1) $K_p = -3\text{ dB}$ @ $\omega_p = 0.3\pi$

$K_s = -38\text{ dB}$ @ $\omega_s = 0.32\pi$

a) Hanning (-44 dB), Hamming (-53 dB), and Blackman (-74 dB) all meet the stopband spec

$$\Delta\omega = 0.32\pi - 0.3\pi = 0.02\pi$$

for Hanning + Hamming $\Rightarrow$ $0.02\pi \geq \frac{8\pi}{N} \rightarrow N_H \geq 400 \rightarrow \alpha_H = \frac{400-1}{2} = 199.5$

for Blackman $\Rightarrow$ $0.02\pi \geq \frac{12\pi}{N} \rightarrow N_B \geq 600 \rightarrow \alpha_B = \frac{600-1}{2} = 299.5$

set $\omega_c = \frac{0.3\pi + 0.32\pi}{2} = 0.31\pi$

So

Hanning

$$h_{HAN}(n) = \frac{\sin[0.31\pi(n-199.5)]}{\pi(n-199.5)} \frac{1}{2}\left(1-\cos\left(\frac{2\pi n}{399}\right)\right)$$

Hamming

$$h_{HAM}(n) = \frac{\sin[0.31\pi(n-199.5)]}{\pi(n-199.5)}\left(0.54 - 0.46\cos\left(\frac{2\pi n}{399}\right)\right)$$

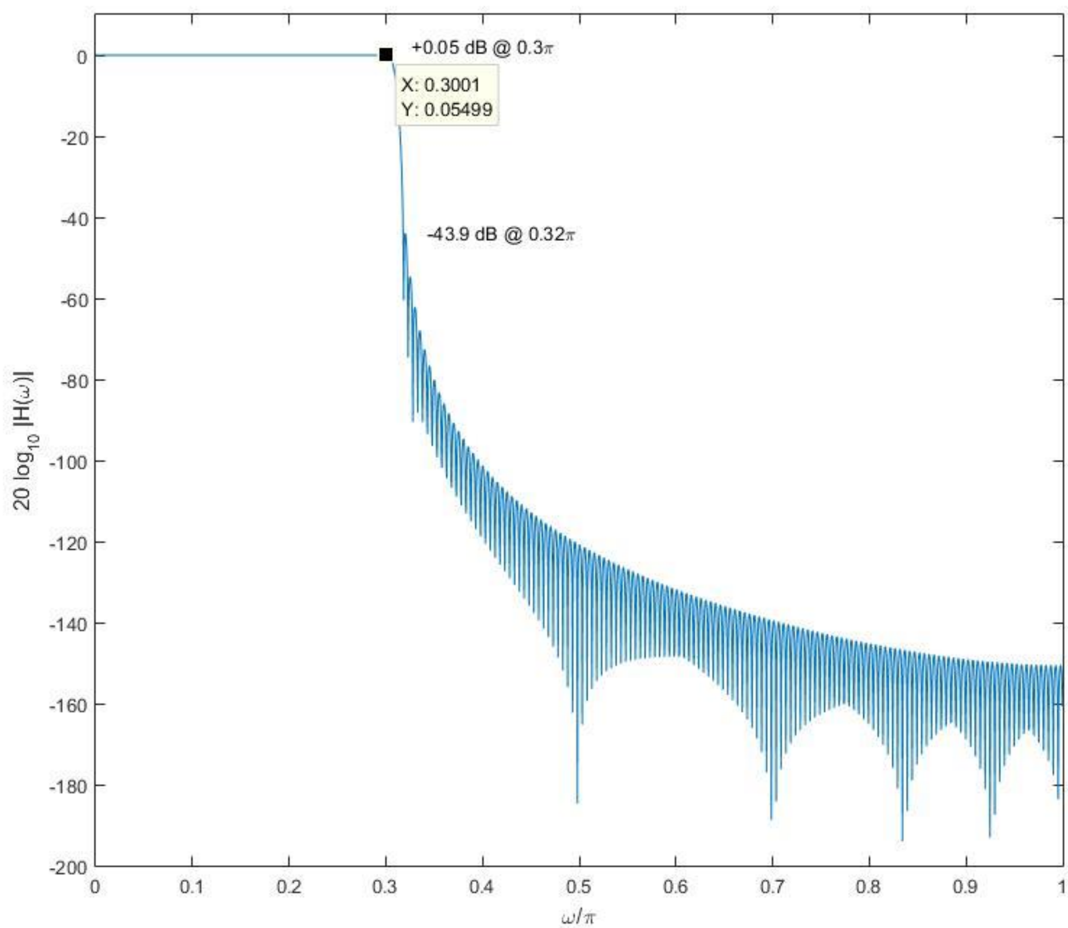
* Hamming meets the spec with the fewest coefficients

Blackman

$$h_{BLK}(n) = \frac{\sin[0.31\pi(n-299.5)]}{\pi(n-299.5)}\left(0.42 - 0.5\cos\left(\frac{2\pi n}{599}\right) + 0.08\cos\left(\frac{4\pi n}{599}\right)\right)$$

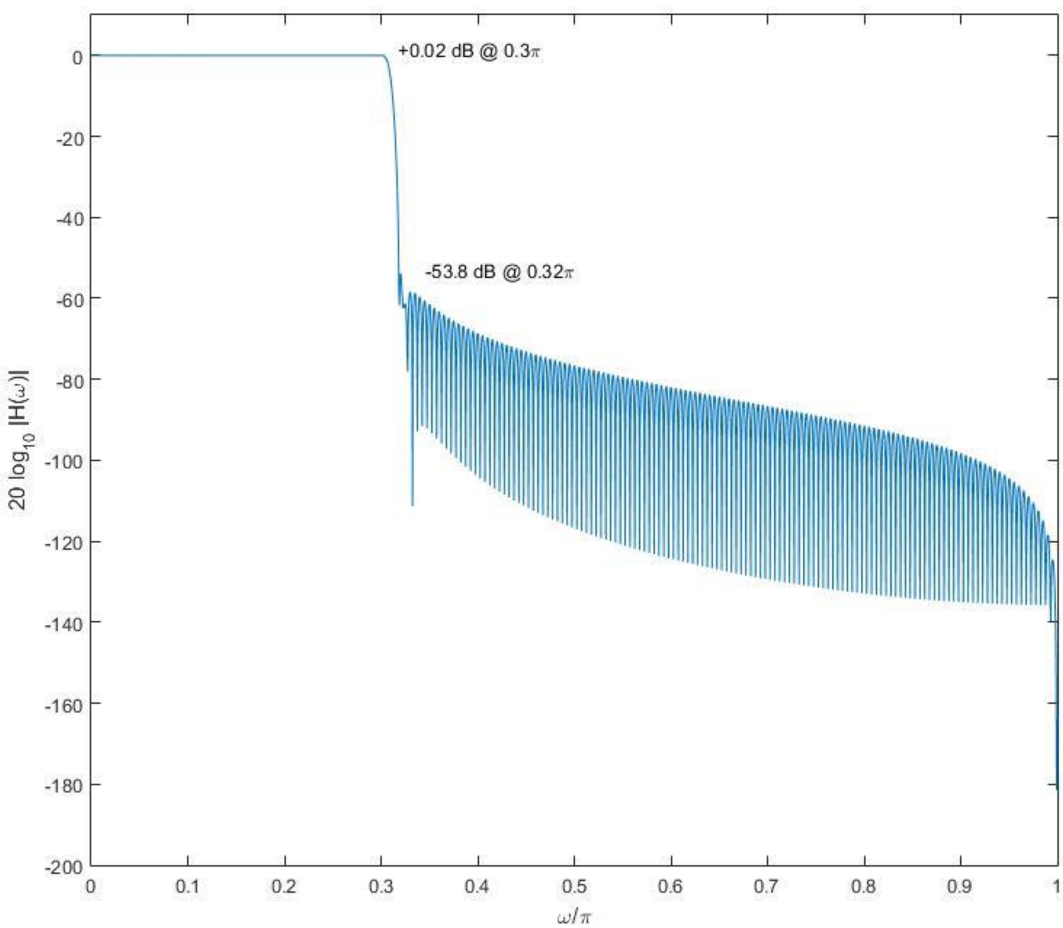
Hanning

```
Nh = 400;
n = 0:1:Nh-1;
h_Han = sin(0.31*pi.*(n-199.5))./(pi.*(n-199.5)).*(0.5 - 0.5.*cos(2*pi.*n./399));
w = linspace(0,2*pi,10*Nh)./pi;
H_Han = fft(h_Han,10*Nh);
plot(w(1:10*Nh/2),20*log10(abs(H_Han(1:10*Nh/2))));   % plots 0 to pi
axis([0 1 -200 10])
xlabel('\omega/\pi')
ylabel('20 log_{10} |H(\omega)|')
```



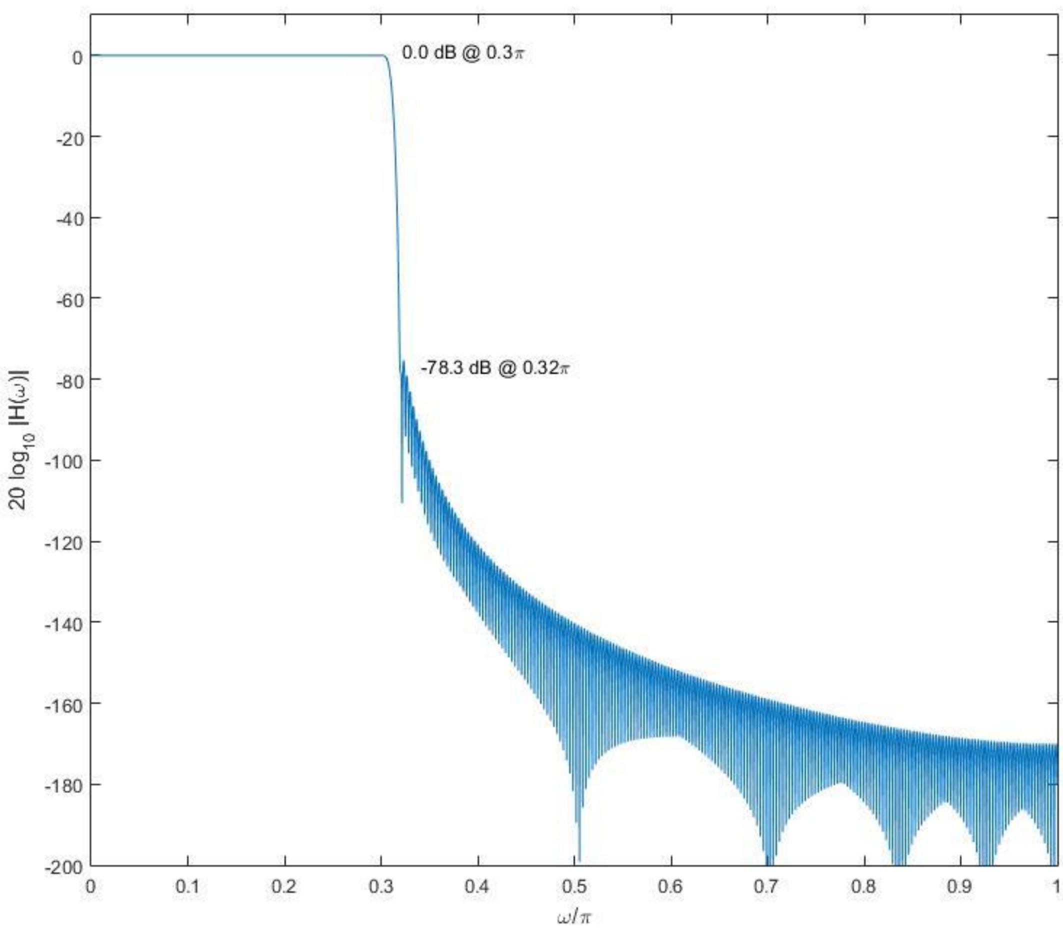
Hamming

```
Nh = 400;
n = 0:1:Nh-1;
h_Ham = sin(0.31*pi.*(n-199.5))./(pi.*(n-199.5)).*(0.54 - 0.46.*cos(2*pi.*n./399));
w = linspace(0,2*pi,10*Nh)./pi;
H_Ham = fft(h_Ham,10*Nh);
plot(w(1:10*Nh/2),20*log10(abs(H_Ham(1:10*Nh/2))));   % plots 0 to pi
axis([0 1 -200 10])
xlabel('\omega/\pi')
ylabel('20 log_{10} |H(\omega)|')
```



Blackman

```
Nb = 600;
n = 0:1:Nb-1;
h_Blk = sin(0.31*pi.*(n-299.5))./(pi.*(n-299.5)).*(0.42 - 0.5.*cos(2*pi.*n./599) +
      0.08.*cos(4*pi.*n/599));
w = linspace(0,2*pi,10*Nb)./pi;
H_Blk = fft(h_Blk,10*Nb);
plot(w(1:10*Nb/2),20*log10(abs(H_Blk(1:10*Nb/2))));   % plots 0 to pi
axis([0 1 -200 10])
xlabel('\omega/\pi')
ylabel('20 log_{10} |H(\omega)|')
```



2) for half-wavelength antenna element spacing

$$d = \lambda/2 \Rightarrow \theta = \pi \sin(\phi)$$

so $\phi = 0° \rightarrow \theta = 0°$

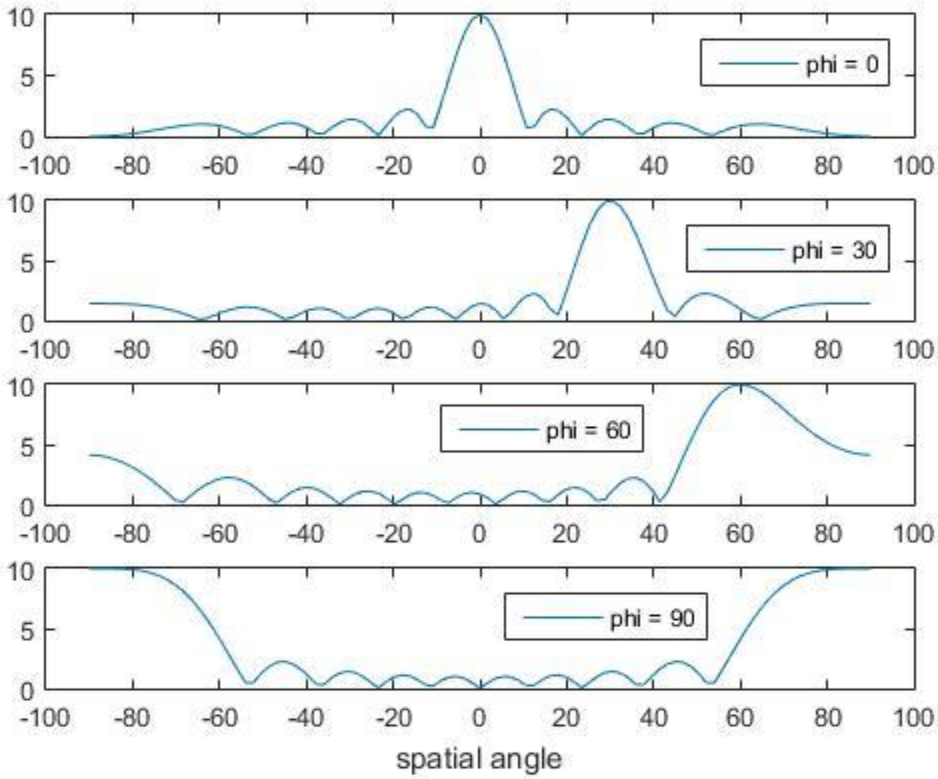
$\phi = 30° \rightarrow \theta = 90°$

$\phi = 90° \rightarrow \theta = 180°$

also including $\phi = 60° \rightarrow \theta = 155.9°$

$\Rightarrow$ because of the nonlinear relationship between spatial angle ϕ and electrical angle θ we see a marginal mainlobe broadening from $\phi = 0°$ to $30°$ steering. However, $60°$ and $90°$ show considerably more.

```
N = 10;
phi_deg = linspace(-90, +90, 101);
phi_rad = phi_deg.*pi/180;
thet = pi.*sin(phi_rad);

for n = 1:101
   S(:,n) = exp(-j.*thet(n).*[0:N-1].');
end;

s0 = exp(-j.*pi.*sin(0*pi/180).*[0:N-1].');
s30 = exp(-j.*pi.*sin(30*pi/180).*[0:N-1].');
s60 = exp(-j.*pi.*sin(60*pi/180).*[0:N-1].');
s90 = exp(-j.*pi.*sin(90*pi/180).*[0:N-1].');

p0 = abs(S'*s0);
p30 = abs(S'*s30);
p60 = abs(S'*s60);
p90 = abs(S'*s90);

subplot(4,1,1)
plot(phi_deg,p0)
legend('phi = 0')

subplot(4,1,2)
plot(phi_deg,p30)
legend('phi = 30')

subplot(4,1,3)
plot(phi_deg,p60)
legend('phi = 60')

subplot(4,1,4)
plot(phi_deg,p90)
legend('phi = 90')
xlabel('spatial angle')
```



phi = 0
phi = 30
phi = 60
phi = 90
spatial angle

3) if the center freq. doubles as $f_{new} = 2 f_{old}$

then $\lambda_{new} = \frac{c}{f_{new}} = \frac{c}{2 f_{old}} = \frac{1}{2}\lambda_{old}$

$\Rightarrow$ wavelength of the new signal is half of that for the old signal

$\Rightarrow$ since the antenna does not change

$d = \frac{\lambda_{old}}{2} = \lambda_{new}$ $\Leftarrow$ now <u>full</u>-wavelength spacing

therefore $\theta = \frac{2\pi d}{\lambda_{new}} \sin(\phi) = \underline{2\pi \sin(\phi)}$ not there before

so now $\phi = 0° \rightarrow \theta = 0°$

$\phi = 30° \rightarrow \theta = 180°$

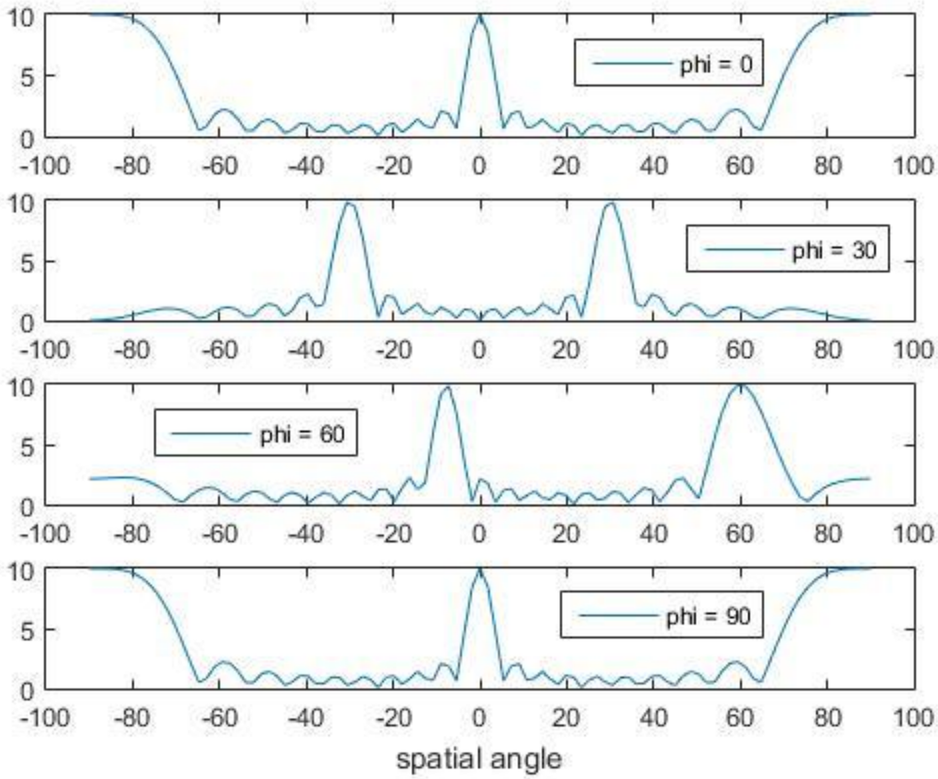
$\phi = 60° \rightarrow \theta = 311.8°$

$\phi = 90° \rightarrow \theta = 360°$

$\Rightarrow$ Because we are now undersampling spatially (not meeting the Nyquist requirement of $d \le \frac{\lambda}{2}$) there is a "grating lobe" that appears along with the correct mainlobe in each case

$\Rightarrow$ <u>spatial aliasing</u>

```
N = 10;
phi_deg = linspace(-90, +90, 101);
phi_rad = phi_deg.*pi/180;
thet = 2*pi.*sin(phi_rad);

for n = 1:101
   S(:,n) = exp(-j.*thet(n).*[0:N-1].');
end;

s0 = exp(-j.*2.*pi.*sin(0*pi/180).*[0:N-1].');
s30 = exp(-j.* 2.*pi.*sin(30*pi/180).*[0:N-1].');
s60 = exp(-j.* 2.*pi.*sin(60*pi/180).*[0:N-1].');
s90 = exp(-j.* 2.*pi.*sin(90*pi/180).*[0:N-1].');

p0 = abs(S'*s0);
p30 = abs(S'*s30);
p60 = abs(S'*s60);
p90 = abs(S'*s90);

subplot(4,1,1)
plot(phi_deg,p0)
legend('phi = 0')

subplot(4,1,2)
plot(phi_deg,p30)
legend('phi = 30')

subplot(4,1,3)
plot(phi_deg,p60)
legend('phi = 60')

subplot(4,1,4)
plot(phi_deg,p90)
legend('phi = 90')
xlabel('spatial angle')
```


phi = 0
phi = 30
phi = 60
phi = 90
spatial angle