z(t):

a. Z(t) = 3 + 2X(t) +Y(t)

b. Z(t) = X(t)Y(t)

c. Let X(t) and Y(t) be Gaussian SI WSS random processes with zero mean and unit variance, for Z(t) = 2X(t) +Y(t) find P(Z(1)<2.85).

4. (Concept: Autocorrelation at the output of a simple system)

Z(t) = X(t) + 0.5 $X(t-\alpha)$

Where $E[X(t)] = \mu_X=0$ and the autocorrelation function of X(t) is $R_{XX}(t_1, t_2)$.

a. Find E[Z(t)]

b. Find $R_{ZZ}(t_1, t_2)$.

c. Repeat b. assuming that X(t) is a wide sense stationary random process.

d. Assume X(t) = Acos(2 $\pi f_1 t+\theta$) where A is a Gaussian random variable with $E[A]=0$, $\sigma_A=1$, θ is uniformly distributed between $-\pi$ and π, also A and θ are SI.

Note that X(t) is a wide sense stationary random process. Find $R_{ZZ}(\tau)$, simplify your result.

5. (Concepts: Autocorrelation function of sums of random cosines, Gaussian random Fourier series or a Gaussian cyclostationary process, Output of system with Gaussian random Fourier series input)

Assume X(t) = Acos(2 $\pi f_1 t+\theta$) + Bcos(2 $\pi(2 f_1)t+\Phi$) where A and B are iid Gaussian random variables with $E[A] = E[B]=0$, $\sigma_A=\sigma_B=1$; θ and Φ are uniformly distributed between $-\pi$ and π, and A, B, θ, and Φ are SI.

Note: a random process that is the infinite sum of harmonically related cosines with independent Gaussian random amplitudes and uniform phases is called Gaussian random Fourier series or a Gaussian cyclostationary process. This problem is a two-tone random Fourier series with frequencies f_1 and $2 f_2$.

a. Find $R_{XX}(\tau)$, simplify your result.

b. Let Z(t) = X(t) + 0.5 $X(t-\alpha)$ Find $R_{ZZ}(\tau)$, simplify your result to show $R_{ZZ}(\tau)$ is the sum of two cosines, $\cos(2\,\pi f_1 \tau)$ and $\cos(4\,\pi f_1 \tau)$.

c. Plot $R_{ZZ}(\tau)$ with f_1=3 and α=0.35.

6. (Concept: Stationarity)
For this problem use the data in this file.
http://www.ittc.ku.edu/~frost/EECS_861/EECS_861_HW_Fall_2025/data_HW_5_Prob_2.xls

Each Sheet contains data from one discrete time random process,

Case 1 X[n],

Case 2 Y[n],

Case 3 Z[n].

Each row is a sample function of that discrete time random process.

a. Consider column i and i+1 as a pair of random samples (e.g., X[i], X[i+1]); calculate the correla tion coefficient between these samples for all i. Calculate the average of the correlation coeff cient. That is, column 1 & 2 is random sample one (e.g., X[1], X[2]), column 2 & 3 is random sample 2 (e.g., X[2], X[3]), column 3 & 4 (e.g., X[3], X[4]), is random sample 3. This is a lag of 1. Report that average of the Lag 1 correlations.

b. Repeat part a) for a lag of 2 column i and i+2 (e.g., X[3], X[5]). Report that average of the Lag 2 correlations.

c. Repeat part a) for a lag of 3 column i and i+3 (e.g., X[3], X[6]). Report that average of the Lag 3 correlations.

d. Repeat a. - c. for Y[n] and Z[n]

e. Comment on the stationary of these random processes.

7. (Concept: Autocorrelation function of a discrete time RP)
Find the autocorrelation function of the discrete time sequence X[n]=Acos(ωn+θ), where A is normally distributed with mean 0 and variance σ^2, and the phase θ is uniformly distributed between $-\pi$ and π.

8. (Concept: Autocorrelation function of a random sequence with memory)
Let a_k be a random sequence with $a_k = 0.75\,A_k + 0.25\,A_{k-1}$. A_k' s are statistically independent random variables with $P(A_k = A)$=0.5, $P(A_k = -A)$=0.5, and A=1.
Find the autocorrelation of the a_k random sequence.

a. Find $E[a_k]$.

b. Find Var[a_k].

c. Find R_a[m]=E[$a_k a_{k+m}$]