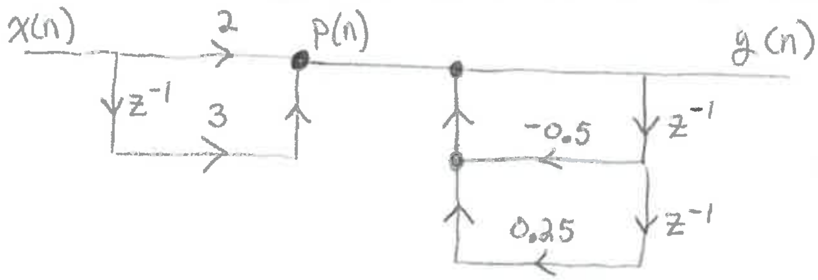
1) $y(n) = 2x(n) + 3x(n-1) - 0.5\,y(n-1) + 0.25\,y(n-2)$



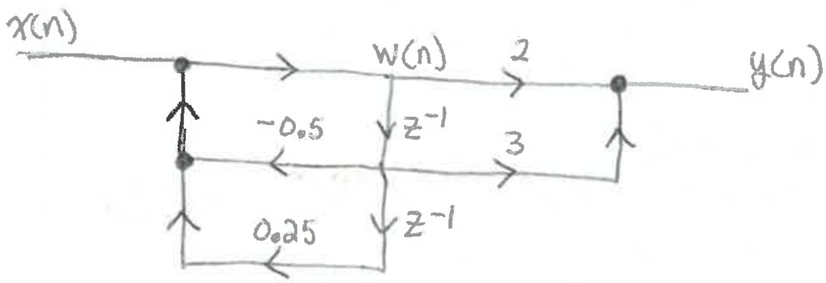
Direct Form I

note multiply by -1 is relative to [redacted] H(z) form, not this form

algorithm:

$p(n) = 2x(n) + 3x(n-1)$

$y(n) = p(n) - 0.5\,y(n-1) + 0.25\,y(n-2)$

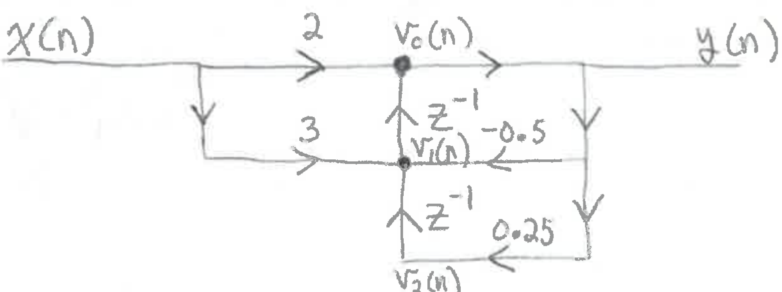


Direct Form II

algorithm:

$w(n) = x(n) - 0.5\,w(n-1) + 0.25\,w(n-2)$

$y(n) = 2\,w(n) + 3\,w(n-1)$



Transposed Direct Form II

algorithm:

$v_0(n) = 2x(n) + v_1(n-1)$

also $v_2(n)$

$v_1(n) = 3x(n) - 0.5\,v_0(n) + 0.25\,v_0(n-1)$

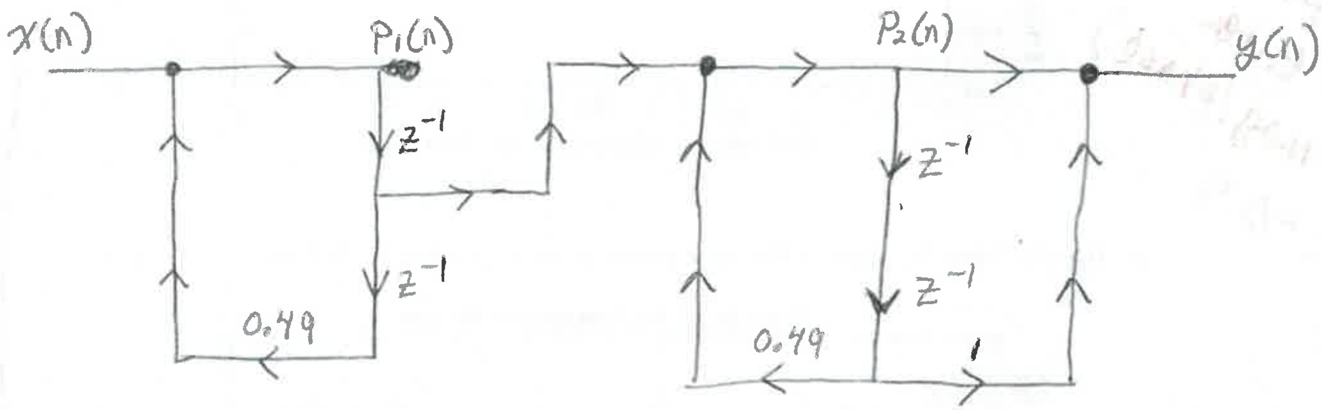
$y(n) = v_0(n)$

2)

$$H(z) = \frac{z(z-j)(z+j)}{(z-0.7)^2(z+0.7)^2} = \frac{z}{(z+0.7)(z-0.7)} \cdot \frac{(z^2+1)}{(z+0.7)(z-0.7)}$$

$$= \frac{z}{(z^2-0.49)} \cdot \frac{(z^2+1)}{(z^2-0.49)} \times \frac{z^{-4}}{z^{-4}}$$

$$= \frac{z^{-1}}{(1-0.49z^{-2})} \cdot \frac{(1+z^{-2})}{(1-0.49z^{-2})}$$



2 multiplies
3 additions
4 delays

$$p_1(n) = x(n) + 0.49\,p_1(n-2)$$

$$p_2(n) = p_1(n-1) + 0.49\,p_2(n-2)$$

$$y(n) = p_2(n) + p_2(n-2)$$

3) $$F(z) = \frac{H(z)}{z} = \frac{(z^2+1)}{(z-0.7)^2(z+0.7)^2} = \frac{A}{(z-0.7)^2} + \frac{B}{z-0.7} + \frac{C}{(z+0.7)^2} + \frac{D}{(z+0.7)}$$

$$A = F(z)(z-0.7)^2\Big|_{z=0.7} = \frac{0.49+1}{(1.4)^2} \cong \underline{\underline{0.76}}$$

$$B = \frac{(z+0.7)^2(2z) - (z^2+1)(2z+1.4)}{(z+0.7)^4}\Bigg|_{z=0.7} = \frac{(1.4)^2(1.4) - (1.49)(2.8)}{(1.4)^4} \cong \underline{\underline{-0.37}}$$

$$C = F(z)(z+0.7)^2\Big|_{z=-0.7} = \frac{0.49+1}{(-1.4)^2} \cong \underline{\underline{0.76}}$$

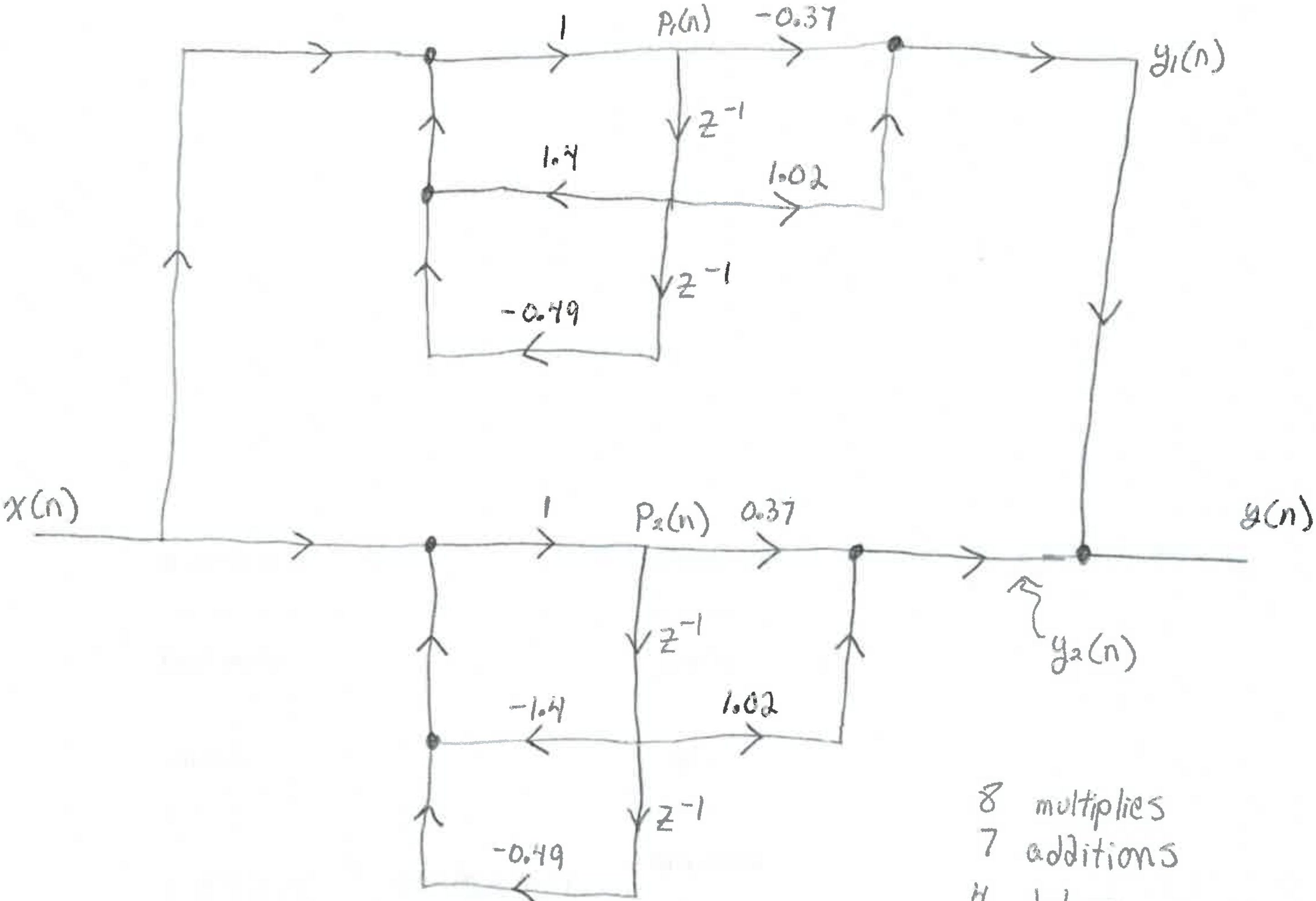
$$D = \frac{(z-0.7)^2(2z) - (z^2+1)(2z-1.4)}{(z-0.7)^4} = \frac{(-1.4)^2(-1.4) - (1.49)(-2.8)}{(-1.4)^4} \cong \underline{\underline{0.37}}$$

so $$H(z) = \underbrace{\frac{0.76z}{(z-0.7)^2} + \frac{-0.37z}{z-0.7}} + \underbrace{\frac{0.76z}{(z+0.7)^2} + \frac{0.37z}{z+0.7}}$$

recombining terms:

$$= \left.\frac{-0.37z^2 + 1.02z}{(z-0.7)^2} + \frac{0.37z^2 + 1.02z}{(z+0.7)^2}\right\} \times \frac{z^{-4}}{z^{-4}}$$

$$= \frac{-0.37 + 1.02z^{-1}}{1 - 1.4z^{-1} + 0.49z^{-2}} + \frac{0.37 + 1.02z^{-1}}{1 + 1.4z^{-1} + 0.49z^{-2}}$$



8 multiplies
7 additions
4 delays

algorithm:

$$p_1(n) = x(n) + 1.4\, p_1(n-1) - 0.49\, p_1(n-2)$$
$$y_1(n) = -0.37\, p_1(n) + 1.02\, p_1(n-1)$$

$$p_2(n) = x(n) - 1.4\, p_2(n-1) - 0.49\, p_2(n-2)$$
$$y_2(n) = 0.37\, p_2(n) + 1.02\, p_2(n-1)$$

$$y(n) = y_1(n) + y_2(n)$$

4) $$H(s) = \frac{1}{s+(3+j0.75)} + \frac{1}{s+(3-j0.75)}$$

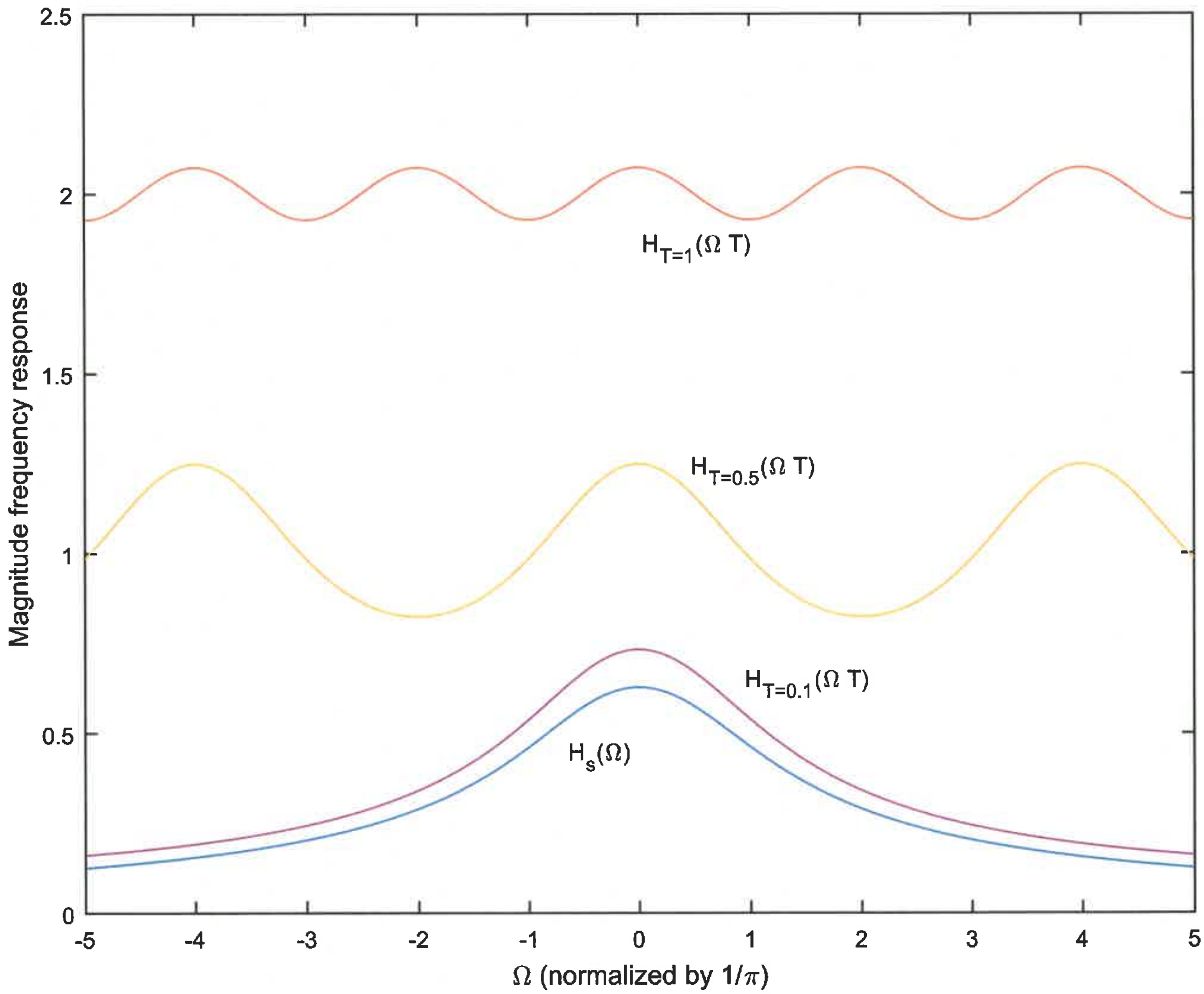
a) $$H(z) = \frac{T}{1-e^{-(3+j0.75)T}z^{-1}} + \frac{T}{1-e^{-(3-j0.75)T}z^{-1}}$$

$$= T\left[\frac{1-e^{-(3-j0.75)T}z^{-1} + 1-e^{-(3+j0.75)T}z^{-1}}{1-e^{-3T}z^{-1}\left(e^{j0.75T}+e^{-j0.75T}\right)+e^{-6T}z^{-2}}\right]$$

$$= T\left[\frac{2-2e^{-3T}\cos(0.75T)z^{-1}}{1-2e^{-3T}\cos(0.75T)z^{-1}+e^{-6T}z^{-2}}\right] = \frac{Y(z)}{X(z)}$$

so $$y(n) = \left[2e^{-3T}\cos(0.75T)\right]y(n-1) - \left[e^{-6T}\right]y(n-2)$$
$$+\left[2T\right]x(n) - \left[2Te^{-3T}\cos(0.75T)\right]x(n-1)$$

b) as T decreases, $|H(\Omega T)|$ better approximates $|H_s(\Omega)|$



2.5
2
1.5
1
0.5
0
-5
-4
-3
-2
-1
0
1
2
3
4
5
Magnitude frequency response
Ω (normalized by 1/π)
$H_{T=1}(\Omega T)$
$H_{T=0.5}(\Omega T)$
$H_{T=0.1}(\Omega T)$
$H_s(\Omega)$

5) $\Omega_p = 100\pi$, $K_p = -3\,dB$ $F_s = 1.2\ kHz$

$\Omega_s = 400\pi$, $K_s = -30\ dB$

$$\omega_p = \frac{100\pi}{1200} = \frac{\pi}{12} \rightarrow \tilde{\Omega}_p = 2\tan\left(\tfrac{1}{2}\cdot\tfrac{\pi}{12}\right) \cong 0.263$$

$$\omega_s = \frac{400\pi}{1200} = \frac{\pi}{3} \rightarrow \tilde{\Omega}_s = 2\tan\left(\tfrac{1}{2}\cdot\tfrac{\pi}{3}\right) \cong 1.155$$

$$N = \left\lceil \frac{\log[(10^{0.3}-1)/(10^3-1)]}{2\log(0.263/1.155)} \right\rceil \cong \lceil 2.34 \rceil = 3$$

satisfy K_s requirement: $\Omega_c = \dfrac{1.155}{(10^3-1)^{1/2N}} \cong 0.365$

from Butterworth prototype table:

$$H(s) = \frac{1}{(s+1)(s^2+s+1)}$$

$$H_a(s) = H(s)\Big|_{s=\frac{0.365}{s}} = \frac{1}{\left(\frac{0.365}{s}+1\right)\left(\frac{0.133}{s^2}+\frac{0.365}{s}+1\right)}$$

$$= \frac{s^3}{(0.365+s)(0.133+0.365s+s^2)}$$

$$= \frac{s^3}{s^3+0.730s^2+0.266s+0.049}$$

$$H(z) = H_a(s)\Big|_{s=2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$$

$$= \frac{8(1-z^{-1})^3/(1+z^{-1})^3}{8\frac{(1-z^{-1})^3}{(1+z^{-1})^3} + 2.92\frac{(1-z^{-1})^2}{(1+z^{-1})^2} + 0.532\frac{(1-z^{-1})}{(1+z^{-1})} + 0.049} \times \frac{(1+z^{-1})^3}{(1+z^{-1})^3}$$

$$= \frac{8(1-z^{-1})^3}{8(1-z^{-1})^3 + 2.92(1-z^{-1})^2(1+z^{-1}) + 0.532(1-z^{-1})(1+z^{-1})^2 + 0.049(1+z^{-1})}$$

$$= \frac{8(1-3z^{-1}+3z^{-2}-z^{-3})}{\left[8(1-3z^{-1}+3z^{-2}-z^{-3}) + 2.92(1-z^{-1}-z^{-2}+z^{-3}) + 0.532(1+z^{-1}-z^{-2}-z^{-3}) + 0.049(1+3z^{-1}+3z^{-2}+z^{-3})\right]}$$

$$= \frac{8-24z^{-1}+24z^{-2}-8z^{-3}}{11.501 - 26.241z^{-1} + 20.695z^{-2} - 5.563z^{-3}} = \frac{Y(z)}{X(z)}$$

normalizing by 11.501:

$$\boxed{\begin{aligned} y(n) = {} & 2.282\,y(n-1) - 1.799\,y(n-1) + 0.484\,y(n-2) \\ & + 0.696\,x(n) - 2.087\,x(n-1) + 2.087\,x(n-2) \\ & - 0.696\,x(n-3) \end{aligned}}$$