1) a) $x(n) = 8e^{j0.3\pi n} = 8\cos(0.3\pi n) + j8\sin(0.3\pi n)$

$$\frac{2\cancel{\pi}k}{N} = 0.3\cancel{\pi} = \frac{3}{10} \Rightarrow \frac{k}{N} = \frac{3}{20} \rightarrow \text{rational}$$

<u>periodic with period N=20 samples</u>

b) $x(n) = 2\cos(0.01n)$

$$\frac{2\pi k}{N} = 0.01 \Rightarrow \frac{k}{N} = \frac{1}{200\pi} \rightarrow \text{not rational}$$

<u>not periodic</u>

c) $x(n) = 3\cos(0.08\pi n) + 2\cos(0.11\pi n)$

$$\left.\frac{2\cancel{\pi}k_1}{N_1} = 0.08\cancel{\pi} \rightarrow \frac{k_1}{N_1} = \frac{8}{200} = \frac{1}{25}\right\} \text{periodic with } N_1 = 25$$

$$\left.\frac{2\cancel{\pi}k_2}{N_2} = 0.11\cancel{\pi} \rightarrow \frac{k_2}{N_2} = \frac{11}{200}\right\} \text{periodic with } N_2 = 200$$

so $N = \text{Least Common Multiple}\{N_1, N_2\} = \underline{\underline{200}}$ samples

1) d) $(\cdot)^{-1}$ is a memoryless, nonlinear function
$\Rightarrow$ and is time-invariant

thus if $\cos(0.5\pi n + \pi/4)$ is periodic, then so is its inverse

phase shift has no impact on periodicity

so $\frac{2\cancel{\pi} k}{N} = 0.5\cancel{\pi} \Rightarrow \frac{5}{20} = \frac{k}{N} = \frac{1}{4}$

periodic with $N = 4$ samples

2) a) $(\cdot)^3$ is a nonlinear function that does not change with time

i) not linear

ii) time invariant

b) $y(n) = b_1 y(n-1) + b_2 y(n-2) + x(n)$

$\Rightarrow$ LCCDE is LTI if initial conditions are zero (i.e. causal)

c) $y(n) = 2x(n)$

$y_1(n) = 2x_1(n)$
$y_2(n) = 2x_2(n)$

so $y(n) = y_1(n) + y_2(n) = 2x_1(n) + 2x_2(n)$

same so linear

or $y(n) = T\{x_1(n) + x_2(n)\} = 2[x_1(n) + x_2(n)]$

$y(n-\ell) = 2x(n-\ell)$ $\rightarrow$ same so time invariant

2) d) $y(n) = x(n) + 2$

$y_1(n) = x_1(n) + 2$
$y_2(n) = x_2(n) + 2$

$y(n) = y_1(n) + y_2(n) = x_1(n) + x_2(n) + 4$

not same
so <u>not linear</u>

or $y(n) = T\{x_1(n) + x_2(n)\} = x_1(n) + x_2(n) + 2$

$y(n-l) = x(n-l) + 2 \rightarrow$ same so <u>time invariant</u>

2) e) $$y(n) = x(n)\cos(0.01\pi n)$$

$$y_1(n) = x_1(n)\cos(0.01\pi n)$$
$$y_2(n) = x_2(n)\cos(0.01\pi n)$$

$$y(n) = y_1(n) + y_2(n) = \underbrace{x_1(n)\cos(0.01\pi n) + x_2(n)\cos(0.01\pi n)}$$

same

$$\text{or } y(n) = T\{x_1(n) + x_2(n)\} = \overbrace{[x_1(n) + x_2(n)]\cos(0.01\pi n)}$$

i) <u>linear</u>

$$y(n) = x(n)\cos(0.01\pi n)$$
$$y(n-\ell) = x(n-\ell)\cos(0.01\pi(n-\ell))$$

not same

$\Rightarrow$ because $\cos(0.01\pi n)$ varies with time

ii) <u>not time invariant</u>

3) For $x(n) = \{\underset{\uparrow}{4}, 1, 3\}$ and $h(n) = \{2, \underset{\uparrow}{3}, 1\}$

a) $y(n) = x(n) * h(n) = \{8, \underset{\uparrow}{14}, 13, 10, 3\}$

b) $y(n) = h(n-1) * x(n) = \{\underset{\uparrow}{8}, 14, 13, 10, 3\}$

c) $y(n) = h(-n) * x(n) = \{4, \underset{\uparrow}{13}, 14, 11, 6\}$

d) $y(n) = h(-n) * h(n) * x(n) = \{8, 38, \underset{\uparrow}{71}, 77, 59, 29, 6\}$

4) $x(n) = \delta(n) \Rightarrow y(n) = h(n)$

$h(0) = 0.5$

$h(1) = -(0.5)(0.5) + (0.25)(1) = 0$

$h(2) = -(0.5)(0) + (0.25)(0.5) = 0.125$

$h(3) = -(0.5)(0.125) + (0.25)(0) = -0.0625$

$h(4) = -(0.5)(-0.0625) + (0.25)(0.125) = 0.0625$

$h(5) = -(0.5)(0.0625) + (0.25)(-0.0625) = -0.046875$

$h(6) = -(0.5)(-0.046875) + (0.25)(0.0625) = 0.0390625$

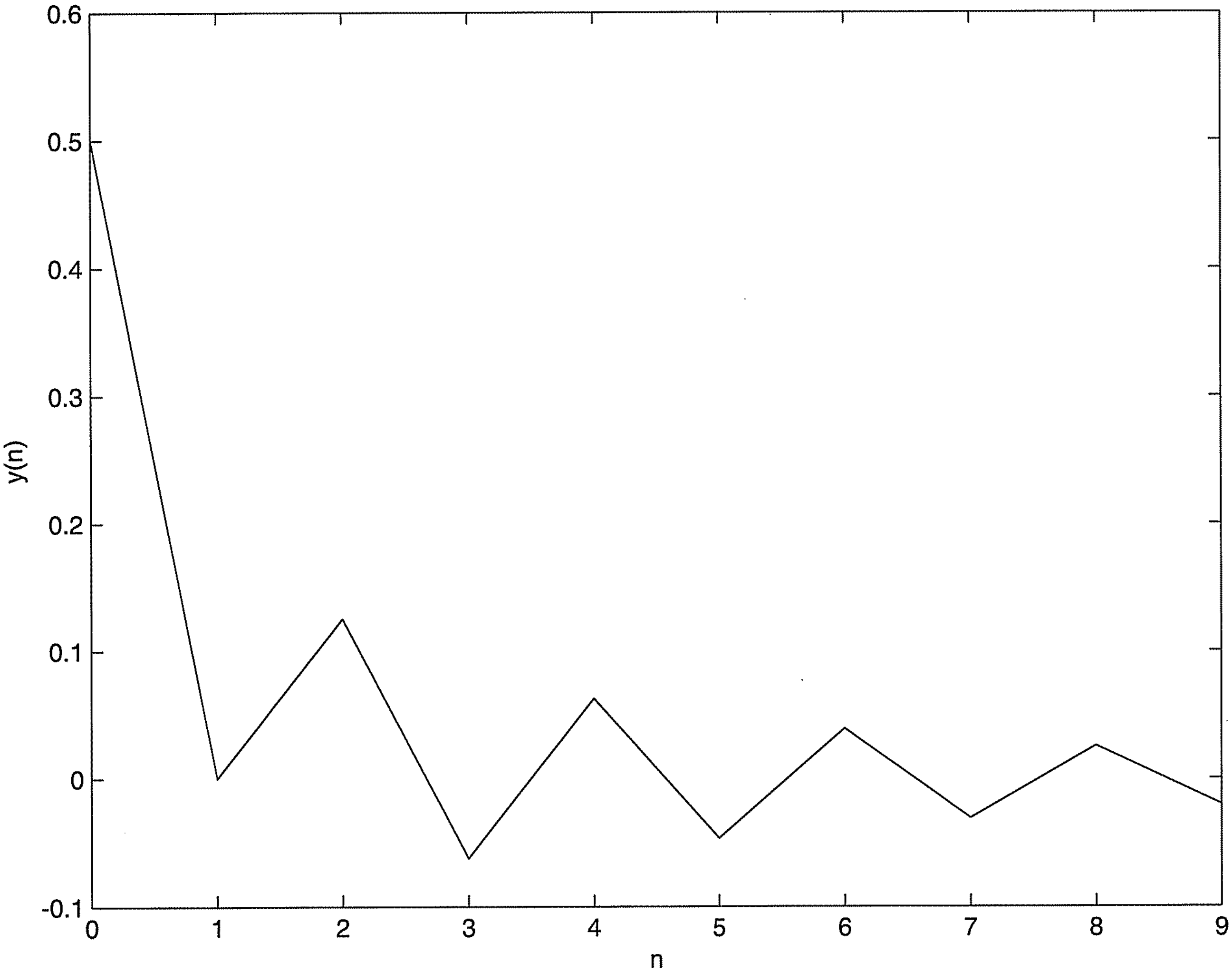
$h(7) = -(0.5)(0.0390625) + (0.25)(-0.046875) = 0.03125$

$h(8) = -(0.5)(0.03125) + (0.25)(0.0390625) = 0.025390625$

$h(9) = -(0.5)(0.025390625) + (0.25)(0.03125)$

$= -0.0205078125$

$\Rightarrow$ the IIR filter has an oscillating decay



0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
y(n)
0
1
2
3
4
5
6
7
8
9
n

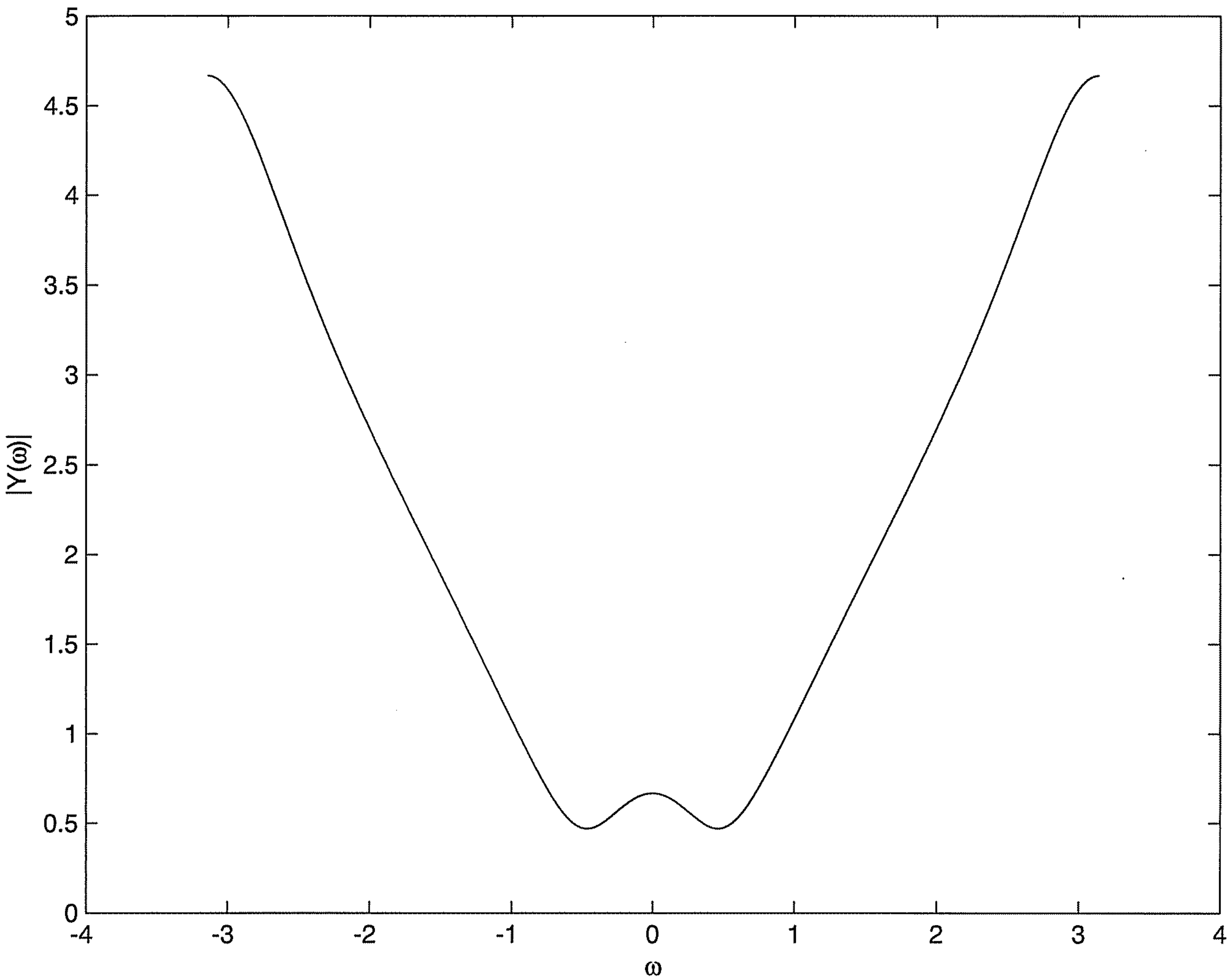
5) $y(n) = \alpha^n u(n) + \beta^{-n} u(-n+1)$

$$Y(\omega) = \sum_{n=0}^{\infty} (\alpha e^{-j\omega})^n + \sum_{n=-\infty}^{1} (\beta^{-1} e^{-j\omega})^n$$

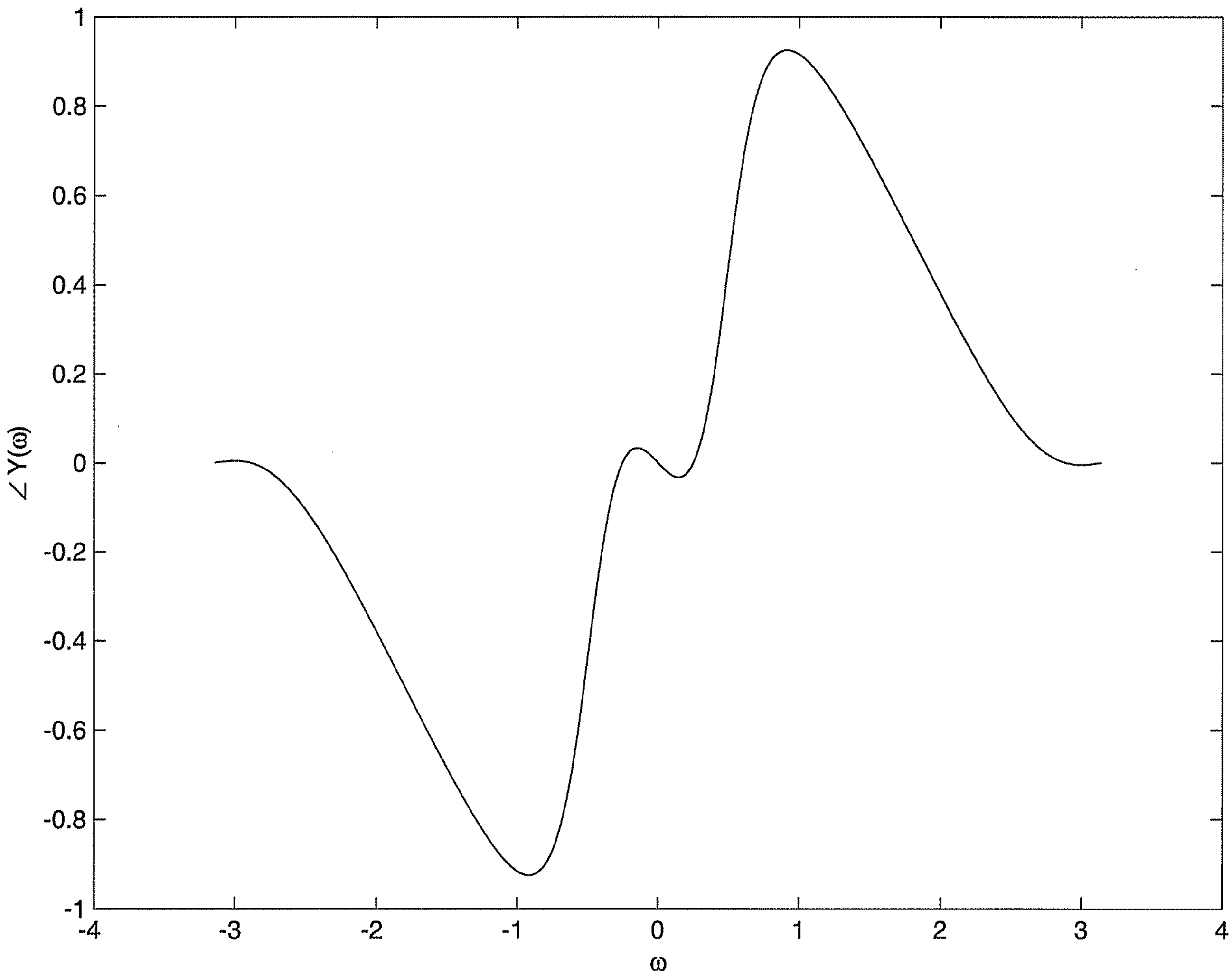
$$= \sum_{n=0}^{\infty} (\alpha e^{-j\omega})^n + \sum_{n=-1}^{\infty} (\beta e^{j\omega})^n$$

$$= \frac{\cancelto{1}{(\alpha e^{-j\omega})^0}\left(1 - \cancelto{0}{(\alpha e^{-j\omega})^\infty}\right)}{1 - \alpha e^{-j\omega}} + \frac{(\beta e^{j\omega})^{-1}\left(1 - \cancelto{0}{(\beta e^{j\omega})^\infty}\right)}{1 - \beta e^{j\omega}}$$

$$= \frac{1}{1 - \alpha e^{-j\omega}} + \frac{1}{\beta e^{j\omega} - \beta^2 e^{j2\omega}}$$



5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
|Y(ω)|
-4
-3
-2
-1
0
1
2
3
4
ω



1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
∠Y(ω)
-4
-3
-2
-1
0
1
2
3
4
ω

6) $x(n) = e^{j0.5\pi n} + 1 + \delta(n+2) - \delta(n-2)$

$x(-n) = e^{-j0.5\pi n} + 1 + \delta(-n+2) - \delta(-n-2)$

so

$$x_e(n) = \frac{1}{2}\left[x(n) + x(-n)\right]$$

$$= \frac{1}{2}\Big[e^{j0.5\pi n} + e^{-j0.5\pi n} + 2 + \cancel{\overset{n=-2}{\delta(n+2)} - \overset{n=-2}{\delta(-n-2)}} \to 0$$
$$+ \cancel{\underset{n=2}{\delta(-n+2)} - \underset{n=2}{\delta(n-2)}}\Big] \to 0$$

$$= \underline{\underline{\cos(0.5\pi n) + 1}}$$

$$x_o(n) = \frac{1}{2}\left[x(n) - x(-n)\right]$$

$$= \frac{1}{2}\Big[e^{j0.5\pi n} - e^{-j0.5\pi n} + \overset{n=-2}{\delta(n+2)} + \overset{n=-2}{\delta(-n-2)}$$
$$- \underset{n=2}{\delta(n-2)} - \underset{n=2}{\delta(-n-2)}\Big]$$

$$= \underline{\underline{j\sin(0.5\pi n) + \delta(n+2) - \delta(n-2)}}$$