# Trees

- Linear Vs non-linear data structures
- Types of binary trees
- Binary tree traversals
- Representations of a binary tree
- Binary tree ADT
- Binary search tree



# Overview

- We have discussed *linear* data structures
  - arrays, linked lists, stacks, queues
- Some other data structures we will consider
  - trees, tables, graphs, hash-tables
- Trees are extremely useful and suitable for a wide range of applications
  - sorting, searching, expression evaluation, data set representation
  - especially well suited to recursive algorithm implementation



# Terminology

- A Tree T is a set of n >= 0 elements:
  - if n == 0, T is an empty tree
  - if n > 0 then there exists some element called $r \in T$ called the root of T such that $T$ - {r} can be partitioned into zero or more disjoint sets $T_1$ ,$T_2$ , ... where each subset forms a tree
- Trees are composed of nodes and edges
- Trees are hierarchical
  - parent-child relationship between two nodes
  - ancestor-descendant relationships among nodes
- Subtree of a tree: Any node and its descendants



# Terminology

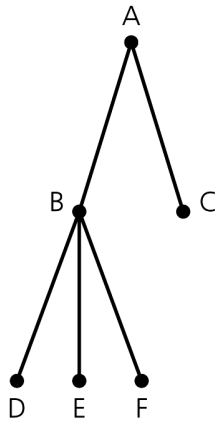



***Figure 10-1*** A general tree

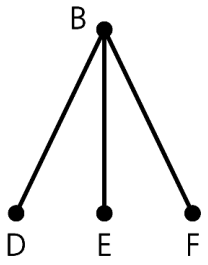



***Figure 10-2***

A subtree of the tree in Figure 10-1

# Terminology

- Parent of node n
  - The node directly above node n in the tree
- Child of node n
  - A node directly below node n in the tree
- Root
  - The only node in the tree with no parent
- Subtree of node n
  - A tree that consists of a child (if any) of node n and the child's descendants



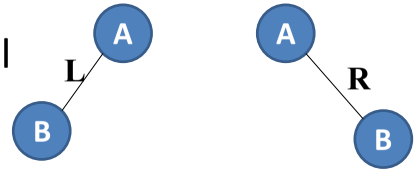
# Terminology

- Leaf
  - A node with no children
- Siblings
  - Nodes with a common parent
- Ancestor of node n
  - A node on the path from the root to n
- Descendant of node n
  - A node on a path from n to a leaf



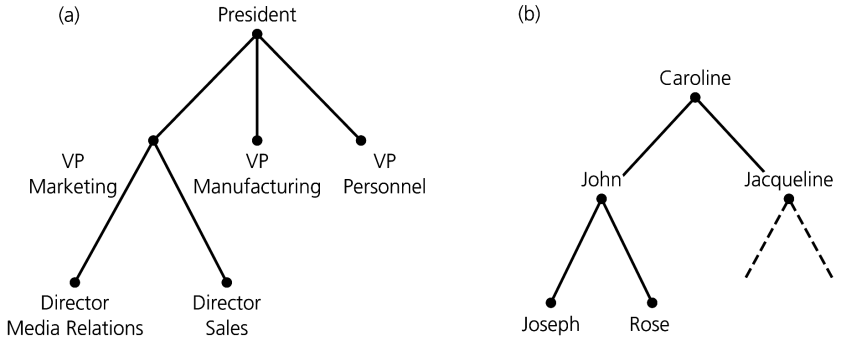
# A Binary Tree

- A binary tree is a set T of nodes such that
  - T is empty, or
  - T is partitioned into three disjoint subsets:
    - a single node r, the root
    - two possibly empty sets that are binary trees, called the left subtree of r and the right subtree of r
- Binary trees are ordered
- These trees are not equal






# A General Tree & A Binary Tree



# More Binary Trees

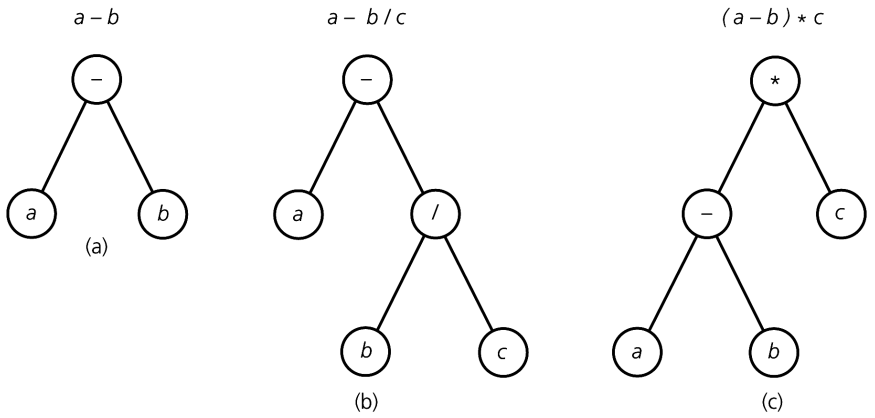
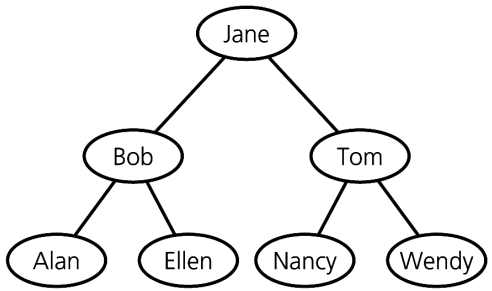



***Figure 10-4*** Binary trees that represent algebraic expressions



# A Binary Search Tree

- A binary search tree is a binary tree that has the following properties for each node n
  - n's value is > all values in n's left subtree TL
  - n's value is < all values in n's right subtree TR
  - both TL and TR are binary search trees






# The Height of Trees

- Height of a tree
  - Number of nodes along the longest path from the root to a leaf

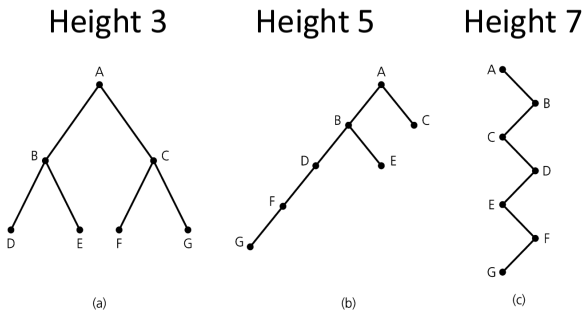



***Figure 10-6***
Binary trees with the same nodes but different heights

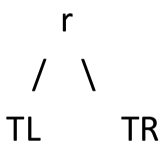


# The Height of Trees

- Level of a node n in a tree T
  - If n is the root of T, it is at level 1
  - If n is not the root of T, its level is 1 greater than the level of its parent
- Height of a tree T defined in terms of the levels of its nodes
  - If T is empty, its height is 0
  - If T is not empty, its height is equal to the maximum level of its nodes

# The Height of Trees

- A recursive definition of height
  - If T is empty, its height is 0
  - If T is not empty,
  - height(T) = 1 + max{height(TL), height(TR)}






# Full Binary Trees

- A binary tree of height h is full if
  - Nodes at levels < h have two children each
- Recursive definition
  - If T is empty, T is a full binary tree of height 0
  - If T is not empty and has height h > 0, T is a full binary tree if its root's subtrees are both full binary trees of height h – 1

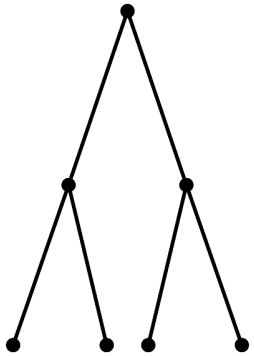


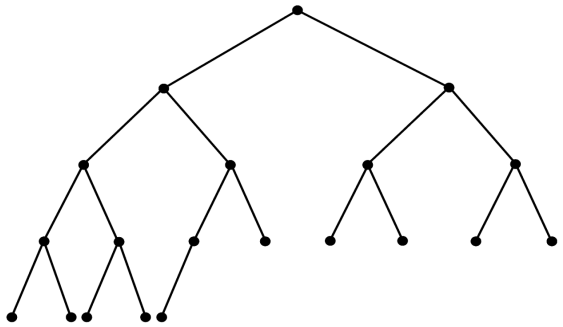
*Figure 10-7*

A full binary tree of height 3



# Complete Binary Trees

- A binary tree of height h is complete if
  - It is full to level h – 1, and
  - Level h is filled from left to right





# Complete Binary Trees

- Another definition:
- A binary tree of height h is complete if
  - All nodes at levels <= h – 2 have two children each, and
  - When a node at level h – 1 has children, all nodes to its left at the same level have two children each, and
  - When a node at level h – 1 has one child, it is a left child

# Balanced Binary Trees

- A binary tree is balanced if the heights of any node's two subtrees differ by no more than 1
- Complete binary trees are balanced
- Full binary trees are complete and balanced



# Traversals of a Binary Tree

- A traversal visits each node in a tree
  - to do something with or to the node during a visit
  - for example, display the data in the node
- General form of a recursive traversal algorithm

```
traverse (in binTree:BinaryTree)
   if (binTree is not empty)
   {  traverse(Left subtree of binTree's root)
      traverse(Right subtree of binTree's root)
   }
```

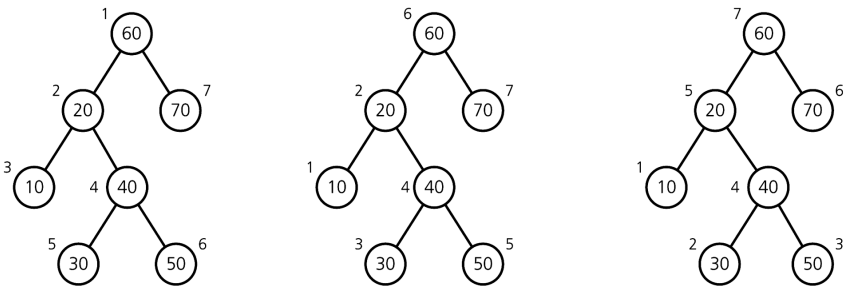


# Traversals of a Binary Tree

- Preorder traversal
  - Visit root before visiting its subtrees
    - i. e. Before the recursive calls
- Inorder traversal
  - Visit root between visiting its subtrees
    - i. e. Between the recursive calls
- Postorder traversal
  - Visit root after visiting its subtrees
    - i. e. After the recursive calls



# Traversals of a Binary Tree




(a) Preorder: 60, 20, 10, 40, 30, 50, 70

(b) Inorder: 10, 20, 30, 40, 50, 60, 70

(c) Postorder: 10, 30, 50, 40, 20, 70, 60

(Numbers beside nodes indicate traversal order.)

***Figure 10-10***

Traversals of a binary tree: (a) preorder; (b) inorder; (c) postorder

# Traversals of a Binary Tree

- A traversal operation can call a function to perform a task on each item in the tree
  - this function defines the meaning of "visit"
  - the client defines and passes this function as an argument to the traversal operation
- Tree traversal orders correspond to algebraic expressions
  - infix, prefix, and postfix



# The ADT Binary Tree

| Binary tree |
| --- |
| root<br>left subtree<br>right subtree |
| createBinaryTree()<br>destroyBinaryTree()<br>isEmpty()<br>getRootData()<br>setRootData()<br>attachLeft()<br>attachRight()<br>attachLeftSubtree()<br>attachRightSubtree()<br>detachLeftSubtree()<br>detachRightSubtree()<br>getLeftSubtree()<br>getRightSubtree()<br>preorderTraverse()<br>inorderTraverse()<br>postorderTraverse() |

```
+createBinaryTree()
+createBinaryTree(in rootItem: TreeItemType)
+createBinaryTree(in rootItem: TreeItemType,
                  inout leftTree: BinaryTree,
                  inout rightTree: BinaryTree)
+destroyBinaryTree()
+isEmpty(): boolean {query}
+getRootData(): TreeItemType                        throw TreeException
+setRootData(in newItem: TreeItemType)              throw TreeException
+attachLeft(in newItem: TreeItemType)               throw TreeException
+attachRight(in newItem: TreeItemType)              throw TreeException
+attachLeftSubtree(inout leftTree: BinaryTree)      throw TreeException
+attachRightSubtree(inout rightTree: BinaryTree)    throw TreeException
+detachLeftSubtree(out leftTree: BinaryTree)        throw TreeException
+detachRightSubtree(out rightTree: BinaryTree)      throw TreeException
+getLeftSubtree(): BinaryTree
+getRightSubtree(): BinaryTree
+preorderTraverse(in visit:FunctionType)
+inorderTraverse(in visit:FunctionType)
+postorderTraverse(in visit:FunctionType)
```



# The ADT Binary Tree

- Building the ADT binary tree in Fig. 10-6b

```
tree1.setRootData('F')
tree1.attachLeft('G')
tree2.setRootData('D')
tree2.attachLeftSubtree(tree1)
tree3.setRootData('B')
tree3.attachLeftSubtree(tree2)
tree3.attachRight('E)
tree4.setRootData('C')
tree10_6.createBinaryTree('A',tree3,tree4)
```



# Possible Representations of a Binary Tree

- An array-based representation
  - Uses an array of tree nodes
  - Requires the creation of a free list that keeps track of available nodes
  - only suitable for *complete* binary trees
- A pointer-based representation
  - Nodes have two pointers that link the nodes in the tree

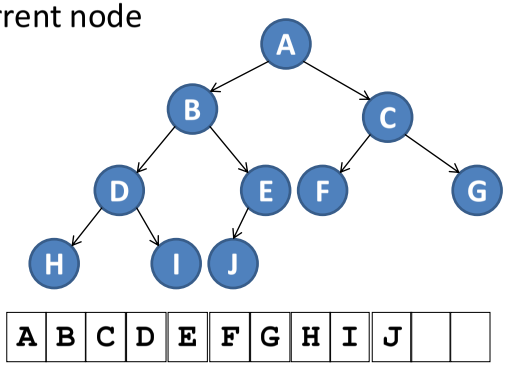
# Array Based Binary Tree

- Given a complete binary tree T with n nodes, T can be represented using an array A[0:n-1] such that
  - root of T is in A[0]
  - for node A[i], its left child is at A[2i+1] and its right child at A[2i+2] if it exists
- Completeness of the tree is important because it minimizes the size of the array required
- Note that
  - parent of node A[i] is at A[(i-1)/2]
  - for n > 1, A[i] is a leaf node iff n <= 2i
- Balanced requirement makes an array representation unsuitable for binary search tree implementation



# Array Based Binary Tree

- Complete tree fits in minimum size array
  - space efficient
- Nodes do not need child or parent pointers
  - index of these can be calculated from the index of the current node






# Array Based Binary Tree

- Advantages
  - space saving through direct computation of child and parent indices rather than pointers
  - O(1) access time through direct computation
    - pointers are also O(1) access but with larger K
- Disadvantages
  - only useful when tree is complete
    - or, complete enough that unused cells do not waste much memory
  - sparse tree representation is too memory intensive
- If a complete tree is of height h, it requires an array of size $2^h$-1
  - a skewed BST of 10 nodes is of height 10, requiring an array of size $2^{10}$-1 = 1023



# Pointer-based ADT Binary Tree

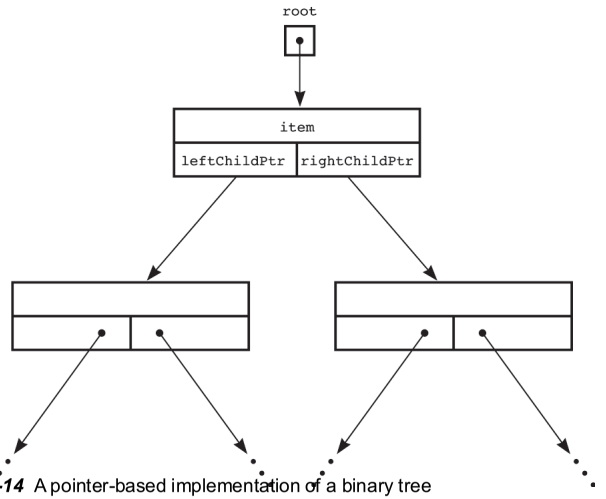



***Figure 10-14*** A pointer-based implementation of a binary tree

# Pointer-based ADT Binary Tree

- `TreeException` and `TreeNode` classes
- `BinaryTree` class
  - Several constructors, including a
    - Protected constructor whose argument is a pointer to a root node; prohibits client access
    - Copy constructor that calls a private function to copy each node during a traversal of the tree
  - Destructor



# Binary Tree ADT – TreeNode.h

```
// TreeNode.h

typedef string TreeItemType;
// node in the tree
class TreeNode {
private:
   TreeNode() {};
   TreeNode(const TreeItemType& nodeItem, TreeNode *left = NULL,
            TreeNode *right = NULL): item(nodeItem),
                                     leftChildPtr(left),
                                     rightChildPtr(right) {}
   TreeItemType item;         // data portion
   TreeNode *leftChildPtr;    // pointer to left child
   TreeNode *rightChildPtr; // pointer to right child
   friend class BinaryTree;  // friend class
};
```



# Binary Tree ADT – TreeException.h

```
// TreeException.h

#include <stdexcept>
#include <string>
using namespace std;

Class Tree Exception : public logic_error {
public:
   TreeException(const string& message = “”) :
                     logic_error(message.c_str())
   {}
};
```



# Binary Tree ADT – BinaryTree.h

```
//Begin BinaryTree.h

#include "TreeException.h"
#include "TreeNode.h"

// This function pointer is used by the client
// to customize what happens when a node is visited
typedef void (*FunctionType)(TreeItemType& anItem);

class BinaryTree {
public:
   // constructors and destructor:
   BinaryTree();
   BinaryTree(const TreeItemType& rootItem);
   BinaryTree(const TreeItemType& rootItem, BinaryTree& leftTree,
                                            BinaryTree& rightTree);
   BinaryTree(const BinaryTree& tree);
   virtual ~BinaryTree();
```

# Binary Tree ADT – BinaryTree.h

```
// binary tree operations:
virtual bool isEmpty() const;

virtual TreeItemType getRootData() const                    throw(TreeException);
virtual void setRootData(const TreeItemType& newItem) throw (TreeException);

virtual void attachLeft(const TreeItemType& newItem)     throw(TreeException);
virtual void attachRight(const TreeItemType& newItem)    throw(TreeException);
virtual void attachLeftSubtree(BinaryTree& leftTree)     throw(TreeException);
virtual void attachRightSubtree(BinaryTree& rightTree)   throw(TreeException);

virtual void detachLeftSubtree(BinaryTree& leftTree)     throw(TreeException);
virtual void detachRightSubtree(BinaryTree& rightTree)   throw(TreeException);

virtual BinaryTree getLeftSubtree() const;
virtual BinaryTree getRightSubtree() const;

virtual void preorderTraverse(FunctionType visit);
virtual void inorderTraverse(FunctionType visit);
virtual void postorderTraverse(FunctionType visit);
```



# Binary Tree ADT – BinaryTree.h

```
// overloaded assignment operator:
   virtual BinaryTree& operator=(const BinaryTree& rhs);

protected:
   BinaryTree(TreeNode *nodePtr);                  // constructor

   // Copies the tree rooted at treePtr into a tree rooted
   // at newTreePtr. Throws TreeException if a copy of the
   // tree cannot be allocated.
   void copyTree(TreeNode *treePtr, TreeNode* & newTreePtr) const
                        throw(TreeException);;

   // Deallocate memory for a tree.
  void destroyTree(TreeNode * &treePtr);

   // The next two functions retrieve and set the value
   // of the private data member root.
   TreeNode *rootPtr( ) const;
   void setRootPtr(TreeNode *newRoot);
```



# Binary Tree ADT – BinaryTree.h

```
   // The next two functions retrieve and set the values
   // of the left and right child pointers of a tree node.
   void getChildPtrs(TreeNode *nodePtr, TreeNode * &leftChildPtr,
                        TreeNode * &rightChildPtr) const;
   void setChildPtrs(TreeNode *nodePtr, TreeNode *leftChildPtr,
                        TreeNode *rightChildPtr);

   void preorder(TreeNode *treePtr, FunctionType visit);
   void inorder(TreeNode *treePtr, FunctionType visit);
   void postorder(TreeNode *treePtr, FunctionType visit);

private:
   TreeNode *root; // pointer to root of tree

}; // end class
// End of header file. BinaryTree.h
```



# Binary Tree ADT – BinaryTree.cpp

```
// Implementation file BinaryTree.cpp for the ADT binary  tree.
#include "BinaryTree.h"            // header file
#include <cstddef>                 // definition of NULL
#include <cassert>                 // for assert()

BinaryTree::BinaryTree() : root(NULL) { }

BinaryTree::BinaryTree(const TreeItemType& rootItem) {
   root = new TreeNode(rootItem, NULL, NULL);
   assert(root != NULL);
}

BinaryTree::BinaryTree(const TreeItemType& rootItem,
                       BinaryTree& leftTree, BinaryTree& rightTree) {
   root = new TreeNode(rootItem, NULL, NULL);
   assert(root != NULL);
   attachLeftSubtree(leftTree);
   attachRightSubtree(rightTree);
}
```

# Binary Tree ADT – BinaryTree.cpp

```
BinaryTree::BinaryTree(const BinaryTree& tree) {
   copyTree(tree.root, root);
}

BinaryTree::BinaryTree(TreeNode *nodePtr): root(nodePtr) { }

BinaryTree::~BinaryTree() {
   destroyTree(root);
}

bool BinaryTree::isEmpty() const {
   return (root == NULL);
}

TreeItemType BinaryTree::getRootData() const {
   if (isEmpty())
      throw TreeException("TreeException: Empty tree");
   return root>item;
}
```



# Binary Tree ADT – BinaryTree.cpp

```
void BinaryTree::setRootData(const TreeItemType& newItem) {
   if (!isEmpty()) {
      root>item = newItem;
   } else {
      root = new TreeNode(newItem, NULL, NULL);
      if (root == NULL)
         throw TreeException("TreeException: Cannot allocate memory");
   }
}

void BinaryTree::attachLeft(const TreeItemType& newItem) {
   if (isEmpty()) {
      throw TreeException("TreeException: Empty tree");
   } else if (root>leftChildPtr != NULL) {
      throw TreeException("TreeException: Cannot overwrite left subtree");
   } else {          // Assertion: nonempty tree; no left child
      root>leftChildPtr = new TreeNode(newItem, NULL, NULL);
      if (root>leftChildPtr == NULL)
         throw TreeException("TreeException: Cannot allocate memory");
   }
}
```



# Binary Tree ADT – BinaryTree.cpp

```
void BinaryTree::attachRight(const TreeItemType& newItem) {
   if (isEmpty())
      throw TreeException("TreeException: Empty tree");
   else if (root>rightChildPtr != NULL)
      throw TreeException("TreeException: Cannot overwrite right subtree");
   else { // Assertion: nonempty tree; no right child
      root>rightChildPtr = new TreeNode(newItem, NULL, NULL);
      if (root>rightChildPtr == NULL)
         throw TreeException("TreeException: Cannot allocate memory");
   }
}
void BinaryTree::attachLeftSubtree(BinaryTree& leftTree) {
   if (isEmpty())
      throw TreeException("TreeException: Empty tree");
   else if (root>leftChildPtr != NULL)
      throw TreeException("TreeException: Cannot overwrite left subtree");
   else {            // Assertion: nonempty tree; no left child
      root>leftChildPtr = leftTree.root;
      leftTree.root = NULL;
   }
}
```



# Binary Tree ADT – BinaryTree.cpp

```
void BinaryTree::attachRightSubtree(BinaryTree& rightTree) {
   if (isEmpty())
      throw TreeException("TreeException: Empty tree");
   else if (root>rightChildPtr != NULL)
      throw TreeException("TreeException: Cannot overwrite right subtree");
   else {                // Assertion: nonempty tree; no right child
      root>rightChildPtr = rightTree.root;
      rightTree.root = NULL;
   }
}

void BinaryTree::detachLeftSubtree(BinaryTree& leftTree) {
   if (isEmpty())
      throw TreeException("TreeException: Empty tree");
   else {
      leftTree = BinaryTree(root>leftChildPtr); // constructor taking node * not
tree *
      root>leftChildPtr = NULL;
   }
}
```

# Binary Tree ADT – BinaryTree.cpp

```
void BinaryTree::detachRightSubtree(BinaryTree& rightTree) {
   if (isEmpty())
      throw TreeException("TreeException: Empty tree");
   else {
      rightTree = BinaryTree(root>rightChildPtr); // node * to tree
conversion
      root>rightChildPtr = NULL; // this tree no longer holds that subtree
   }
}

BinaryTree BinaryTree::getLeftSubtree() const {
   TreeNode *subTreePtr;
   if (isEmpty())
      return BinaryTree();
   else {
      copyTree(root>leftChildPtr, subTreePtr);
      return BinaryTree(subTreePtr);
   }
}
```



# Binary Tree ADT – BinaryTree.cpp

```
BinaryTree BinaryTree::rightSubtree() const {
   TreeNode *subTreePtr;
   if (isEmpty())
      return BinaryTree();
   else {
      copyTree(root>rightChildPtr, subTreePtr);
      return BinaryTree(subTreePtr);
   }
}

void BinaryTree::preorderTraverse(FunctionType visit) {
   preorder(root, visit); // preorder written with respect to a tree ptr
}

void BinaryTree::inorderTraverse(FunctionType visit) {
   inorder(root, visit);
}
```



# Binary Tree ADT – BinaryTree.cpp

```
void BinaryTree::postorderTraverse(FunctionType visit) {
   postorder(root, visit);
}
BinaryTree& BinaryTree::operator=(const BinaryTree& rhs) {
   if (this != &rhs) {
      destroyTree(root);            // deallocate lefthand side
      copyTree(rhs.root, root);     // copy righthand side
   }
   return *this;
}
void BinaryTree::destroyTree(TreeNode *& treePtr) {
   if (treePtr != NULL) {
      destroyTree(treePtr>leftChildPtr);
      destroyTree(treePtr>rightChildPtr);
      delete treePtr;                          // postorder traversal
      treePtr = NULL;
   }
}
```



# Binary Tree ADT – BinaryTree.cpp

```
void BinaryTree::copyTree(TreeNode *treePtr, TreeNode *& newTreePtr) const
{
   // preorder traversal
   if (treePtr != NULL) {
      // copy node
      newTreePtr = new TreeNode(treePtr>item, NULL, NULL);
      if (newTreePtr == NULL)
         throw TreeException("TreeException: Cannot allocate memory");
      copyTree(treePtr>leftChildPtr, newTreePtr>leftChildPtr);
      copyTree(treePtr>rightChildPtr, newTreePtr>rightChildPtr);
   } else
      newTreePtr = NULL;     // copy empty tree
}

TreeNode *BinaryTree::rootPtr() const {
    return root;
}
```

# Binary Tree ADT – BinaryTree.cpp

```
void BinaryTree::setRootPtr(TreeNode *newRoot) {
   root = newRoot;
}

void BinaryTree::getChildPtrs(TreeNode *nodePtr, TreeNode *& leftPtr,
                              TreeNode *& rightPtr) const {
   leftPtr = nodePtr>leftChildPtr;
   rightPtr = nodePtr>rightChildPtr;
}

void BinaryTree::setChildPtrs(TreeNode *nodePtr, TreeNode *leftPtr,
                              TreeNode *rightPtr) {
   nodePtr>leftChildPtr = leftPtr;
   nodePtr>rightChildPtr = rightPtr;
}
```



# Binary Tree ADT – BinaryTree.cpp

```
void BinaryTree::preorder(TreeNode *treePtr, FunctionType visit) {
   if (treePtr != NULL) {
      visit(treePtr>item);
      preorder(treePtr>leftChildPtr, visit);
      preorder(treePtr>rightChildPtr, visit);
   }
}

void BinaryTree::inorder(TreeNode *treePtr, FunctionType visit) {
   if (treePtr != NULL) {
      inorder(treePtr>leftChildPtr, visit);
      visit(treePtr>item);
      inorder(treePtr>rightChildPtr, visit);
   }
}
```



# Binary Tree ADT – BinaryTree.cpp

```
void BinaryTree::postorder(TreeNode *treePtr, FunctionType visit) {
   if (treePtr != NULL) {
      postorder(treePtr>leftChildPtr, visit);
      postorder(treePtr>rightChildPtr, visit);
      visit(treePtr>item);
   }
}
// End of implementation file.
```



# Binary Tree ADT – Client Code

```
// Example client code
#include <iostream>
#include "BinaryTree.h"
using namespace std;
void display(TreeItemType& anItem);

int main()
{
  BinaryTree tree1, tree2, left;
  // tree with only a root 70
  BinaryTree tree3(70);

  // build the tree in Figure 10-10
  tree1.setRootData(40);
  tree1.attachLeft(30);
  tree1.attachRight(50);
  tree2.setRootData(20);
  tree2.attachLeft(10);
  tree2.attachRightSubtree(tree1);

  // tree in Fig 10-10
  BinaryTree binTree(60, tree2, tree3);

  binTree.inorderTraverse(display);
  binTree.getLeftSubtree().inorderTraverse
                                (display);
  binTree.detachLeftSubtree(left);
  left.inorderTraverse(display);
  binTree.inorderTraverse(display);

  return 0;
} // end main
```

# Pointer-based ADT Binary Tree: Tree Traversals

- `BinaryTree` class (continued)
  - Public methods for traversals so that visiting a node remains on the client's side of the wall

```
void inorderTraverse(FunctionType visit);
typedef void (*FunctionType)(TreeItemType&
                             item);
```

  - Protected methods, such as `inorder`, that enable the recursion

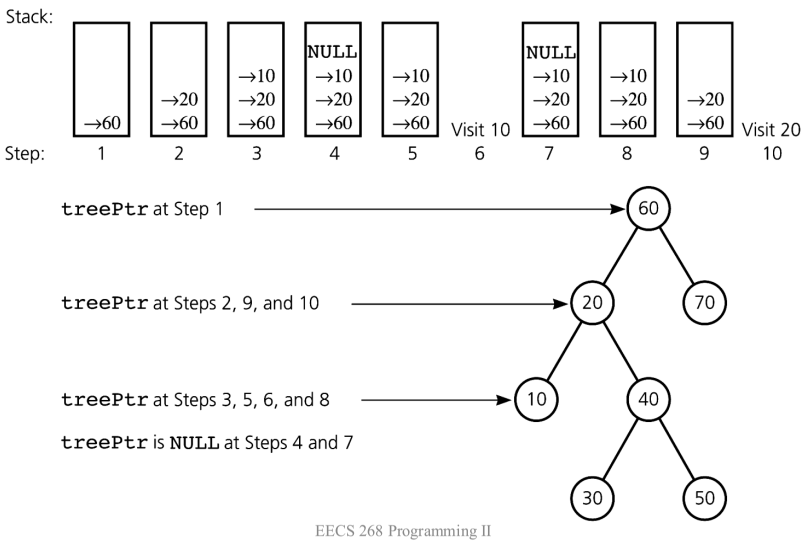
```
void inorder(TreeNode *treeptr,
             FunctionType visit);
```

  - `inorderTraverse` calls `inorder`, passing it a node pointer and the client-defined function `visit`



# Recursive Inorder Traversal

(The notation →60 means "a pointer to the node containing 60.")






# Nonrecursive Inorder Traversal

- An iterative method and an explicit stack can mimic the actions of a return from a recursive call to `inorder`

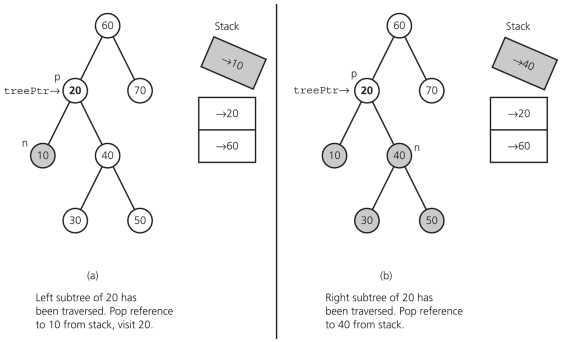



***Figure 10-16***

Traversing (a) the left and (b) the right subtrees of 20



# Copying a Binary Tree

- To copy a tree
  - traverse it in preorder
  - insert each item visited into a new tree
  - use in copy constructor
- To deallocate a tree
  - traverse in postorder
  - delete each node visited
  - "visit" follows deallocation of a node's subtrees
  - use in destructor

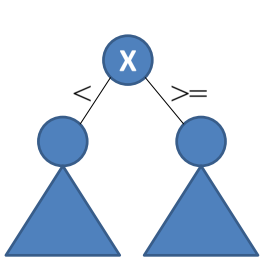
# The ADT Binary Search Tree

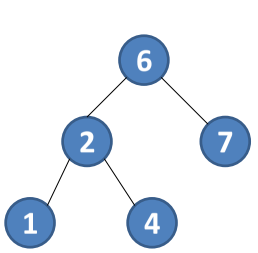
- The ADT binary tree is not suitable when you need to search for a particular item
  - binary *search* tree (BST) is more suitable
- A data item in a BST has specially designated search key
  - search key is the part of a record that identifies it within a collection of records
- Assume that the set of all keys can be linearly ordered
  - a comparison function for two keys cmp(k2, k2) distinguishes 3 cases: (1) k1 < k2, (2) k1 == k2, or (3) k1 > k2
- If we use a binary search tree to organize the set of records, then each record must be a node in the tree
  - Record is a class instance held by tree node
  - Record field is a member variable
  - Key is the record field used as search tag

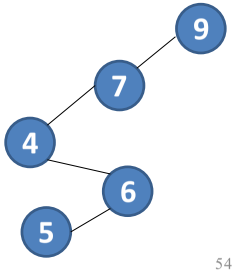


# Binary Search Trees

- Binary tree H such that key of any node x, *key(x)*, is greater than the keys of all nodes in its *left* subtree and is less than or equal to keys of all nodes in its *right* sub-tree
  - often called the BST property
- Equal elements could as easily be in the left subtree
  - but some standard definition is required!






# Binary Search Trees – Observations

- BST may not be a *balanced* binary tree
  - choice of root node is important with respect to the set of all key values present in the tree
- Leftmost descendant of root = minimum item
- Rightmost descendant of root = maximum item
- Inorder traversal of BST = sorted key order
- BST strongly analogous to binary search of an array in sorted order
- Pointer based implementation dynamically allocating tree nodes is the most obvious approach
  - nodes are wrappers for records, might point to records
  - BST **template** would use record **type** as parameter



# The ADT Binary Search Tree

- Simple BST API
  - similar to 10-18 in book
- Assumes method RecordT.get_key( ) exists for all possible record types
- Logic of BST find( ) closely resembles binary search in an array
- Logic of insertion is essentially search for the right place for the inserted record in the tree

```
class BST {
public:
   BST();
   ~BST();
   boolean is_empty();
   boolean insert( RecordT& r);
   RecordT* find( KeyT key);
   boolean delete(KeyT key);
   void preorder();
   void inorder();
    void postorder();
private:
   BST_Node *lchild;
   BST_Node *rchild;
   RecordT *record;
};
```

# ADT Binary Search Tree – find

- find the record with search key *skey*
- first checks the current node and then recursively searches the relevant subtree if it exists
- If relevant subtree does not exist, the search has fails

```
RecordT * BST::find(const KeyT& skey) {
    if ( record == NULL ) {
        return(NULL);
    } else if ( record>get_key() == skey ) {
        return(record);  // key found
    } else if ( record>get_key() > skey ) {
        // search left tree
        if ( lchild == NULL ) {
            return(NULL);
        }
        return(lchild>find(skey));
    } else {
        // search right tree
        if ( rchild == NULL ) {
            return(NULL);
        }
        return(rchild>find(skey));
    }
}
```



# ADT Binary Search Tree: Insertion

- BST::insert() method looks for proper place and adds the record in the right spot
  - insert 7, 3, 1, 8, 13 15, 6, 9, 10 using this algorithm

```
boolean BST::insert(constRecordT& inr) {
    if ( record == NULL ) {
        // This will be the first record in empty tree
        record = &inr;
        return True;
    } else if ( inr>get_key() < record>get_key() ) {
        if ( lchild == NULL )  lchild = new BST;
            return(lchild>insert(inr));
    } else {
        if ( rchild == NULL )   rchild = new BST;
            return(rchild>insert(inr));
    }
}
```



# ADT Binary Search Tree: Insertion

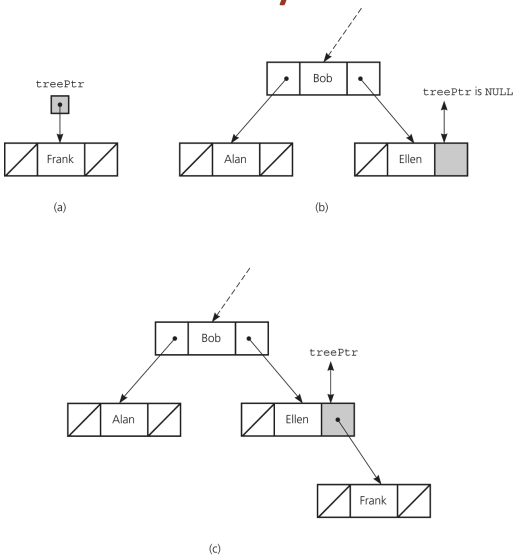



***Figure 10-23***

(a) Insertion into an empty tree;

(b) search terminates at a leaf;

(c) insertion at a leaf



# ADT Binary Search Tree – Delete

- Delete operation on node N is a bit more complicated
- If N is a leaf
  - both lchild and rchild are NULL
  - parent node pointer referring to N should be set to NULL
    - need a pointer to parent node to do this
- If N has only 1 child
  - replace N with its only child
- If N has two children
  - replace N with minimum item of its right subtree

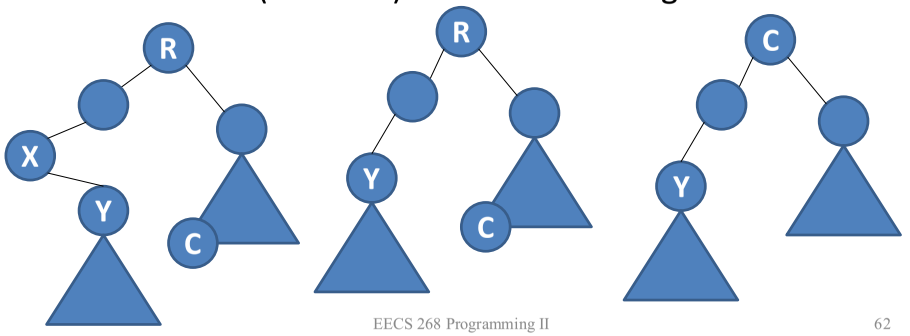
# ADT Binary Search Tree: Delete

- Deleting the item in node N when N has two children (continued)
  - locate another node M that is easier to delete
    - M is the leftmost node in N's right subtree
    - M will have no more than one child
    - M's search key is called the inorder successor of N's search key
  - copy the item that is in M to N
  - remove the node M from the tree



# ADT Binary Search Tree: Delete

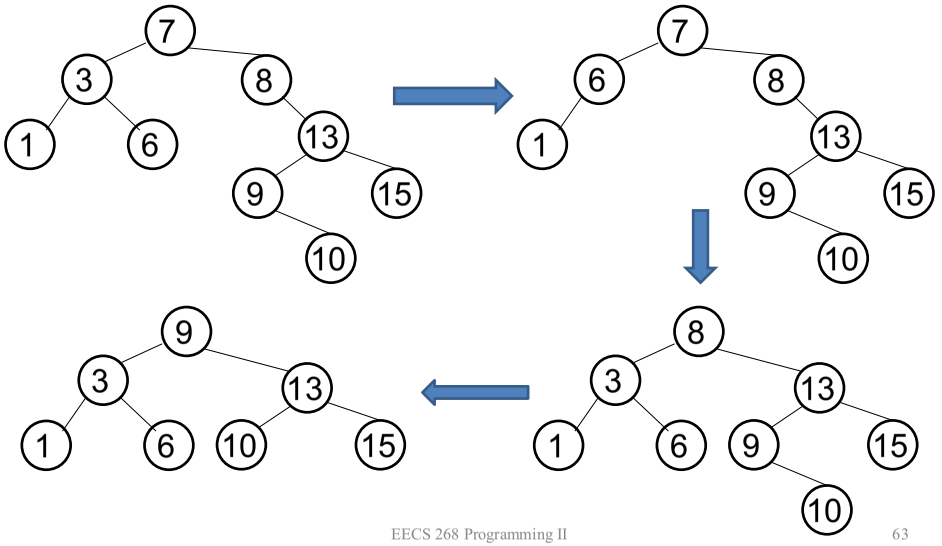
- Deleting node x is simple because it has only one child and can be replaced by the root of its child without violating any of the BST constraints
- Deleting R is harder, but c can replace it because it is the smallest (leftmost) element of the right sub-tree






# ADT Binary Search Tree: Delete

- Delete 3, 7, 8 in order






# ADT Binary Search Tree: Retrieval and Traversal

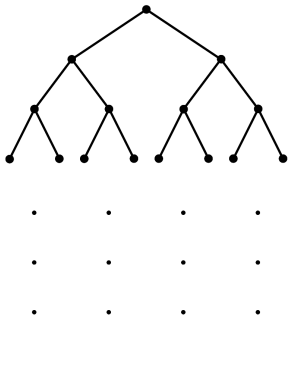
- The retrieval operation can be implemented by refining the search algorithm
  - return the item with the desired search key if it exists
  - otherwise, throw TreeException
- Traversals for a binary search tree are the same as the traversals for a binary tree
- Theorem 10-1
  - the inorder traversal of a binary search tree T will visit its nodes in sorted search-key order

# Height of a Binary Tree

- Theorem 10-2
  - A full binary tree of height $h \geq 0$ has $2h - 1$ nodes
- Theorem 10-3
  - The maximum number of nodes that a binary tree of height h can have is $2h - 1$
- Theorem 10-4
  - The minimum height of a binary tree with n nodes is $\lceil \log_2(n+1) \rceil$
  - Complete trees and full trees have minimum height
- The maximum height of a binary tree with n nodes is n



# Height of a Binary Tree



| Level | Number of nodes at this level | Number of nodes at this and previous levels |
|---|---|---|
| 1 | $1 = 2^0$ | $1 = 2^1 - 1$ |
| 2 | $2 = 2^1$ | $3 = 2^2 - 1$ |
| 3 | $4 = 2^2$ | $7 = 2^3 - 1$ |
| 4 | $8 = 2^3$ | $15 = 2^4 - 1$ |
| . | . | . |
| . | . | . |
| . | . | . |
| h | $2^{h-1}$ | $2^h - 1$ |

***Figure 10-32*** Counting the nodes in a full binary tree of height *h*



# The Efficiency of Binary Search Tree Operations

- The maximum number of comparisons required by any b. s. t. operation is the number of nodes along the longest path from root to a leaf—that is, the tree's height
- The order in which insertion and deletion operations are performed on a binary search tree affects its height
- Insertion in random order produces a binary search tree that has near-minimum height



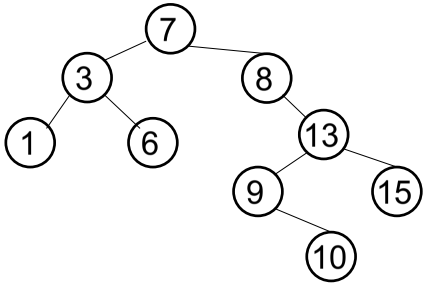
# The Efficiency of Binary Search Tree Operations

| Operation | Average case | Worst case |
|---|---|---|
| Retrieval | O(log *n*) | O(*n*) |
| Insertion | O(log *n*) | O(*n*) |
| Deletion | O(log *n*) | O(*n*) |
| Traversal | O(*n*) | O(*n*) |

***Figure 10-34*** The order of the retrieval, insertion, deletion, and traversal operations for the pointer-based implementation of the ADT binary search tree

# Saving and Restoring a BST

- Saving/restoring any data structure to/from a file requires us to serialize the data structure
- files store data linearly
- arrays and linked lists are linear




- Preorder, postorder and inorder traversals produce a linear tree listings
  - what order makes restoration easiest?
- Preorder: 7,3,1,6,8,13,9,10,15
- Insert nodes in an empty BST in this order and it reproduces the original



# Applications

- Treesort
  - Uses the ADT binary search tree to sort an array of records into search-key order
    - Average case: O(n * log n)
    - Worst case: O(n2)



# *n*-ary Trees

- An *n*-ary tree is a general tree whose nodes can have no more than *n* children each
  - a generalization of a binary tree

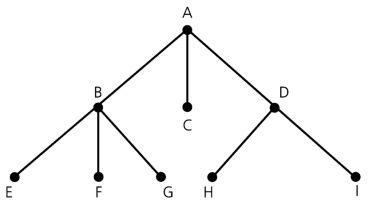



***Figure 10-38*** A general tree

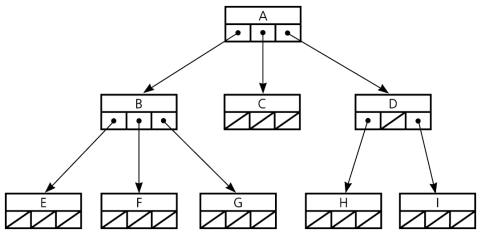



***Figure 10-41***

An implementation of the *n*-ary tree in Figure 10-38



# *n*-ary Trees

- A binary tree can represent an *n*-ary tree
  - seems a bit odd, but good when the number of children is highly variable and especially when there is no upper bound on the number of children
- Lchild is used to point to the first of its children
  - Rchild pointers are used to link siblings together

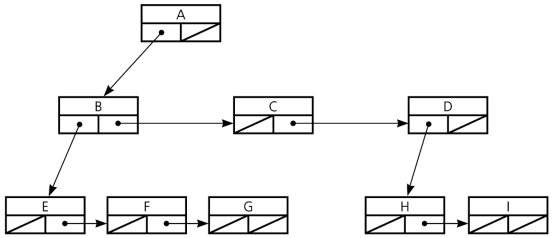



***Figure 10-39*** Another implementation of the tree in Figure 10-38

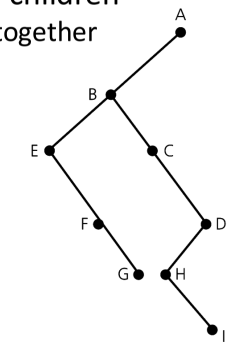



***Figure 10-40*** The binary tree that Figure 10-39 represents

# Summary

- Binary trees provide a hierarchical organization of data
- The implementation of a binary tree is usually pointer-based
- If the binary tree is complete, an efficient array-based implementation is possible
- Traversing a tree to "visit"—that is, do something to or with—each node is useful
- You pass a client-defined "visit" function to the traversal operation to customize its effect on the items in the tree



# Summary

- The binary search tree allows you to use a binary search-like algorithm to search for an item having a specified value
- Binary search trees come in many shapes
  - The height of a binary search tree with n nodes can range from a minimum of $\lceil \log_2(n + 1) \rceil$ to a maximum of n
  - The shape of a binary search tree determines the efficiency of its operations



# Summary

- An inorder traversal of a binary search tree visits the tree's nodes in sorted search-key order
- The treesort algorithm efficiently sorts an array by using the binary search tree's insertion and traversal operations



# Summary

- Saving a binary search tree to a file while performing
  - An inorder traversal enables you to restore the tree as a binary search tree of minimum height
  - A preorder traversal enables you to restore the tree to its original form