21) $y(n) = -0.5\,y(n-1) + x(n) - 0.5\,x(n-1)$ for $x(n)=\delta(n)$

$h(0) = 1$

$h(1) = (-0.5)(1) - 0.5(1) = -1$

$h(2) = -0.5(-1) = 0.5 = 1/2$

$h(3) = -0.5(0.5) = -0.25 = -1/4$

$h(4) = -0.5(-0.25) = 0.125 = 1/8$

$$Y(z)\left[1 + 0.5z^{-1}\right] = X(z)\left[1 - 0.5z^{-1}\right]$$

so $$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - 0.5z^{-1}}{1 + 0.5z^{-1}}$$

$$\begin{array}{r|l}
1 + 0.5z^{-1} & 1 - z^{-1} + 0.5z^{-2} - 0.25z^{-3} + 0.125z^{-4} \\
 & 1 - 0.5z^{-1} \\
 & -(1 + 0.5z^{-1}) \\
 & \hline
 & -z^{-1} \\
 & -(-z^{-1} - 0.5z^{-2}) \\
 & \hline
 & 0.5z^{-2} \\
 & -(0.5z^{-2} + 0.25z^{-3}) \\
 & \hline
 & -0.25z^{-3} \\
 & -(-0.25z^{-3} - 0.125z^{-4}) \\
 & \hline
 & 0.125z^{-4} \\
 & -(0.125z^{-4} + 0.0625z^{-5}) \\
 & \hline
 & -0.0625z^{-5}
\end{array}$$

2) $y(n) = 0.5\,y(n-1) - 0.5\,y(n-2) + 0.75\,x(n) - 0.75\,x(n-1)$

$\Rightarrow$ determine $y(n) = h(n)$ when $x(n) = \delta(n)$

$h(0) = 0.75 = 3/4$

$h(1) = (0.5)(0.75) - 0.75 = -0.375 = -3/8$

$h(2) = (0.5)(-0.375) - (0.5)(0.75) = -0.5625 = -9/16$

$h(3) = (0.5)(-0.5625) - (0.5)(-0.375) = -0.09375 = -3/32$

$h(4) = (0.5)(-0.09375) - (0.5)(-0.5625) = 0.234375 = 15/64$

$Y(z)\left[1 - 0.5z^{-1} + 0.5z^{-2}\right] = X(z)\left[0.75 - 0.75z^{-1}\right]$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\frac{3}{4}\left(1 - z^{-1}\right)}{1 - \frac{1}{2}z^{-1} + \frac{1}{2}z^{-2}}$$

PSE:

$$\begin{array}{l}
\frac{3}{4} - \frac{3}{8}z^{-1} - \frac{9}{16}z^{-2} - \frac{3}{32}z^{-3} + \frac{15}{64}z^{-4} \\
1 - \frac{1}{2}z^{-1} + \frac{1}{2}z^{-2} \;\overline{)\; \frac{3}{4} - \frac{3}{4}z^{-1}} \\
\quad -\left(\frac{3}{4} - \frac{3}{8}z^{-1} + \frac{3}{8}z^{-2}\right) \\
\quad\quad -\frac{3}{8}z^{-1} - \frac{3}{8}z^{-2} \\
\quad\quad -\left(-\frac{3}{8}z^{-1} + \frac{3}{16}z^{-2} - \frac{3}{16}z^{-3}\right) \\
\quad\quad\quad -\frac{9}{16}z^{-2} + \frac{3}{16}z^{-3} \\
\quad\quad\quad -\left(-\frac{9}{16}z^{-2} + \frac{9}{32}z^{-3} - \frac{9}{32}z^{-4}\right) \\
\quad\quad\quad\quad -\frac{3}{32}z^{-3} + \frac{9}{32}z^{-4} \\
\quad\quad\quad\quad -\left(-\frac{3}{32}z^{-3} + \frac{3}{64}z^{-4} - \frac{3}{64}z^{-5}\right) \\
\quad\quad\quad\quad\quad \frac{15}{64}z^{-4} + \frac{3}{64}z^{-5} \\
\quad\quad\quad\quad\quad -\left(\frac{15}{64}z^{-4} - \frac{15}{128}z^{-5} + \frac{15}{128}z^{-6}\right) \\
\quad\quad\quad\quad\quad\quad \frac{21}{128}z^{-5} + \frac{15}{128}z^{-6}
\end{array}$$

3) $H(z) = \dfrac{(z+1)}{(z-\frac{1}{4})(z+\frac{1}{2})} = \dfrac{z+1}{z^2+\frac{1}{4}z-\frac{1}{8}}$ ⇝ order 1 ⇝ order 2

$$F(z) = \frac{H(z)}{z} = \frac{z+1}{z(z-\frac{1}{4})(z+\frac{1}{2})} = \frac{A}{z} + \frac{B}{z-1/4} + \frac{C}{z+1/2}$$

$$A = z\,F(z)\Big|_{z=0} = \frac{1}{(-0.25)(0.5)} = -8$$

$$B = (z-0.25)\,F(z)\Big|_{z=0.25} = \frac{1.25}{(0.25)(0.75)} = 6.6\overline{67} = \frac{20}{3}$$

$$C = (z+0.5)\,F(z)\Big|_{z=-0.5} = \frac{0.5}{(-0.5)(-0.75)} = 1.3\overline{33} = \frac{4}{3}$$

so $H(z) = z\,F(z) = -8 + \dfrac{(20/3)z}{z-0.25} + \dfrac{(4/3)z}{z+0.5}$

stable: ROC includes $|z|=1$ ⇒ ROC is $|z|>0.5$

$$h(n) = -8\delta(n) + \frac{20}{3}\left(\frac{1}{4}\right)^n u(n) + \frac{4}{3}\left(\frac{-1}{2}\right)^n u(n)$$

4) $H(z) = \dfrac{(z-1)(z+1)}{(z-\frac{1}{4})(z+\frac{1}{2})} = \dfrac{z^2-1}{z^2+\frac{1}{4}z-\frac{1}{8}}$ $\rightsquigarrow$ order = 2, $\rightsquigarrow$ order = 2 } need PSE before PFE

$$\begin{array}{r|l} z^2+\frac{1}{4}z-\frac{1}{8} & 1 \\ & z^2-1 \\ & -(z^2+\frac{1}{4}z-\frac{1}{8}) \\ \hline & -\frac{1}{4}z-\frac{7}{8} \end{array}$$

$$\Rightarrow \quad H(z) = 1 - \underbrace{\frac{\frac{1}{4}z+\frac{7}{8}}{z^2+\frac{1}{4}z-\frac{1}{8}}}_{\text{denote } B(z)}$$

$$F(z) = \frac{B(z)}{z} = \frac{\frac{1}{4}z+\frac{7}{8}}{z(z-\frac{1}{4})(z+\frac{1}{2})} = \frac{A}{z} + \frac{B}{z-\frac{1}{4}} + \frac{C}{z+\frac{1}{2}}$$

$$A = zF(z)\Big|_{z=0} = \frac{\frac{7}{8}}{(-\frac{1}{4})(\frac{1}{2})} = -7$$

$$B = (z-\tfrac{1}{4})F(z)\Big|_{z=\frac{1}{4}} = \frac{(\frac{1}{4})(\frac{1}{4})+\frac{7}{8}}{(\frac{1}{4})(\frac{3}{4})} = 5$$

$$C = (z+\tfrac{1}{2})F(z)\Big|_{z=-\frac{1}{2}} = \frac{(\frac{1}{4})(-\frac{1}{2})+\frac{7}{8}}{(-\frac{1}{2})(-\frac{3}{4})} = 2$$

so $H(z) = 1 - B(z) = 1 - zF(z) = \underbrace{1-(-7)}_{8}\frac{z}{z} - \frac{5z}{z-\frac{1}{4}} - \frac{2z}{z+\frac{1}{2}}$

to include $|z|=1$, same ROC as before $\rightarrow$ $|z| > \frac{1}{2}$

$$\Rightarrow \quad h(n) = 8\delta(n) - 5\left(\tfrac{1}{4}\right)^n u(n) - 2\left(-\tfrac{1}{2}\right)^n u(n)$$

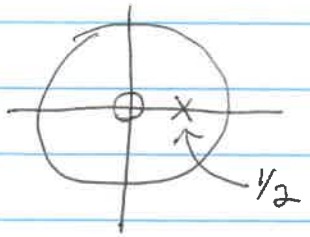
5) $y(n) = 0.5\,y(n-1) + x(n)$

$$Y(z)\left[1 - 0.5z^{-1}\right] = X(z) \Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-0.5z^{-1}} = \frac{z}{z-0.5}$$

$|z| > 1/2$

a) stable, ROC includes unit circle for causal input

b) zero @ 0, pole @ 1/2



c) $X(z) = \frac{z}{z-1} \Rightarrow Y(z) = H(z)X(z)$

$$= \frac{z^2}{(z-1)(z-0.5)}$$ ← order = 0; ← order = 2

$$F(z) = \frac{Y(z)}{z} = \frac{z}{(z-1)(z-0.5)} = \frac{A}{z-1} + \frac{B}{z-0.5}$$

$$A = (z-1)F(z)\Big|_{z=1} = \frac{1}{0.5} = 2$$

$$B = (z-0.5)F(z)\Big|_{z=1/2} = \frac{0.5}{-0.5} = -1$$

so $Y(z) = zF(z) = \frac{2z}{z-1} - \frac{z}{z-1/2} \Rightarrow y(n) = 2u(n) - (1/2)^n u(n)$

d) $\lim_{n\to\infty} h(n) = \lim_{z\to 1}\left[(z-1)H(z)\right] = \lim_{z\to 1}\left[\frac{z(z-1)}{z-0.5}\right] = \underline{\underline{0}}$

6) $y(n) = 0.5\,y(n-1) + 0.5\,y(n-2) + 2x(n) - 2x(n-1)$

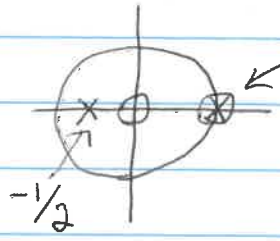
$$Y(z)\left[1 - 0.5z^{-1} - 0.5z^{-2}\right] = X(z)\left[2 - 2z^{-1}\right]$$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = \frac{2(1-z^{-1})}{1-0.5z^{-1}-0.5z^{-1}} = \frac{2z(z-1)}{z^2-0.5z-0.5} = \frac{2z\cancel{(z-1)}}{\cancel{(z-1)}(z+1/2)}$$

$$= \frac{2z}{z+1/2} \quad \text{for ROC: } |z| > 1/2$$

a) stable, ROC includes unit circle for causal input

b) zeros @ 0, +1 , poles @ +1, $-1/2$



c) $X(z) = \frac{z}{z-1} \Rightarrow Y(z) = H(z)X(z) = \frac{2z^2}{(z-1)(z+1/2)}$ ← order = 2, ← order = 0

$$F(z) = \frac{Y(z)}{z} = \frac{2z}{(z-1)(z+1/2)} = \frac{A}{z-1} + \frac{B}{z+1/2}$$

$$A = (z-1)F(z)\Big|_{z=1} = \frac{2}{3/2} = 4/3$$

$$B = (z+1/2)F(z)\Big|_{z=-1/2} = \frac{2(-1/2)}{-3/2} = 2/3$$

so $Y(z) = zF(z) = \frac{(4/3)z}{z-1} + \frac{(2/3)z}{z+1/2} \Rightarrow y(n) = \frac{4}{3}u(n) + \frac{2}{3}(-1/2)^n u(n)$

d) $\lim_{n\to\infty} h(n) = \lim_{z\to 1}\left[(z-1)H(z)\right] = \lim_{z\to 1}\left[\frac{2z(z+1)}{z+1/2}\right] = \underline{\underline{0}}$