EECS 861

Homework #9

1. (Concept: Variance of a time average and applying the large B_{eff}T approximation)

A zero mean WSS random process, X(t) has the following autocorrelation function

$R_{XX}(\tau)= 4\, e^{-100|\tau|}$

a. Find E[X(t)]

b. Find the variance of X(t).

c. Find the effective bandwidth, B_{eff} of X(t).

d. Find the correlation time τ_c in ms

Let

$$Y = \frac{1}{T}\int_0^T X(\eta)\, d\eta$$

e. For T = 6.5 ms find the variance of Y and the ratio of Var[X]/Var[Y].

f. For T = 2000 ms find the variance of Y and the ratio of Var[X]/Var[Y]; then compare the Var[Y] to the large B_{eff}T approximation of the of Var[Y].

g. For T = 2000 ms find P(Y> 0.4)

2. (Concept: Comparing ensemble and time averages)

Let X(t) = A sin(2 πf_ct) where A is a random variable with E[A]=m and $Var[A] = \sigma^2$.

a. Find E[X(t)]

b. Find $\langle X(t)\rangle_T = \frac{1}{T}\int_{-T/2}^{T/2} X(t)\, dt$

c. Find $\lim_{T\to\infty} \langle X(t)\rangle_T$

d. Compare the results from part a and part b.

e. Repeat a. and d. with E[A]=0 and $Var[A] = \sigma^2$

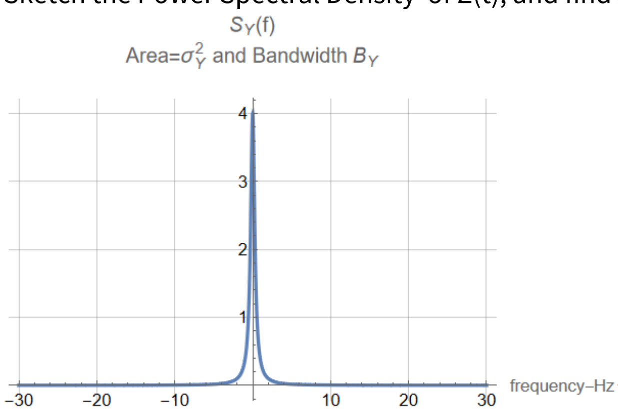
3. (Concepts: SSS and ergodicity)

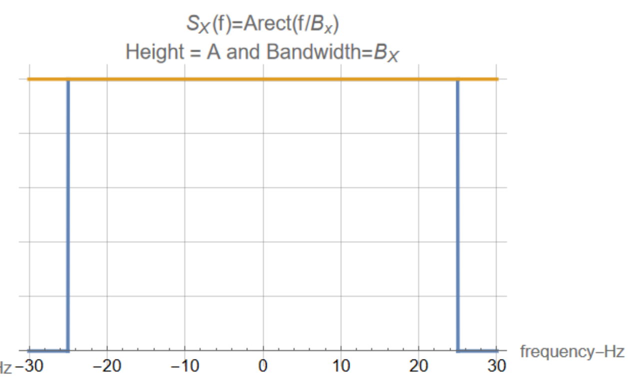
Explain the concepts of strict sense stationarity and ergodicity.

4. (Concept: PSD of Product of RP with significantly different bandwidths)

X(t) and Y(t) are two independent WSS random processes with the power spectral density functions shown below. Let Z(t) = X(t)Y(t).

Sketch the Power Spectral Density of Z(t), and find $S_Z(0)$.





5. (Concept: Properties of a Gaussian RP at different times)

A zero mean Gaussian WSS random process, X(t) has the following PSD

$S_x(f) = \frac{2\,\alpha}{1+4\,f^2\,\pi^2\,\alpha^2}$

a. Plot $R_{XX}(\tau)$ for α=0.002.

b Find E[X(t)]

c. Find Var[X(t)]

d. Are the random variables X(t) and X(t-2.5ms) uncorrelated (Yes or No); justify?

e. Are the random variables X(t) and X(t-2.5ms) statistically independent (Yes or No); justify?

f. What is the bivariate pdf for the random variables X(t) and X(t-2.5ms), Specify the pdf, mean vector and covariance matrix.

g. Can the random variables X(t) and X(t-25ms) be assumed to be uncorrelated (Yes or No); justify?

h. Can the random variables X(t) and X(t-25ms) statistically independent (Yes or No); justify?

i. What is P(X(t)>1| X(t-25ms)=3.29]? Justify any assumptions.

6. (Concept: Variance of samples of RP and finding the observation time to achieve a variance goal.) Given the random process X(t) has the following PSD $S_x(f) = \frac{4}{1000}\Lambda\left(\frac{f}{1000}\right)$. 20 samples of X(t) are collected at a sample rate of 1000 samples/sec the average of these 20 samples is $Y = \frac{1}{20}\sum_{k=1}^{20} X\,(t - \mathrm{k}\Delta\mathrm{t})$ where $\Delta\mathrm{t} = 1$ ms

a. Find the variance of Y.

b. How long to you need to sample X(t) (that observe the signal X(t) at a sample rate of 1000 samples/sec such that the variance of the time average Var[Y]=0.01

7. (Concept: finding integration time to achieve a variance goal)

Let $R_{XX}(\tau)= 9 + e^{-\frac{|\tau|}{10}}$ and Y $= \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} X(t)\,dt$.

Find T such that the $\mathrm{Var}[Y] = \frac{\mathrm{Var}[X]}{10}$.

Hint: Plot Var[Y] as a function of T and find the intersection of that curve with the target Var[Y].