EECS 861

Homework #7

1. (Concept: Criteria for being an autocorrelation function)
Determine whether the following functions can be the autocorrelation function for a WSS real values random process (YES or NO):

a. $2\Lambda(\tau)$ +$\sin(2\pi 100\tau)$
b. $\Lambda(\tau\text{-}2)$
c. $3\Lambda(\tau)$
d. $4\,e^{-10|t|}$
e. $1000\text{sinc}(1000\tau)$
f. $16\,e^{\frac{-\pi\tau^2}{16}}$

2. (Concept: Autocorrection of an RF signal with WSS iid Gaussian in-phase and quadrature components)
X(t) and Y(t) are wide sense stationary, independent, zero mean, and jointly Gaussian random processes Z(t) = X(t)cos($2\pi f_c$t)+Y(t)sin($2\pi f_c$t) with f_c a constant and $R_{XX}(\tau)$= $R_{YY}(\tau)$

a. Find E[Z(t)]
b. Find $R_{ZZ}(\tau)$

3. (Concept: finding means and variances given an autocorrelation function.)
Find the E[X(t)] and Var[X(t)] for a wide sense stationary random process with the following autocorrelation functions:

a. $R_{XX}(\tau)$=9 $e^{-5|\tau|}$
b. $R_{XX}(\tau)$ = 9 + 9 $e^{-5|\tau|}$
c. $R_{XX}(\tau)$ = 9 $e^{\frac{-\pi\tau^2}{25}}$
d. $R_{XX}(\tau)$ = 9+9 $e^{\frac{-\pi\tau^2}{25}}$

4. (Concept: Conditional probabilities of RP's given autocorrelation functions)
X(t) is a wide sense stationary Gaussian random processes with $R_{XX}(\tau)$ = $10\,e^{-\frac{\pi\tau^2}{2}}$.

a. Find E[X(0.1)], Var[X(0.1)], E[X(0.2)], and Var[X(0.2)]
b. What is the distribution of X(0.1), i.e., name of pdf and its parameters?
c. Find P(X(0.1)>4)
d. What is the covariance matrix for X(0.1) and X(0.2)?
e. What is the joint distribution of X(0.1) and X(0.2), i.e., name of pdf and its parameters?
f. What is the correlation coefficient between X(0.1) and X(0.2)?
g. Find P(X(0.2)>4|X(0.1)=3.5)
h. Approximate P(X(2)>4|X(0.1)=3.5)

Hint: Check the result for part g. with **Study of the Conditional probability P(X(t+-Tau)>L|X(t)=y) for Gaussian Random Process given Different Autocorrelation Functions**

5. (Concept: RP at the output of square law detector)
X(t) is a wide sense stationary zero mean, Gaussian random processes with $R_{xx}(\tau)$=$16\,e^{\frac{-\pi\tau^2}{8}}$.

Define Z(t) = X^2(t). This is the case of Gaussian noise as input to a square law detector.

a. Find E[Z(t)]

b. Find $R_{ZZ}(t_1, t_2)$

c. Is Z(t) a wide sense stationary random processes (YES or NO)?

d. Is Z(t) a Gaussian random processes (YES or NO)?

Hint: If X and Y are jointly Gaussian random variables then

$E\left[X^2 Y^2\right] = E\left[X^2\right] E\left[Y^2\right] + 2\,(E[XY])^2$

e. What is the pdf of Z(t)? Hint: See Homework 4 problem 1c.