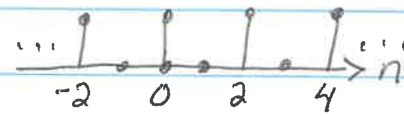
1) a) $x(n) = \sum_{k=-\infty}^{\infty} \delta(n-2k) \Rightarrow$ 

$$E_x = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{2n=-\infty}^{\infty} 1 = \infty$$ → periodic with $N=2$

only even values

$$P_x = \frac{1}{2} \sum_{n=0}^{1} |x(n)|^2 = \frac{1}{2}(1+0) = \frac{1}{2}$$ → power signal

b) $x(n) = a^n u(n) + b^n u(n)$ for $|a|<1$, $|b|<1$

$$E_x = \sum_{n=-\infty}^{\infty} |a^n u(n) + b^n u(n)|^2 = \sum_{n=0}^{\infty} |a^n + b^n|^2$$

$$= \sum_{n=0}^{\infty} \left(a^{2n} + 2a^n b^n + b^{2n}\right)$$

$$= \sum_{n=0}^{\infty} (a^2)^n + 2\sum_{n=0}^{\infty} (ab)^n + \sum_{n=0}^{\infty} (b^2)^n$$

$$= \frac{1}{1-a^2} + \frac{2}{1-ab} + \frac{1}{1-b^2} < \infty$$ for ⇒ energy signal

2) $x(n) = a^n u(n)$ $\quad y(n) = \{1, b, b^2\}$ for $|a|<1$

express $y(n) = \delta(n) + b\delta(n-1) + b^2\delta(n-2)$

so $r_{xy}(\ell) = \sum_{n=-\infty}^{\infty} a^n u(n)\left[\delta(n-\ell) + b\delta(n-1-\ell) + b^2\delta(n-2-\ell)\right]$

for $\ell \leq -3 \Rightarrow$ no overlap, so $r_{xy}(\ell) = \underline{\underline{0}}$

for $\ell = -2 \Rightarrow r_{xy}(-2) = \sum_{n=-\infty}^{\infty}\left[a^n u(n)\right]\left[b^2\delta(n)\right] = \underline{\underline{b^2}}$

for $\ell = -1 \Rightarrow r_{xy}(-1) = \sum_{n=-\infty}^{\infty}\left[a^n u(n)\right]\left[b\delta(n) + b^2\delta(n-1)\right]$

$$= \underline{\underline{b + ab^2}}$$

for $\ell \geq 0 \Rightarrow r_{xy}(\ell) = \sum_{n=-\infty}^{\infty}\left[a^n u(n)\right]\left[\delta(n-\ell) + b\delta(n-1-\ell) + b^2\delta(n-2-\ell)\right]$

$$= a^\ell + ba^{\ell+1} + b^2 a^{\ell+2}$$

so in general

$$r_{xy}(\ell) = a^\ell u(\ell) + ba^{\ell+1}u(\ell+1) + b^2a^{\ell+2}u(\ell+2)$$

for all ℓ

3) $y(n) = \{1, b, b^2\}$

$$w(n) = x(-n) = a^{-n} u(-n) \qquad |a| < 1$$

so $r_{xy}(\ell) = \sum_{n=-\infty}^{\infty} [a^{-n} u(-n)][\delta(n-\ell) + b\delta(n-1-\ell) + b^2\delta(n-2-\ell)]$

for $\ell \geq 1 \Rightarrow$ no overlap, so $r_{xy}(\ell) = \underline{\underline{0}}$

for $\ell = 0 \Rightarrow r_{xy}(0) = \sum_{n=-\infty}^{0} a^{-n} \delta(n) = \underline{\underline{1}}$

for $\ell = -1 \Rightarrow r_{xy}(-1) = \sum_{n=-\infty}^{0} a^{-n} [\delta(n+1) + b\delta(n)]$

$$= \underline{\underline{a + b}}$$

for $\ell \leq -2 \Rightarrow r_{xy}(\ell) = \sum_{n=-\infty}^{0} a^{-n} [\delta(n-\ell) + b\delta(n-1-\ell) + b^2\delta(n-2-\ell)]$

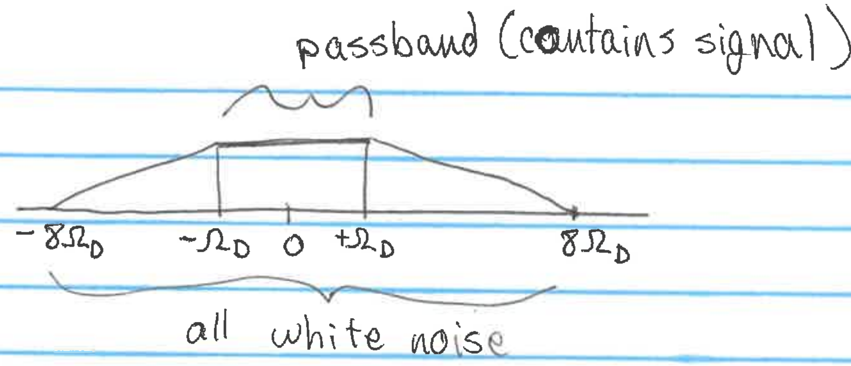
$$= \underline{\underline{a^{-\ell} + b a^{-(\ell+1)} + b^2 a^{-(\ell+2)}}}$$

so in general

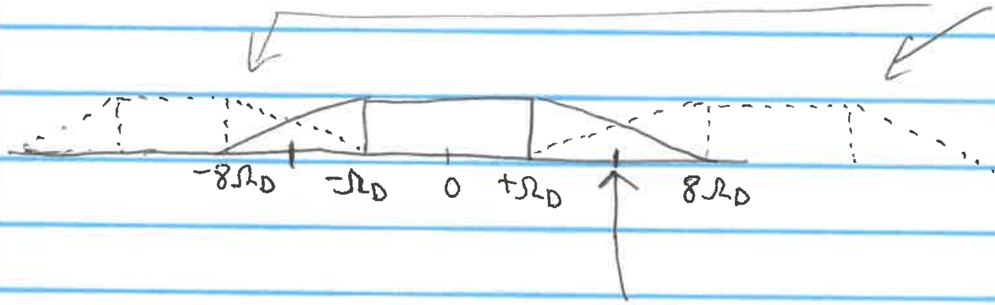
$$r_{xy}(\ell) = a^{-\ell} u(-\ell) + b\, a^{-\ell-1} u(-\ell-1) + b^2 a^{-\ell-2} u(-\ell-2)$$

for all ℓ

4)



$\Downarrow$ sample such that the image appear like



So the lowest foldover frequency for which no more noise is aliased into the passband is when $8\Omega_D$ is folded onto $1\Omega_D$ $\Rightarrow$ so we want the halfway point:

$$\text{foldover freq.} = \frac{8\Omega_D + 1\Omega_D}{2} = 4.5\,\Omega_D$$

the sampling rate is double the foldover freq. so

$$\Omega_S = 9\Omega_D \quad \text{or} \quad F_S = \frac{9}{2\pi}\Omega_D$$

5) $x(n) = \sum_{k=-\infty}^{\infty} \left[\delta(n-4k) + 7\delta(n-4k-1) - 7\delta(n-4k-2) - \delta(n-4k-3)\right]$

signal power $= \sigma_x^2 = \frac{1}{4}\left[1 + 49 + 49 + 1\right] = 25$

so $\sigma_x = 5$

for $X_m = 7 \Rightarrow SQNR = 6.02B + 10.8 - 20\log_{10}\left(\frac{7}{5}\right)$
$= 6.02B + 7.88$

for $SQNR \geq 28$ dB $\Rightarrow B = \left\lceil \frac{28-7.88}{6.02} \right\rceil = \underline{\underline{4 \text{ bits}}}$

+1 for sign bit
= 5 bits

$\Rightarrow$ for a Gaussian signal, set $\underline{\underline{X_m = 3\sigma_x}}$

so $SQNR = 6.02B + 10.8 - 20\log_{10}(3)$
$= 6.02B + 1.26$

then for $SQNR \geq 28 \rightarrow B = \left\lceil \frac{28-1.26}{6.02} \right\rceil$

$= \underline{\underline{5 \text{ bits}}}$

+1 for sign bit
= 6 bits

6) $e_0(n) = \{a, b, c\}$

$e_1(n) = \{d, e, f\}$

$e_2(n) = \{g, h, i\}$

$e_3(n) = \{j, k, l\}$

from $M=4$ polyphase decomposition

$\Rightarrow h(n) = \{a, d, g, j, b, e, h, k, c, f, i, l\}$