EECS 644 HW 4: due 10/21/2025

1. For the following filters, determine the phase response as a function of *ω*. If the filter is linear phase, what is the group delay τ in samples?

   a) $h(n) = \{-0.25, -0.5, 0.25, 0.5\}$

   b) $h(n) = \{1, -2, 1, 2, -1\}$

   c) $h(n) = \sum_{k=1}^{16} \left(\frac{k}{16}\right) \delta(n-k) - \sum_{k=0}^{15} \left(\frac{16-k}{16}\right) \delta(n-k)$

2. Use the Matlab command 'freqz' to plot the magnitude & phase response of the LCCDE

   $$y(n) = 0.8y(n-1) + x(n) + 3x(n-1) + x(n-2) - x(n-4) + 2x(n-5)$$

   What type of filter is this (low-pass vs. high-pass)? Transform the filter such that the opposite is obtained and plot its magnitude and phase response as well.

3. Determine the null depth (in dB) with respect to the peak of the magnitude frequency response for the system function

   $$H(z) = \frac{z^2 + 1.44}{z^3 - 0.2z^2 - 0.48z}.$$

   Hint: The null depth relative to the peak is the ratio of the largest value of |H(ω)| to the smallest value of |H(ω)|. The Matlab 'roots' command may be useful as well.

4. Show that the even and odd components of the filter $h(n) = \{-1, 2, 1, -2, 1\}$ are linear phase. What Type (1, 2, 3, or 4) is each? Why is $h(n)$ not linear phase even though its components are? *Note: "even" and "odd" here refer to the symmetry properties starting on page 2-54 of the notes.*

5. Given the stable transfer function

   $$H(z) = \frac{\left(z + \frac{1}{4}\right)\left(z - \frac{1}{4}\right)(z + 2)(z - 4)}{z^2 \left(z + j\frac{1}{4}\right)\left(z - j\frac{1}{4}\right)},$$

   what is the <u>stable</u> transfer function $G(z)$ that would produce $|G(\omega)\,H(\omega)| = \text{constant}$?