EECS 861

Homework #8

1. (Concept: Conditional probabilities of RP's given PDS's)

X(t) is a wide sense stationary zero mean, Gaussian random processes with a power spectral density of $S_x(f) = 14.14\, e^{-2\pi f^2}$. [Hint: see Homework 7-problem 4]

a. Find E[X(t)], Var[X(t)], E[X(t+0.1)], and Var[X(t+0.1)]

b. What is the distribution of X(t), i.e., name of pdf and its parameters?

c. Find P(X(t)>4)

d. What is the covariance matrix for X(t) and X(t+0.1)?

e. What is the joint distribution of X(t) and X(t+0.1), i.e., name of pdf and its parameters?

f. What is the correlation coefficient between X(t) and X(t+0.1)?

g. Find P(X(t+0.1)>4|X(t)=3.5)

h. Approximate P(X(t+20)>4|X(t)=3.5)

Hint: Check the result for part g. with **Study of the Conditional probability P(X(t+-Tau)>L|X(t)=y) for Gaussian Random Process given Different Autocorrelation Functions**

2. (Concept: Comparison of different definitions of the bandwidth of a RP)

For a random process with a PSD of $S_x(f) = 10\, e^{\frac{-f^2}{2\alpha}}$

a. Find the B_{eff}

b. Find the $B_{3\,\text{dB}}$

c. Find root mean square bandwidth, B_{RMS} (Hint: See Homework 1 Problem 2)

d. Compare the above definitions of bandwidth.

3. (Concept: Correlation time)

Given the random process from problem 2,

a. Find the correlation time τ_c

b. Compare the correlation time to $\frac{1}{2B_{\text{eff}}}$ $\frac{1}{2B_{3\,\text{dB}}}$ and $\frac{1}{2B_{\text{RMS}}}$, and which bandwidth definition best matches the correlation time.

c. What is the approximate correlation coefficient between X(t) and X(t+$\frac{1}{2\sqrt{\alpha}}$)?

4. (Concept: Criteria for being a PSD)

Determine whether the following functions can be the power spectral density for a WSS real valued random process (YES or NO).

a. $\text{rect}\left(\frac{f}{10}\right)$

b. $e^{+\pi f^2}$

c. $\Lambda(10f)$

d. $10e^{-(f+0.10)}$

e. $e^{-\pi f^2}$

f. $5\,\delta(f) + \sin(200\,\pi f)$

g. $\delta(f) + 4\,\delta(f+20\ 0) + 4\,\delta(f-200)$

5. (Concept: Finding autocorrelation functions from PSDs)
The random process X(t) is WSS. For each of the autocorrelation functions below find and plot the corresponding power spectral density, $S_x(f)$.

a) $R_{XX}(\tau) = 16\cos(2\,\pi 1000\,\tau)$

b) $R_{XX}(\tau) = \frac{8}{1+4\,\pi^2\,\tau^2}$

c) $R_{XX}(\tau) = 16\,\Lambda\left(\frac{\tau}{4}\right)$

d) $R_{XX}(\tau) = 16\,e^{-\left|\frac{\tau}{2}\right|}$

e) $R_{XX}(\tau) = 16\,e^{-\pi\left(\frac{\tau}{2}\right)^2}$

6. (Concept: total power and power in band)
A power spectral density for a WSS random process X(t) is
$S_X(f) = 0.001\,\Lambda\left(\frac{f}{100\text{ kHz}}\right)$

a. Find $E[X^2(t)]$.

b. Find the Average power.

c. Find the % power the band [0, 50 kHz]

7. (Concept: Correlation properties of RV’s defined at different times of a RP)
Given X(t) is Gaussiand RP with $S_X(f) = \frac{2L}{1+4f^2L^2\pi^2}$.

a. Find $R_{XX}(\tau)$.

b. Given $R_{XX}(\tau)$ from part a. and set $L = 5.0$, τ_1=1.0, τ_2=2.0, and τ_3=3 find the covariance matrix for X(t), X(t-τ_1), X(t-τ_2), X(t-τ_3).

c. Given X(t)=0 and X(t-τ_1)=0, find the correlation coefficient between X(t-τ_2), X(t-τ_3).

To confirm your result use: **Interactive Gaussian Process Visualization**

Hint: see **Conditional Bivariate distributions from a 4-Dimensional Mulitvariate Gaussian**